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Choosing a lawyer as a special case of self-insurance-cum-protection

Abstract We consider the problem of choosing a lawyer as a particular case of self-insurance-cum-protection (SICP) when the lawyer's costs are covered in the case of victory. This problem was introduced by Sevi and Yafil (2005) in the context of self-protection (SP), with the assumption that the size of a loss does not depend on the level of effort (expenditure on a lawyer). In this paper we drop that assumption and our model considers the possibility that both the loss and probability of incurring a loss depend on effort. We compare the optimal effort in our case with the standard one and prove that, according to the modified model of SICP, repayment is a good incentive to invest more. We also show that, unlike in the standard cases of SP and SICP, the level of effort is monotone in the level of risk aversion. We prove that, according to our model, decreasing absolute risk aversion (DARA) implies that a lawyer's service is a normal good, which is intuitive. We show that for a certain type of increase in risk aversion, the reimbursement effect is stronger than the risk aversion effect. For other changes in risk aversion, there is a probability threshold such that if the probability of a loss is below that level, then the risk-aversion effect prevails. For higher initial probabilities, the reimbursement effect is stronger.

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1. Introduction Individuals facing the risk of a possible loss may invest in preventative actions that reduce the size of a loss (self-insurance) or the probability of a loss (self-protection). Ehrlich and Becker [5] were first to define and analyze these concepts. However, there are examples combining self-insurance (SI) with self-protection (SP). Ehrlich and Becker give the example of a good lawyer who is able to reduce both the probability of a conviction and the punishment for a crime when convicted. This concept is called self-insurance-cum-protection (SICP). Investment in SICP reduces both the probability and size of a loss. Other examples of SICP are investments in high-quality brakes, appropriate medical checkups, helmets for cyclist, etc. Sevi

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and Yafil in [12] observed that in some cases the law provides for repayment of the lawyer's costs. The basic model of SP does not include such a case and should be modified due to indemnity. Investment (effort) changes the probability distribution in a different manner than in the standard SP setup. As a consequence, the optimal effort is monotone with respect to risk-aversion. This result differentiates the problem of reimbursement from standard SP. It is well-known that the optimal level of self-protection is not necessarily increasing in the level of risk-aversion (Dionne and Eeckhoudt [4], Briys and Schlesinger [1]). It is related to the fact that self-protection in general does not reduce riskiness of final wealth and may be attractive to both risk averse and risk lovers. However, there is critical probability of loss, below which highly risk-averse individuals invest more in self-protection (Julien, Salanie, Salanie [6]). In [7] Lee proved analogous results in the case of self-insurance-cum-protection, but he also emphasized its dependence on the shape of the loss function. If losses decrease fast (compared to the increase in costs), then the optimal effort is increasing in risk-aversion. Otherwise, there exists a probability threshold separating two regions of monotonicity. Sevi and Yafil [12] proved that the above results are no longer valid in the case of self-protection with reimbursement. They showed that with repayment, effort declines when risk-aversion increases. This means that the introduction of indemnity into the standard model of SP is significant. However, Sevi and Yafil do not consider the possibility of reducing the size of a loss, despite the fact that they quote Ehrlich and Becker's example as inspiration. Using a model of self-protection (modified by indemnity or not) requires the assumption that the size of a loss does not depend on effort (expenditure on a lawyer).
In this paper we drop that assumption and our model includes possibility that both loss and probability of incuring a loss depend on effort. It means that we use self-insurance-cum-protection model modified by possibility of repayment in case of success. Our aim is to prove that the result still holds in SICP setup, but again it depends on shape of the loss function.

Another well-known fact is that in the standard case, self-insurance is an inferior good under decreasing absolute risk aversion (DARA), but the effect of wealth on self-protection and self-insurance-cum-protection is ambiguous (Sweeney and Beard [11], Lee [8]). We will show that in the context of SICP with indemnity, legal services are a normal good under DARA.

One of Sevi and Yafil's most important observations is that neither the reimbursement effect nor the risk aversion effect systematically prevails. A more risk-averse individual, who may benefit from an indemnity, will not necessarily exert a higher effort than a less risk-averse one with no indemnity. We will prove that this is only true for certain types of increases in risk-aversion and the initial probability of loss. We show that if the increase in risk-aversion satisfies a certain condition, then the reimbursement effect is unambiguously stronger. For other types of changes in risk-aversion, there exists a critical probability level separating the results of combined effects. If (and only if) the probability of a loss is below this level, then the reimbursement effect prevails. For higher initial probabilities, the risk-aversion effect is stronger. We also prove that this probability threshold is strictly between 0 and 1, so it is impossible for one of the effects to systematically prevail.

