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BOOK REVIEW
Nonlinear Dynamical Systems and Chaos

Proceedings of the Dynamical Systems Conference held at the University of Groningen, Groningen, December 1995. Edited by H. W. Broer, S. A. van Gils, I. Hoveijn and F. Takens. Progress in Nonlinear Differential Equations and Their Applications, 19. *Birkäuser Verlag, Basel, 1996*, viii + 459 pp., $149.00

This book contains the proceedings of the December 1995 **Dynamical Systems Conference** held in Groningen, in honour of Johann Bernoulli, and dealing with the following topics:
(i) Symmetries in dynamical systems
(ii) KAM theory and other perturbation theories
(iii) Infinite dimensional systems
(iv) Time series analysis
(v) Numerical continuation and bifurcation analysis.

The book contains papers both reviewing the state of art in Topics (i)–(v) and dealing with recent results of current research in these topics.

The first 8 papers (pp. 1–170) belong to Topic (i). In the first one *Symplecticity, reversibility and elliptic operators*, pp. 1–20, by T. J. Bridges, the concepts of symplecticity and reversibility are generalised and then used in analysis of some gradient elliptic operators on $\mathbb{R}^n$. In the paper *The rolling disc*, pp. 21–60, by R. Cushman, J. Hermans and D. Kemppainen, a homogeneous disc, which rolls without slipping on a horizontal plane under the vertical gravitational force field, is considered. Due to some symmetries, the problem is reduced to a four dimensional dynamical system. A rigorous qualitative description of the dynamics of this system is provided (both periodic and non-periodic orbits are described). Finally the motion of the disc without reduction is reconstructed. The aim of the paper *Testing for S_n-symmetry with a recursive detective*, pp. 61–78, by K. Gatermann, is to present an efficient numerical method to obtain a set of so-called detectives (which is the set of functions that enables one to determine symmetries of a certain attractor) by applying methods of representation theory and of invariant theory. For the symmetric group S_n the detectives are defined recursively. I. U. Bronstein and A. Ya. Kopanskii, in the paper *Normal forms of vector fields satisfying certain geometric conditions*, pp. 79–101, describe normal forms of vector fields (near a hyperbolic singular point), which satisfy some geometric conditions. The paper *On symmetric ω-limit sets in reversible flow*, pp. 103–120, by J. S. W. Lamb and M. Nicol, deals with symmetry properties of ω-limit sets of reversible flows (i.e. the flow $f^t : \mathbb{R}^n \to \mathbb{R}^n$ which possess reversing symmetries: a homeomorfism $\rho : \mathbb{R}^n \to \mathbb{R}^n$ is a reversing symmetry of the flow f^t if $f^t \circ \rho = \rho \circ f^{-t}$ for all t). In dimensions $n = 1, 2$, for reversible flows, they describe all symmetry groups of the ω-limit sets and in all dimensions they give restrictions on possible symmetry groups. Moreover, they give a full description of the possible symmetries of periodic orbits. Finally, they show that Liapunov stable ω-limit sets that are symmetric with respect to a reversing symmetry must be transitive. R. Lauterbach, in the paper *Symmetry breaking in dynamical systems*, pp. 121–144, discusses symmetry breaking bifurcations in dynamical systems with continuous time. This is related to the occurrence of stucturally stable heteroclinic cycles in equivariant systems. Author presents some of the difficulties of investigating the behaviour of dynamical systems near points where the symmetry was perturbed by outside influences. Moreover, he reviews some recent results on a geometric approach. M. Rumberger and J. Scheurle, in the paper *Invariant C^j functions and center manifold reduction*, pp. 145–154„ present an extension of G. W. Schwarz's result, concerning C^∞-functions invariant under the action of a compact Lie group which acts orthogonally on $\mathbb{R}^n$, to the class of C^j-functions. The C^j-theorem (proved in the thesis by the first author) is useful in the context of orbit space reduction on center manifold. This is illustrated in the case of the Swift–Hohenberg equation which models certain hydrodynamical instability phenomena. J. Knobloch and A. Vanderbauwhede, in the paper *Hopf bifurcation at k-fold resonances in conservative systems*, pp. 155–170 determine the bifurcation set in parameter space and describe changes of the set of small periodic orbits of parameter-dependent conservative systems as the parameter crosses the bifurcation set.

