

# PARTIAL DIFFERENTIAL EQUATIONS

[2]

S.14665  
[19]

*Editors of the Volume*

BOGDAN BOJARSKI  
ROMAN DWILEWICZ

WARSZAWA 1987

## PREFACE

The present volume includes some of the papers prepared by the participants of the activities of the Semester on Partial Differential Equations held at the Banach Center (Warsaw, September 12–December 12, 1984).

The main areas of activity were: microlocal analysis of pseudodifferential operators and boundary value problems, spectral theory of differential operators, homogenization methods and various aspects of nonlinear problems in P.D.E. As a matter of fact, the nonlinear problems occupied a significant place among the topics discussed in numerous seminar lectures. In particular, research and survey papers on existence and regularity theory of elliptic nonlinear equations and variational inequalities, bifurcation problems and topological methods in nonlinear differential equations, nonlinear equations of elasticity theory, hydrodynamics, mathematical biology were presented by many participants. Unfortunately, only a relatively small part of these are published in the proceedings. However, for information, we include a full list of lectures presented during the Semester and the list of participants.

We hope that this volume will be of some interest to specialists in P.D.E., complex analysis, mathematical physics, etc.

*Bogdan Bojarski*  
*Roman Dwilewicz*

## CONTENTS

Preface . . . . .	5
List of lectures . . . . .	9–13
List of participants . . . . .	14–15
T. B. BOEV, Uniqueness and regularity properties of linear operators and their applications . . . . .	17–24
B. BOJARSKI and T. IWANIEC, $p$ -Harmonic equation and quasiregular mappings . . . . .	25–38
P. BRUNOVSKÝ and B. FIEDLER, Heteroclinic connections of stationary solutions of scalar reaction-diffusion equations . . . . .	39–47
M. BURNAT, J. HERCZYŃSKI and B. ZAWISZA, Perturbation properties of the spectrum of periodic Schrödinger operators . . . . .	49–58
D. CAILLERIE and B. DINARI, A perturbation problem with two small parameters in the framework of the heat conduction of a fiber reinforced body . . . . .	59–78
R. DWILEWICZ and P. M. GAUTHIER, Some aspects of holomorphic approximations from the point of view of PDEs . . . . .	79–88
M. FILA, Stabilization of solutions of a reaction-diffusion equation . . . . .	89–93
P. GODIN, A discrete phenomenon in propagation of $C^\infty$ singularities . . . . .	95–99
T. V. GRAMCHEV, The stationary phase method in Gevrey classes and Fourier integral operators on ultradistributions . . . . .	101–112
U. HAMANN and G. WILDENHAIN, Approximation by solutions of general boundary value problems for elliptic equations of arbitrary order . . . . .	113–119
J. HERCZYŃSKI, Schrödinger operators with almost periodic potentials in nonseparable Hilbert spaces . . . . .	121–142
E. HORST, Global strong solutions of Vlasov's equation – necessary and sufficient conditions for their existence . . . . .	143–153
P.-A. IVERT, Continuity of quasiminima under the presence of irregular obstacles . . . . .	155–167
N. KUTEV, Existence and nonexistence of classical solutions of the Dirichlet problem for a class of fully nonlinear nonuniformly elliptic equations . . . . .	169–178
A. LEHTONEN, On the Euler–Lagrange inequality of a convex variational integral in Orlicz spaces . . . . .	179–191
O. LISS, Microlocal solvability of the Cauchy problem and boundary regularity . . . . .	193–204
A. PIERZCHALSKI, Some differential operators connected with quasiconformal deformations on manifolds . . . . .	205–212
P. R. POPIVANOV, Microlocal properties of a class of pseudodifferential operators with double involutive characteristics . . . . .	213–224
J. A. REMPALA, A remark on symbols of an operator . . . . .	225–230
S. REMPEL and B.-W. SCHULZE, Branching of asymptotics for elliptic operators on manifolds with edges . . . . .	231–236
G. F. ROACH, Boundary integral equations and modified Green's functions . . . . .	237–261
L. RODINO, Pseudodifferential operators with multiple characteristics and Gevrey classes . . . . .	263–267