2. The model and a comparison with the standard case Consider an individual (legal claimant or defendant) who has initial wealth w. This wealth is subject to a possible loss, which means that a trial can be lost or won. The individual invests e (the effort) in SICP by choosing a lawyer. This affects both the probability of a loss, $p(e) \in [0,1]$ and the size of a loss, $l(e) > 0$. The final wealth depends on whether or not a loss occurs. We assume that the lawyer's costs are repaid in the case of victory. Thus final wealth equals:

$A = w - e - l(e)$ if the loss occurs, or

$B = w$ otherwise.

We assume that both $l(\cdot)$ and $p(\cdot)$ are positive, decreasing, convex and continuously twice-differentiable. Obviously, $A < B$. The individual's problem is to choose e to maximize his expected utility:

$$Eu = p(e)u(A) + (1 - p(e))\, u(B),$$

where u is the von Neumann-Morgenstern utility function. We assume that it is continuously twice-differentiable, strictly increasing and concave.

We assume that there exists an interior solution to the problem. We will denote it by e^*. It satisfies the first-order condition (FOC)

$$\frac{\partial Eu}{\partial e} = p'(e)[u(A) - u(B)] - (l'(e) + 1)p(e)u'(A) = 0. \tag{1}$$

It is also assumed that the second-order condition (SOC) is satisfied, that is $\frac{\partial^2 Eu}{\partial e^2} \leqslant 0$. A sufficient condition for SOC is $p''p \geqslant (p')^2$ (condition C1 in [6]).

Observe that for an interior solution to exist, it is necessary that $l'(e) > -1$. This means that the loss function is decreasing, but not too fast. Moreover, it seems that assuming the convexity of the loss with respect to effort is not restrictive. Of course, other cases are possible; sometimes, a small effort can lead to a big drop in the loss, but we assume that this is not the case. On the other hand, if the loss decreases fast, i.e. $l' \leqslant -1$ uniformly, then $\frac{\partial Eu}{\partial e} > 0$ and the problem is not interesting. So from now on we shall assume that $l' > -1$ uniformly.

We want first to answer the following question: is repayment a good incentive to invest more in SICP? Sevi and Yafil proved it is so in the case of SP. Therefore, we can rephrase this question: is the assumption of the size of a loss being independent of effort essential or not? We will prove that the answer depends on shape of the loss function. If the loss function decreases not too fast, then the assumption made by Sevi and Yafil can be omitted. In order to do this, we recall the standard model of SICP. The final wealth is

$w - e - l(e)$ with probability $p(e)$ or

$w - e$ with probability $1 - p(e)$.

The problem is to maximize

$$p(e)u(w - e - l(e)) + (1 - p(e))u(w - e).$$

The FOC for this problem is

$$p'u(w-e-l(e)) - pu'(w-e-l(e))(l'+1) - p'u(w-e) - (1-p)u'(w-e) = 0,$$

where p, p' and l' are evaluated at e.

Again, we assume that the second-order condition is satisfied and there is an internal solution, which we will by denote e_s. We are interested in determining the type of inequality between e_s and e^* (if one exists). Both problems are concave in e, so all we need to know is the sign of $\frac{\partial Eu}{\partial e}|_{e=e_s}$. We prove that it is positive if the loss is not decreasing too fast.