The 6 papers (pp. 171–324) belong to Topic (ii). The paper *Families of quasi-periodic motions in dynamical systems depending on parameters*, pp. 171–211, by H. W. Broer, G. B. Huitema and M. B. Sevryuk, review existence of invariant tori with parallel dynamics, in dynamical systems depending on parameters, in the following four "contexts": dissipative, volume preserving, Hamiltonian and reversible. The paper *Towards a global theory of singularly perturbed dynamical systems*, pp. 213–225, by J. Guckenheimer, is a step to construct a systematic global theory of singularly perturbed systems. The author sketches some topological aspects of the theory. H. Hanssmann, in the paper *Equivariant perturbations of the Euler top*, pp. 227–252, considers the motion of the rigid body in a small non-constant force field. The force field is assumed to be invariant under two spacial reflections. The author identifies periodic and quasi-periodic motions of the rigid body. The paper *On stability loss delay for a periodic trajectory*, by A. I. Neishtadt, C. Simó and D. V. Treschev, pp. 253–278, deals with stability loss delay for periodic trajectories of a system of differential equations depending on a slowly varying parameter. The authors estimate the time of delay and under some assumptions describe asymptotics of the escape time. M. Ruijgrok and F. Verhulst, in the paper *Parametric and autoparametric resonance*, pp. 279–298, discuss both a one degree of freedom, parametrically excited system (the force is varying periodically) and its generalisation — a two degree of freedom, autoparametric system (it consists of two constituing subsystems: an oscillator and an excited system). The latter admits a rich bifurcation structure and chaotic dynamics. M. Viana, in the paper *Global attractors and bifurcations*, pp. 299–324, gives a survey of recent results concerning the attractors of smooth dynamical systems. He analyses the basin of Hénon–like attractors, describes the ergodic properties of certain nonuniformly hyperbolic unimodal maps of the interval, discusses of a geometric model for the behaviour of the Lorenz system (this part was written jointly with S. Luzzatto) and finally shows the occurence of "chaotic" attractors with spiraling geometry in certain families of vector fields.

The 4 papers (pp. 325–404) belong to Topic (iii). The paper *Modulated waves in perturbed Korteweg–de Vries equation*, by S. A. van Gils and E. Soewono, pp. 325–346, deals with the analysis of modulated traveling waves in the one–dimensional perturbed Korteweg–de Vries equation. In the paper *Hamiltonian perturbation theory for concentrated structures in inhomogeneous media*, pp. 347–372, E. R. Fledderus and E. van Groesen consider spatially inhomogeneous Hamiltonian system with small rate of change of the inhomogeneity. They describe deformations of structures, that are characteristic for the homogeneous case, as a consequence of the inhomogeneity. The theory is applied to the motion of a Bloch wall in an inhomogeneous ferro–magnetic crystal. X. Huang, in the paper *On instability of minimal foliations for a variational problem on T^2*, pp. 373–383, studies a variational problem on a torus T^2. He shows that the Z^2–invariant C^m–minimal foliation can be destroyed by a small perturbation. The paper *Local and global existence of multiple waves near formal approximations*, pp. 385–404, by

X.–B. Lin, deals with a general singularly perturbed parabolic system. The author shows that, if a formal approximation is precise enough, then there exists a solution near the formal approximation for a short time. Then he shows that, with some restrictions, the solution exists globally, if the formal approximation is global.

The paper *Estimation of dimension and order of time series*, pp. 405–422, by F. Takens, belongs to Topic (iv). The author gives a survey of two approaches of analyzing time series — in terms of the attractor dimensions of (deterministic) dynamical systems and — in terms of the order of nonlinear autoregressive models. In this context he discusses some numerical examples (for the Hénon map, the logistic map and the Cheng–Tong model).

Topic (v) contains 2 papers. The first one is *On the computation of normally hyperbolic invariant manifolds*, pp. 423–447, by H. Broer, H. M. Osinga and G. Vegter. It deals with the numerical computation of normally hyperbolic invariant manifolds of dynamical systems. The method is applied to some examples (the fattened Thom map, the fattened Arnold family and the forced Van der Pol oscillator). The second one is *The computation of unstable manifolds using subdivision and continuation*, by M. Dellnitz and A. Hohmann. The authors develop a method for constructing a covering of unstable manifolds up to a given accuracy. As an example they compute of two–dimensional invariant manifolds in the Lorenz system.

The book can be recommended to researchers specializing in dynamical systems.

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