E. SANCHEZ-PALENCIA, On the edge singularities in composite media. Influence of anisotropy . . . . .	269–286
J. SOKOŁOWSKI, Sensitivity analysis of the Signorini variational inequality . .	287–300
H. STETKAER, Representations of groups on eigenspaces of invariant differential operators . . . . .	301–309
L. VÉRON, Limit behaviour of singular solutions of some semilinear elliptic equations . . . . .	311–350
I. I. VRABIE, Some compactness methods in the theory of nonlinear evolution equations with applications to P.D.E. . . . .	351–361
W. WATZLAWEK, Two examples of an alternative approach to systems analogous to the heat polynomials . . . . .	363–369
A. YANUSHAUSKAS, Well-posedness of the Dirichlet problem and homotopy classification of elliptic systems of second-order partial differential equations . .	371–381
W. M. ZAJĄCZKOWSKI, Some leakage problems for ideal incompressible fluid motion in domains with edges . . . . .	383–397

---

## LIST OF LECTURES

- H. D. Alber (FRG), High frequency asymptotics of the scattering matrix and the inverse problem of acoustics.  
–, Local existence of solutions to the quasilinear wave equation for large initial values.
- G. Anzellotti (Italy), Noncoercive variational problems and elasticity with unilateral constraint on the stress.  
–, Functions whose derivatives are measures and functionals with linear growth in the gradient.
- H. Berestycki (France), New methods in critical point theory with applications to nonlinear PDE.
- Yu. M. Berezanskii (USSR), Integration of nonlinear difference equations by means of inverse spectral problem.
- T. Boev (Bulgaria), Uniqueness and regularity properties of linear operators and their applications.
- J. Brilla (Czechoslovakia), Spaces of analytic functions valued in Sobolev spaces and analysis of linear time dependent problems.  
–, On spectral analysis of nonselfadjoint operators.
- M. Burnat (Poland), Spectral analysis in nonseparable spaces.
- D. Caillerie (France), Conduction problems in fibered bodies.
- W. Chojnacki (Poland), Cocycles and spectral analysis on Bohr groups.
- B. Dacorogna (Switzerland), Relaxation theorems in the calculus of variations.
- R. Denčev (Bulgaria), The method of descent in the study of the singularities of solutions to the Schrödinger equation.
- A. A. Dezin (USSR), Concerning the general theory of boundary problems.
- K. Doppel (West Berlin), Existence and regularity of solutions of a nonhypoelliptic Dirichlet problem.
- R. Dwilewicz (Poland), Introduction to Cauchy–Riemann manifolds and functions.  
–, Some problems in the theory of CR manifolds and functions.  
–, Approximation and embedding problems in the theory of Cauchy–Riemann manifolds and functions.
- Yu. V. Egorov (USSR), On the estimates of the first eigenvalue for elliptic boundary value problems in multiply connected domains.
- J. Eichhorn (GDR), Theory of Laplace operators over noncompact manifolds.