PROPOSITION 2.1 *If the absolute value of the slope of the loss function is uniformly less than 1, then a risk-averse individual invests more in SICP when costs are covered in the case of success, compared to the standard case (no repayment).*

PROOF From the FOC for the standard problem, we have

$$p'(e_s) = \frac{p(e_s)[u'(w - e_s - l(e_s))(l'(e_s) + 1) - u'(w - e_s)] + u'(w - e_s)}{u(w - e_s - l(e_s)) - u(w - e_s)}.$$

Substituting this into the FOC yields (we omit the argument e_s in order to simplify the notation)

$$\frac{\partial Eu}{\partial e}\Big|_{e=e_s} = \frac{p[u'(w-e-l)(l'+1) - u'(w-e)] + u'(w-e)}{u(w-e-l)) - u(w-e)}[u(w-e-l) - u(w)]$$
$$-(l'+1)pu'(A).$$

By the monotonicity of u, we have $\frac{u(w-e-l) - u(w)}{u(w-e-l)) - u(w-e)} > 1$. Thus we may write

$$\frac{\partial Eu}{\partial e}\,|_{e=e_s} > p(l'+1)u'(A) - pu'(w-e) + u'(w-e) - p(l'+1)u'(A) =$$
$$= (1-p)u'(w-e) > 0.$$

Reimbursement is thus a sufficient incentive to invest more in legal services in the case of variable loss, compared with constant loss.

3. Effect of an increase in risk aversion

Let us consider a more risk-averse individual. His preferences are represented by the von Neumann-Morgenstern utility $v(\cdot) = T(u(\cdot))$, where T is continuously twice-differentiable, increasing and concave [11]. His expected utility is thus

$$Ev = p(e)T(u(A)) + (1-p(e))\,T(u(B)).$$

Hence, the FOC for the maximization problem of this more risk-averse agent is

$$\frac{\partial Ev}{\partial e} = p'(e)[T(u(A)) - T(u(B))] - p(e)T'(u(A))u'(A)(l'(e)+1) = 0. \quad (2)$$

Let us denote the solution to this problem by e_v.

Proposition 3.1 *If $l' > -1$ uniformly, then an increase in risk-aversion leads to lower effort.*

Proof As above, we have to prove that the sign of $\frac{\partial Ev}{\partial e}$ evaluated at e^* (the optimal effort for u) is negative. Without loss of generality, we may assume that $T(u(A)) = u(A)$ and $T(u(B)) = u(B)$ (see the Appendix). From (1), we have

$$p'(e^*) = \frac{(l'(e^*)+1)p(e^*)u'(A)}{u(A) - u(B)}.$$

Substituting this into (2), we obtain

$$\frac{\partial Ev}{\partial e}\,|_{e=e^*} = (l'(e^*)+1)p(e^*)u'(A)[1 - T'(u(A))]\,.$$

The first three components of this product are positive and the last one is negative (see the Appendix). Hence, $\frac{\partial Ev}{\partial e}\,|_{e=e^*} < 0$, which implies that $e_v < e^*$. ∎

Observe that Proposition 3.1 has a straightforward consequence for the effect of wealth on SICP with possible reimbursement of costs. Let us recall that decreasing absolute risk aversion (DARA) is a non-controversial assumption that wealthier individuals can bear risk better. Formally, it states that if initial wealth increases, then the amount of money that an agent is willing to pay to avoid the same risk is smaller. This is equivalent to the condition that the utility function of an agent with a lower level of initial wealth is a concave transformation of the utility function corresponding to a higher initial level of wealth. Thus we have

Corollary 3.2 *If $l' > -1$ uniformly and the utility function exhibits decreasing absolute risk aversion, then an increase in the initial wealth w leads to an increase in the optimal effort in the case of SICP with possible reimbursement of costs.*

For self-contained proof of the corollary, see the Appendix.

The corollary states that when DARA holds, legal services (given reimbursement of costs in the case of success) is a normal good. This corresponds to the intuition that wealthier individuals usually hire more expensive lawyers. Of course, under the assumption of increasing absolute risk aversion (IARA), the opposite holds. Under constant absolute risk aversion (CARA), a change in the initial wealth has no effect on SICP with possible reimbursement of costs. As mentioned above, DARA is often considered to be a realistic description of behavior towards risk. It is plausible, because it implies, for instance, that wealthier agents invest more in risky assets. Therefore, we consider the consequences of assuming DARA to be the most interesting.

4. Reimbursement versus risk-aversion

Proposition 2.1 states that the possibility of costs being covered leads to higher effort. This is called the *reimbursement effect* (Sevi, Yafil, [12]). On the other hand, an increase in risk-aversion entails a lower effort, the (*risk-aversion effect*). It is interesting to ask whether one of these conflicting effects prevails. Sevi and Yafil claim that, in general, neither effect is stronger than the other one. We will show that this holds only for certain types of increase in risk-aversion and the initial probability of loss. Moreover, the formulation of the problem itself raises some doubts. There is a clear distinction between reimbursement and no reimbursement, whereas increase in risk-aversion is vague. Increases in risk-aversion may be of various types, but Proposition 3 in Sevi, Yafil holds for any type of change in risk-aversion.

Our aim is to prove that if an increase in risk-aversion is of a particular type, the reimbursement effect is unambiguously stronger. We will also prove that for other changes in risk-aversion there exists a certain probability of loss that equalizes the strengths of these effects. The results are no longer

ambiguous either; if the probability of loss is below the threshold, then the reimbursement effect prevails. If this probability is above the threshold, then the risk-aversion effect is stronger.
In order to do this, we consider two individuals: one who is more risk-averse and may benefit from an indemnity and one who is less risk-averse with no indemnity. The first one is subject to two different incentives: to decrease effort, because of increased risk-aversion, and to increase effort, because of the possibility of repayment. The second individual is free of both incentives. To determine which incentive is stronger, we evaluate $\dfrac{\partial Ev}{\partial e}$ at $e = e_s$. For consistency with Sevi and Yafil, we will assume the case of self-protection, hence $l' = 0$. By (2)

$$\frac{\partial Ev}{\partial e} = p'(e)[T(u(A)) - T(u(B))] - p(e)T'(u(A))u'(A).$$

From the FOC for the standard problem, we have

$$p'(e_s) = \frac{p(e_s)[u'(w - e_s - l) - u'(w - e_s)] + u'(w - e_s)}{u(w - e_s - l) - u(w - e_s)}.$$

Therefore,

$$\frac{\partial Ev}{\partial e}\Big|_{e=e_s} = \frac{p(e_s)u'(A) + (1 - p(e_s))u'(w - e_s)}{u(A) - u(w - e_s)}[T(u(A)) - T(u(w))]$$
$$-p(e_s)T'(u(A))u'(A).$$

Let us denote

$$\beta = \frac{T(u(A)) - T(u(w))}{u(A) - u(w - e_s)},$$

which is equal to

$$\frac{T(u(w)) - T(u(w - e_s - l))}{u(w - e_s) - u(w - e_s - l)}.$$

Observe that $\beta > 1$, due to the monotonicity of T and u. It is easily seen that risk-aversion prevails, i. e. $\dfrac{\partial Ev}{\partial e}|_{e=e_s} < 0$ if and only if

$$p(e_s) > \frac{\beta u'(w - e_s)}{u'(A)[T'(u(A)) - \beta] + \beta u'(w - e_s)}. \tag{3}$$

The value of $p(e_s)$ is referred to as the initial probability of loss, before introducing the possibility of reimbursement and increase in risk-aversion. The expression on the right-hand side of (3) could thus be the probability threshold, provided its value is less than or equal to 1. This depends on the sign of $T'(u(A)) - \beta$, which is ambiguous in general. The problem is that both $T'(u(A))$ and β are larger than 1. On the basis of the above considerations, we may formulate the following proposition.

Proposition 4.1 *If an increase in risk-aversion is such that $T'(u(A)) > \beta$, then*

$$p_0 = \frac{\beta u'(w - e_s)}{u'(A)[T'(u(A)) - \beta] + \beta u'(w - e_s)}$$

is the probability that equalizes the risk aversion and reimbursement effects. If the initial probability is below p_0, then the reimbursement effect prevails, i.e., an indemnified, more risk-averse individual exerts more effort in self-protection than a non-indemnified, less risk-averse one. If the initial probability is greater than p_0, then the risk-aversion effect is stronger and the opposite holds.

The condition $T'(u(A)) < \beta$ is hard to verify, so our next aim is to determine a simple sufficient condition for the inequality $T'(u(A)) < \beta$ to hold. First, we prove the following useful lemma.