- B. Fiedler (FRG), Orbits connecting critical points of  $u_t = u_{xx} + f(u)$ .  
 –, Time periodic solutions of parabolic systems.
- P. M. Gauthier (Canada), Uniform approximation by solutions of elliptic equations.
- B. Gaveau (France), Quantum fields and Poisson fields.  
 –, Rigorous Feynman integral method for some hyperbolic equations.
- K. Gęba (Poland), Homotopy and bifurcation.
- E. Giusti (Italy), Regularity of minima of variational integrals.
- P. Godin (Belgium), On singularities of solutions of nonlinear PDE: nonlinear oblique derivative problems, second order nonlinear hyperbolic equations in one and two variables.
- T. Gramchev (Bulgaria), The stationary phase method in Gevrey classes and Fourier integral operators on ultradistributions.
- A. Granas (Canada), Fifty years of Leray–Schauder theory.
- A. Grigis (France), Smoothness and analyticity of solutions of equations with multiple characteristics.  
 –, The FBI transform and the problem of analyticity of solutions for Hörmander's second order equations.
- A. Grodel (GDR), Initial value problems with  $q$ -holomorphic and generalized  $q$ -holomorphic vectors as initial vectors.
- G. Gussi (Romania), Microlocal methods in extendability of CR functions.
- D. K. Gvazava (USSR), On initial and characteristic problems for quasilinear hyperbolic equations.
- L. Habermann (GDR), The Yang–Mills equation on the 2-dimensional sphere  $S^2$ .
- L. Hedberg (Sweden), On Sobolev spaces and potential theory.
- J. Herczyński (Poland), Almost periodic Schrödinger operators in  $B^2$  spaces.
- W. Hoffmann (FRG), Finite element method for hyperbolic initial value problems.
- E. Horst (FRG), On Vlasov's and similar equations (nonlinear equations of plasma physics and stellar dynamics).
- P.-A. Ivert (Sweden), Continuity of quasiminima under the presence of irregular obstacles.
- O. John (Czechoslovakia), On the connection between Liouville property and regularity for nonlinear PDE (elliptic and parabolic).
- A. Juhl (GDR), Symplectic geometry and analysis.
- A. S. Kalashnikov (USSR), On qualitative theory of second order quasilinear degenerate parabolic equations.
- G. Karadzov (Bulgaria), Semiclassical asymptotics of the spectral function for some Schrödinger operators.
- J. Kral (Czechoslovakia), Removable singularities of solutions to PDE.
- M. Kučera (Czechoslovakia), Destabilizing effect of unilateral conditions in reaction-diffusion systems.

- L. D. Kudryavcev (USSR), Existence and uniqueness theorems for generalized solutions of differential equations in weighted spaces.
- T. Küpper (FRG), On infinitely many solutions of a semilinear elliptic equation in  $\mathbb{R}^n$ .
- N. Kutev (Bulgaria), Fully nonlinear nonuniformly elliptic equations and applications to Monge–Ampère type equations.
- O. A. Ladyzhenskaya (USSR), New results about quasilinear elliptic and parabolic equations.
- J. Langer (FRG), The curve shortening problem.
- M. M. Lavrent'ev (USSR), Qualitative properties of parabolic equations.
- A. Lehtonen (Finland), On the Euler–Lagrange inequality of a convex variational integral.
- R. Leis (FRG), Initial boundary value problems in the electromagnetic theory.
- O. Liess (FRG), Microlocality of the Cauchy problem.
- P.-L. Lions (France), On first order Hamilton–Jacobi equations.
- M. Lorenz (GDR), Anisotropic operators with characteristics of constant multiplicity.
- J. Ławrynowicz (Poland), Supercomplex structures and their applications.
- J. Madjarova (Bulgaria), Third boundary value problem for a fully nonlinear two-dimensional elliptic equation.
- G. I. Marchuk (USSR), Mathematical models of immunology.
- M. L. Marinov (Bulgaria), On equations with periodic coefficients of filtration theory.
- E. Martensen (FRG), Application of the horizontal line method to hyperbolic problems of mathematical physics.  
–, Approximation of shocks and rarefaction waves by discretization in time.
- O. Martio (Finland), Boundary regularity of quasilinear elliptic equations.
- E. Meister (FRG), Some classes of singular and integro-differential equations on the half-line.
- A. Melin (Sweden), Pseudodifferential calculus on nilpotent groups.  
–, Some methods in inverse scattering.
- E. Miersemann (GDR), Quasilinear elliptic equations of second order over domains with corners.
- E. I. Moiseev (USSR), On the solvability and uniqueness of a nonlinear equation in gravitation theory.
- J. Nečas (Czechoslovakia), On the trans-sonic flow problem.
- Nguyen Dinh Ngoc (Vietnam), A conceptual scheme and some open questions relevant to PDE.
- Nguyen Thua Hop (Vietnam); Integral representation of solutions of an elliptic system in several variables.  
–, Counterpart of Riemann problems for harmonic functions.
- P. Nistri (Italy), Some topological methods for the controllability of nonlinear control systems.
- O. Oleinik (USSR), On  $G$ -convergence and homogenization problems.  
–, Boundary value problems for nonsmooth domains.