Lemma 4.2 *If T is thrice-differentiable and $-2T''(w-x) + xT'''(w-x) < 0$ for $x \in (0, w]$, then the function $\phi(x) = \dfrac{xT'(x)}{T(w) - T(w-x)}$ is decreasing on the interval $[0, w]$ and its value is less than 1 for $x \in (0, w]$.*

Proof After straightforward calculations

$$\phi'(x) = \frac{(T'(w-x) - xT''(w-x))(T(w) - T(w-x)) - x(T'(w-x))^2}{(T(w) - T(w-x))^2}.$$

Let us denote the numerator of $\phi'(x)$ by $\alpha(x)$. Observe first that $\alpha(0) = 0$. We will prove that the equation $\alpha(x) = 0$ cannot have any positive roots. To see this, we calculate

$$\alpha'(x) = (-2T'' + xT''')(T(w) - T) + (T' - xT'')T' - (T')^2 + 2xT'T'' =$$
$$= (-2T'' + xT''')(T(w) - T) + xT'T'',$$

where $T^{(i)} = T^{(i)}(w - x)$, $i = 0, 1, 2, 3$. From the monotonicity of T, we have $T(w) - T(w - x) > 0$. Our assumption is $-2T'' + xT''' < 0$. Hence, $(-2T'' + xT''')(T(w) - T) < 0$ and $\alpha'(x) < 0$ for $x > 0$. Since $\alpha(0) = 0$, then $\alpha(x) < 0$ for $x > 0$. Therefore, $\phi'(x) < 0$ for $x > 0$ and it follows that ϕ is decreasing on the interval $[0, w]$. Moreover, by de L'Hospital's rule $\lim_{x\to 0} \phi(x) = \lim_{x\to 0} \dfrac{T'(w-x) - xT''(w-x)}{T'(w-x)} = 1$. We conclude that $\phi(x) < 1$ for $x \in (0, w]$. ■

Proposition 4.3 *If an increase in risk-aversion satisfies $-2T''(w - x) + xT'''(w-x) < 0$ for $x \in [0, w]$, then the reimbursement effect is stronger than the risk-aversion effect and a more risk-averse, indemnified individual will exert more effort in self-protection than a non-indemnified, less risk-averse one.*

PROOF Without loss of generality, we may assume that

$$u(A) = A \text{ and } u(w) = w. \tag{4}$$

Hence, $u'(A) > 1$ and $u'(w) < 1$ (see the Appendix). Thus $T'(u(A)) < \beta$ if and only if

$$\frac{(w-A)T'(A)}{T(w)-T(A)} < \frac{w-A}{u(w-e_s)-u(A)}. \tag{5}$$

Let us first examine the right-hand side of (5). We will denote it by γ. It is equal to

$$\frac{e_s+l}{u(w-e_s)-u(w-e_s-l)}.$$

By the monotonicity of u and from (4), we have

$$u(w-e_s)-u(w-e_s-l) < u(w)-u(w-e_s-l) = \\ = w-(w-e_s-l) = e_s+l.$$

Hence,

$$\gamma = \frac{e_s+l}{u(w-e_s)-u(w-e_s-l)} > \frac{e_s+l}{e_s+l} = 1.$$

From the concavity of u and (4), we have $u(w-e_s) > w-e_s$ and therefore

$$u(w-e_s)-u(w-e_s-l) > w-e_s-(w-e_s-l) = l.$$

Consequently,

$$\gamma = \frac{e_s+l}{u(w-e_s)-u(w-e_s-l)} < \frac{e_s+l}{l} = 1+\frac{e_s}{l},$$

so we have proved that

$$1 < \gamma < 1+\frac{e_s}{l}. \tag{6}$$

Observe that the value of γ depends only on u and p. It does not depend on T.

To assess the left-hand side of (5), we will make use of lemma 4.2, which states that if $-2T''(w-x)+xT'''(w-x) < 0$ for $x \in (0,w]$ then the function $\phi(x) = \dfrac{xT'(w-x)}{T(w)-T(w-x)}$ is decreasing on the interval $(0,w]$ and its value is less than 1 for $x \in (0,w]$. Comparing this with (6), we find that inequality (5) holds, which completes the proof. ■

The condition $-2T''(w-x)+xT'''(w-x) < 0$ describes a type of increase in risk-aversion sufficient for the reimbursement effect to prevail. Proposition 4.3 thus may serve as an existence theorem. It shows that in some cases one of the effects is unambiguously stronger than the other and highlights the reason for this.

5. Appendix

LEMMA 5.1 *Assume that the points A, $B \in \mathbf{R}$ are given and $A < B$. Then for every VNM utility function u there is a VNM utility function v such that*
(i) $v(A) = A$ and $v(B) = B$,
(ii) v represents the same preferences as u,
(iii) $v'(A) > 1$ and $v'(B) < 1$.