- , On a problem of E. Sanchez-Palencia.
- B. Ørsted (Denmark), Representations of Lie groups and the connection between spectral geometry and conformally invariant PDE.
- A. P. Oskolkov (USSR), Initial boundary value problems for equations of motion of viscoelastic fluids.
- R. Picard (FRG), The low frequency limit for stationary electromagnetic waves.
- , On the principle of limiting amplitude for Maxwell's initial boundary value problem.
- A. Pierzchalski (Poland), Some differential operators connected with quasiconformal deformations on manifolds.
- V. Pluschke (GDR), Constructive and nonconstructive existence theorems for various types of PDE by semidiscretization in time.
- J. F. Pommaret (France), Differential sequences and their applications to mathematical physics.
- G. Popov (Bulgaria), Spectral asymptotics for elliptic operators on unbounded domains.
- T. V. Rangelov (Bulgaria), Representation of the scattering operators and singularities of the scattering kernel for moving obstacles.
- H. M. Reimann (Switzerland), Quasiconformal mappings on the Heisenberg group.
- S. Rempel and B.-W. Schulze (GDR), Mellin operators and conical manifolds.
- , Pseudodifferential and Mellin operator calculus on nonsmooth domains.
- G. F. Roach (Great Britain), Modified Green's function techniques.
- L. Rodino (Italy), Pseudodifferential operators with multiple characteristics and Gevrey classes.
- W. Rüdprich (GDR), Implicit partial differential equation systems.
- E. Sanchez-Palencia (France), On the edge singularities in composite media. Influence of anisotropy.
- , Holomorphic and singular perturbation of eigenvalues in thermoelasticity.
- A. Sändig (GDR), Regularity theorems for elliptic boundary value problems in domains with edges.
- T. Schmitt (GDR), A comprehensive introduction to superdifferential geometry.
- , Coherent sheaves on analytic supermanifolds.
- B.-W. Schulze (GDR), Pseudodifferential boundary value problems.
- K. Senator (Poland), Unique continuation for elliptic equations with unbounded coefficients.
- M. A. Shubin (USSR), Random elliptic operators.
- , Pseudodifferential operators on unimodular Lie groups.
- , Asymptotic expansion of the spectral function for the Hill operator.
- , Pseudodifference operators.
- J. Sokołowski (Poland), Material derivative method in shape sensitivity analysis of boundary value problems.
- J. Souček (Czechoslovakia), Singular solution to elliptic systems.
- , Regularity of harmonic maps into a hemisphere.



- W. Sprössig (GDR), Application of quaternionic analysis for calculating thermal stresses in real bodies.
- H. Stetkaer (Denmark), Representations of groups on eigenspaces of invariant differential operators.
- A. Szulkin (Sweden), Minimax principles for a class of lower semicontinuous functions and applications to nonlinear boundary value problems.
- F. Tomi (FRG), Existence theorems for minimal surfaces of prescribed topological type in  $R^3$ .
- N. S. Trudinger (Australia), Nonlinear boundary value problems for second order nonlinear elliptic equations.
- W. Tutschke (GDR), The meaning of the concept of holomorphy for initial value problems.  
–, Initial value problems with generalized analytic functions (in the plane and in the space) as initial functions.
- L. Véron (France), Singularities of nonlinear elliptic equations, the power case.  
–, Singularities of nonlinear elliptic equations, the general case.  
–, Symmetry and breaking of symmetry for singular solutions of nonlinear elliptic equations.
- V. S. Vinogradov (USSR), Special complex forms for elliptic systems in the plane.
- I. I. Vrabie (Romania), Some compactness methods in the theory of nonlinear evolution equations with applications to PDE.
- H. Wallin (Sweden), Spaces of functions defined on subsets of  $R^n$ .
- W. Wendland (FRG), On boundary element methods.
- A. I. Yanushauskas (USSR), On the well-posedness of the Dirichlet problem for elliptic systems of second order.
- W. Zajączkowski (Poland), Existence of solutions of the Euler (hydrodynamics) equations in a domain with edges on the boundary.  
–, The Neumann problem for elliptic equations of second order in domains with nonsmooth boundary.
- F. Zickermann (GDR), On the index of transversally elliptic complexes.
- B. Ziemian (Poland), Taylor formula for distributions.