PROOF It is easy to see that the function

$$v(x) = \frac{B-A}{u(B)-u(A)}u(x) + A - \frac{B-A}{u(B)-u(A)}u(A)$$

satisfies $v(A) = A$ and $v(B) = B$. The function v is a positive affine transformation of u, so from the von Neumann-Morgenstern Theorem it represents the same preferences as u. From the Mean Value Theorem there exists $\xi \in (A, B)$ such that $v'(\xi) = 1$. Since $A < \xi$ and v is concave, then $v'(\xi) > 1$. An analogous argument proves that $v'(B) < 1$. ■

Proof of Corollary 1. From the Implicit Function Theorem, equation (1), which can be written in the general form $F(w, e^*) = 0$, defines e^* as a function of w. Total differentiation of this equality leads to $\frac{\partial e^*}{\partial w} = -\frac{\partial F}{\partial w}/\frac{\partial F}{\partial e^*}$. Since $\frac{\partial F}{\partial e^*} = \frac{\partial F}{\partial e}|_{e=e^*}$ is negative by the second order condition, the signs of $\frac{\partial e^*}{\partial w}$ and $\frac{\partial F}{\partial w}$ are the same. Straightforward calculations lead to

$$\frac{\partial F}{\partial w} = p'(e^*)(u'(A) - u'(w)) - p(e^*)(l' + 1)u''(A). \tag{7}$$

From (1), we have $p(e^*)(l'+1) = \frac{p'(e^*)(u(A) - u(w))}{u'(w)}$. Substituting this into (7) gives us

$$\frac{\partial F}{\partial w} = p'(e^*)(u'(A) - u'(w)) - \frac{p'(e^*)(u(A) - u(w))}{u'(w)}u''(A) =$$
$$= p'(e^*)(u(A) - u(w))\left[\frac{u'(A) - u'(w)}{u(A) - u(w)} - \frac{u''(A)}{u'(w)}\right].$$

Applying Cauchy's Mean Value Theorem to the functions u' and u on the interval $[w - e^* - l(e^*), w]$ we have

$$p'(e^*)(u(A) - u(w))\left[\frac{u'(A) - u'(w)}{u(A) - u(w)} - \frac{u''(A)}{u'(w)}\right] =$$

$$= p'(e^*)(u(A) - u(w))\left[\frac{u''(\alpha)}{u'(\alpha)} - \frac{u''(A)}{u'(w)}\right] =$$

$$= p'(e^*)(u(A) - u(w)) \left[\frac{u''(\alpha)}{u'(\alpha)} - \frac{u'(A)}{u'(w)} \frac{u''(A)}{u'(A)} \right]$$

for some $w - e^* - l(e^*) < \alpha < w$.

Thus we may write

$$\frac{\partial F}{\partial w} = p'(e^*)(u(A) - u(w)) \left[\frac{u'(A)}{u'(w)} \lambda_u(A) - \lambda_u(\alpha) \right],$$

where $\lambda_u = -\frac{u''}{u'}$ denotes the Arrow-Pratt index of the absolute risk aversion for the function u.

Observe that $\frac{u'(A)}{u'(w)} > 1$. This follows from the fact that the function u' is decreasing and positive, and $A < w$. Moreover, the factor $p'(e^*)(u(A)-u(w))$ is positive and from the assumption of DARA, λ_u is decreasing (and positive). Since $\alpha > A$, we conclude that $\frac{\partial F}{\partial w} > p'(e^*)(u(A) - u(w)) [\lambda_u(A) - \lambda_u(\alpha)] > 0$.

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Wybór prawnika jako szczególny przypadek samoubezpieczenia z ochroną

Piotr Dudziński

Streszczenie Analizujemy decyzję o wyborze prawnika jako szczególnym przypadku samoubezpieczenia z ochroną, gdy koszt prawnika zostanie spłacony w przypadku wygrania procesu. Problem został wprowadzony przez Sevi i Yafil(2005) w kontekście obrony, która wymaga założenia, że wielkość strat nie zależy od wysiłku (poziom wydatków na adwokata).

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Słowa kluczowe: samoubezpieczenia, niechęć do ryzyka, zwrot kosztów.