## LIST OF PARTICIPANTS

- |                            |                              |                               |
|----------------------------|------------------------------|-------------------------------|
| S. Ackermann (GDR)         | B. Gaveau (France)           | S. Knecht (FRG)               |
| H. D. Alber (FRG)          | M. Geisler (GDR)             | J. H. Kołakowski (Poland)     |
| G. Anzellotti (Italy)      | K. Gęba (Poland)             | R. Kopiecki (Poland)          |
| G. Arsu (Romania)          | H. P. Gittel (GDR)           | J. Kral (Czechoslovakia)      |
| W. Bauhardt (GDR)          | E. Giusti (Italy)            | D. Krastev (Bulgaria)         |
| N. Belalia (Algeria)       | P. Godin (Belgium)           | M. Kučera<br>(Czechoslovakia) |
| F. Benkert (GDR)           | B. Goldschmidt (GDR)         | L. D. Kudryavcev (USSR)       |
| H. Berestycki (France)     | T. Gramchev (Bulgaria)       | T. Kütper (FRG)               |
| Yu. M. Berezanskii (USSR)  | A. Granas (Canada)           | N. Kutev (Bulgaria)           |
| G. Berger (GDR)            | A. Grigis (France)           | O. A. Ladyzhenskaya (USSR)    |
| W. Berndt (GDR)            | I. Grinda (GDR)              | B. Lange (GDR)                |
| D. Bodo (GDR)              | A. Grodel (GDR)              | J. Langer (FRG)               |
| T. Boev (Bulgaria)         | R. Grunwald (GDR)            | M. M. Lavrent'ev (USSR)       |
| B. Bojarski (Poland)       | H. Günzel (GDR)              | A. Lehtonen (Finland)         |
| B. Božek (Poland)          | G. Gussi (Romania)           | R. Leis (FRG)                 |
| F. Brock (GDR)             | D. K. Gvazava (USSR)         | H. Leopold (GDR)              |
| I. Brilla (Czechoslovakia) | L. Habermann (GDR)           | O. Liess (FRG)                |
| J. Brilla (Czechoslovakia) | R. Hagen (GDR)               | P.-L. Lions (France)          |
| Z. Brzeźniak (Poland)      | U. Hamann (GDR)              | M. Lorenz (GDR)               |
| M. Burnat (Poland)         | L. Hedberg (Sweden)          | A. Łada (Poland)              |
| D. Caillerie (France)      | J. Heinrich (GDR)            | J. Ławrynowicz (Poland)       |
| J. Chmaj (Poland)          | E. Herbst (GDR)              | J. Madjarova (Bulgaria)       |
| W. Chojnacki (Poland)      | J. Herczyński (Poland)       | R. Manthey (GDR)              |
| M. Costabel (FRG)          | Hoang Quac Toan<br>(Vietnam) | G. I. Marchuk (USSR)          |
| B. Dacorogna (Switzerland) | W. Hoffmann (FRG)            | M. L. Marinov (Bulgaria)      |
| R. Denčev (Bulgaria)       | E. Horozov (Bulgaria)        | L. Marko (Czechoslovakia)     |
| A. A. Dezin (USSR)         | E. Horst (FRG)               | E. Martensen (FRG)            |
| W. Domański (Poland)       | R. Hünlich (GDR)             | O. Martio (Finland)           |
| K. Doppel (West Berlin)    | I. Iliev (Bulgaria)          | J. Maul (GDR)                 |
| P. Drabek (Czechoslovakia) | B. I. Islamov (USSR)         | K. Maurin (Poland)            |
| S. Dümmel (GDR)            | P.-A. Ivert (Sweden)         | E. Meister (FRG)              |
| R. Dwilewicz (Poland)      | J. Janassary (GDR)           | A. Melin (Sweden)             |
| Yu. V. Egorov (USSR)       | O. John (Czechoslovakia)     | M. Meluř (Czechoslovakia)     |
| J. Eichhorn (GDR)          | J. R. Jonescu (Romania)      | E. Miersemann (GDR)           |
| A. Fabrikant (Bulgaria)    | A. Juhl (GDR)                | E. I. Moiseev (USSR)          |
| B. Fiedler (FRG)           | H. Junek (GDR)               | R. Mosurski (Poland)          |
| M. Fila (Czechoslovakia)   | A. S. Kalashnikov (USSR)     | J. Nečas (Czechoslovakia)     |
| J. Fuka (Czechoslovakia)   | G. Karadzov (Bulgaria)       | M. Nedelev (Bulgaria)         |
| R. Funke (GDR)             | A. Y. Khapalov (USSR)        | A. Nestke (GDR)               |
| P. M. Gauthier (Canada)    |                              |                               |