1. Omówienie wyników

Wybór prawnika jest szczególnym przypadkiem prewencji i samoubezpieczenia. Prewencja (SP) jest zdefiniowana jako inwestycja redukująca prawdopodobieństwo poniesienia finansowej straty, zaś samoubezpieczenie (SI) oznacza inwestycję skutkującą redukcją poniesionej straty (Ehrlich, Becker, 1972). Połączenie SP i SI jest nazywane śelf-insurance-cum-protection"(SICP). Sevi i Yafil (2005) zauważyli, że w sytuacji gdy możliwy jest zwrot kosztów poniesionych na usługi prawne w razie wygranego procesu, to standardowy model prewencji wymaga modyfikacji uwzględniającej taką możliwość. Podstawową konsekwencją tej zmiany jest tzw. efekt zwrotu kosztów, sprowadzający się do obserwacji, że poziom inwestycji w usługi prawne wzrasta w przyadku możliwości zwrotu kosztów w porównaniu z sytuacją gdy takiej możliwości nie ma. Drugą konsekwencją tej modyfikacji jest monotoniczność wydatków na usługi prawne względem poziomu awersji do ryzyka. Jest to dość zaskakujący wynik, zważywszy że ogólnie prewencja nie jest monotoniczna względem awersji do ryzyka (Dionne i Eeckhoudt 1985, Bryis i Schlesinger 1990). Sevi i Yafil modelowali wybór prawnika za pomocą prewencji co w szczególności oznacza założenie o stałej wysokości strat, niezależnej od kosztów obsługi prawnej. Jest to silne założenie i jednym z celĂłw niniejszej pracy było zbadanie, czy jest ono istotne. Jednak opuszczenie tego założenia wymaga użycia modelu SICP. Zostało udowodnione, że jeśli funkcja straty jest malejąca w ograniczonym tempie, to po pierwsze możliwość zwrotu kosztów powoduje wzrost wydatków na usługi prawne, po drugie zaś wzrost awersji do ryzyka (w sensie Arrowa-Pratta) powoduje spadek inwestycji w usługi prawne. Konsekwencją drugiego twierdzenia jest możliwość określenia kierunku efektu dochodowego na SICP w przypadku możliwości zwrotu kosztów poniesionych w trakcie procesu. Przy naturalnym założeniu malejącej bezwzględnej awersji do ryzyka (DARA), usługi prawne są dobrem normalnym, tzn. rosną wraz z poziomem zamożności osoby wynajmującej prawnika do obrony, co jest zgodne z powszechną intuicją. Należy jednak zwrócić uwagę na istotność założenia o

możliwości zwrotu kosztów, bez którego teza twierdzenia jest fałszywa.
Na optymalny wybór prawnika mają więc wpływ dwa przeciwstawne impulsy: efekt zwrotu kosztów podnoszący poziom wydatków prawnych i efekt awersji do ryzyka redukujący te wydatki. Sevi i Yafil w swojej pracy zaobserowowali, że w pełnej ogólności nie można przesądzić który z tych efektów jest silniejszy. Zatem interesujące jest pytanie, jakie warunki są wystarczające na to, aby jeden z efektów przeważał w sposób jednoznaczny. W niniejszej pracy zostało udowodnione twierdzenie wskazujące krytyczny poziom początkowego prawdopodobieństwa poniesienia straty, poniżej którego efekt zwrotu kosztów jest silniejszy, zaś powyżej - efekt wzrostu awersji do ryzyka jest przeważający. Podany wzór na początkowe prawdopodobieństwo jest jednak na tyle skomplikowany, że może nie być możliwy do praktycznej weryfikacji. W pracy został podany i udowodniony warunek wystarczający na to, aby efekt zwrotu kosztów przeważał nad efektem awersji do ryzyka. Warunek ten jest wyrażony nierównością która określa typ wzrostu awersji do ryzyka poprzez drugą i trzecią pochodną funkcji transformacji użyteczności. Taki właśnie typ awersji do ryzyka powoduje, że osoba o większej bezwzględnej awersji do ryzyka, mająca możliwość zwrotu kosztów procesu wynajmie droższego prawnikaniż osoba o mniejszej awersji do ryzyka której nie przysługuje zwrot poniesionych kosztów.

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