- Nguyen Dinh Ngoc  
 (Vietnam)  
 Nguyen Thua Hop  
 (Vietnam)  
 P. Nistri (Italy)  
 O. A. Oleinik (USSR)  
 B. Ørsted (Denmark)  
 A. P. Oskolkov (USSR)  
 P. Oswald (GDR)  
 G. Pasa (Romania)  
 M. Pascu (Romania)  
 A. W. Perzan (USSR)  
 H. Petzeltova  
 (Czechoslovakia)  
 R. Picard (FRG)  
 A. Pierzchalski (Poland)  
 B. Piłat (Poland)  
 A. Piskorek (Poland)  
 V. Pluschke (GDR)  
 D. Polisevski (Romania)  
 J. F. Pommaret (France)  
 G. Popov (Bulgaria)  
 K. Quasthoff (GDR)  
 J. Rakosnik  
 (Czechoslovakia)  
 T. V. Rangelov (Bulgaria)  
 H. M. Reimann  
 (Switzerland)  
 M. Reissig (GDR)  
 J. A. Rempala (Poland)
- S. Rempel (GDR)  
 F. Richter (GDR)  
 G. F. Roach (Great Britain)  
 L. Rodino (Italy)  
 J. Rossmann (GDR)  
 W. Rüprich (GDR)  
 J. Rusinek (Poland)  
 G. Rządkowski (Poland)  
 W. Sadkowski (Poland)  
 E. Sanchez-Palencia  
 (France)  
 A. Sändig (GDR)  
 I. Schmelzer (GDR)  
 G. Schmidt (GDR)  
 T. Schmitt (GDR)  
 B.-W. Schulze (GDR)  
 K. Senator (Poland)  
 M. A. Shubin (USSR)  
 W. Sickel (GDR)  
 P. Simonescu (Romania)  
 M. Slodiček  
 (Czechoslovakia)  
 M. Sofonea (Romania)  
 J. Sokołowski (Poland)  
 D. Sosna (GDR)  
 J. Souček (Czechoslovakia)  
 W. Sprössig (GDR)  
 R. Stawre (Romania)  
 H. Stetkaer (Denmark)  
 I. Straskraba  
 (Czechoslovakia)
- I. Szepesvari (Hungary)  
 Z. Szmydt (Poland)  
 A. Szulkin (Sweden)  
 S. Tarasova (USSR)  
 M. Terbeche (Algeria)  
 D. Tiba (Romania)  
 F. Tomi (FRG)  
 Tran Van Trien (Vietnam)  
 N. S. Trudinger  
 (Australia)  
 W. Tutschke (GDR)  
 P. Urbański (Poland)  
 A. I. Urusov (USSR)  
 B. Vernescu (Romania)  
 L. Véron (France)  
 V. S. Vinogradov (USSR)  
 E. Vízus  
 (Czechoslovakia)  
 Vo Duc Ton (Vietnam)  
 I. I. Vrabie (Romania)  
 H. Wallin (Sweden)  
 W. Wendland (FRG)  
 G. Wildenhain (GDR)  
 K. Wojciechowski (Poland)  
 A. I. Yanushauskas (USSR)  
 E. Zadrzyńska (Poland)  
 W. Zajączkowski (Poland)  
 B. Zawisza (Poland)  
 F. Zickermann (GDR)  
 B. Ziemian (Poland)