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The Milestones in the life course of distinguished mathematicians and mathematically gifted adolescents*

Abstract: The article presents the results of research on milestones (significant events, critical points, and crystallizing experiments) in the course of life of outstanding mathematicians and mathematically talented adolescents. The work covers the period of approximately the last 80 years.

Four distinct group of mathematicians had received their education and pursued their scientific careers at that time:

- The late, distinguished professors of mathematics,
- Distinguished professors of mathematics who are still alive,
- PhD students and doctors of mathematical faculties,
- Laureates of mathematical olympiads.

The author's intention was to indicate the milestones – key events and moments in their history determined by the author (or those indicated by those interested) to reach the highest position and recognition in the field of mathematics. Those are, for example, important experiences in a person's life that played a huge role in choosing mathematics as a direction for further development or reasserted that mathematics is the right choice.

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Key words: crystallizing experiences, outstanding mathematicians, mathematical abilities, the milestones.

The considerations were based on a holistic, humanistic approach and a biographical approach from the perspective of Charlotte Bühler (1999). The techniques that have been used include document analysis and narrative interviews. According to the recommendations of Ch. Bühler results were presented graphically on the timelines. The information that is interesting and at the same time important from the point of view of mathematical education and the development of mathematical abilities is that some milestones are timeless.

It is crucial to properly support children in developing their talents from an early age. Each of the respondents told about the people that were significant in his or her life.

However, one can observe the fact that scientific authorities are becoming less and less important. The respondents from the group of distinguished PhDs and PhD students (apart from only one researched) did not mention scientific authorities who directed their mind towards mathematics, when speaking about the milestones in their lives. Trips abroad and scientific internships were much more important. Another important issue is the age of starting a scientific career. The starting point was the date of the first, significant publication, as mentioned by the researcher himself. Among the late, distinguished professors and outstanding professors living, it happened when they were between 20 and 30 years old. Unfortunately, no PhD student or fresh PhD graduate mentioned the outstanding work published at that time.

Cognitive psychology tells us, however, that young people, due to the stages of intellectual development, can carry out the greatest mathematical discoveries.

1 What made me interested in researching the milestones of outstanding mathematicians in the course of their lives? Briefly about the motives and the most important findings in the literature

Since 2014, while participating in the discussions during the PhD seminar, I noticed that the majority of research projects focus on the negative aspects of education and upbringing. Negative factors are determined in order to support the development of mathematical aptitude gains in children, shortcomings in school education blocking their development, and finally life events that influence mathematically talented individuals in achieving academic success in mathematics.

While studying pedagogical and psychological literature, I noticed that there had not been any research on talents taking into account specific events and

situations that have started the career path and have contributed to achieving extraordinary achievements.

The exceptions are the studies of Howard Gardner and Joseph Walters (Walters, Gardner, 1986, as cited in K. J. Szmidt, 2012) described in the article: "The Crystallizing Experience: Discovering an Intellectual Gift". The authors defined experiences crystallizing as events "that involve a person with an unusual talent or potential abilities in a given field, in which the talent can be revealed" (Szmidt, 2012, p. 77).

According to Gardner and Walters, the crystallising experience is an extraordinary meeting, usually in adolescence, with an authority in a given field of creativity or with its characteristic material, or even hardware and instrumentation, which becomes a breakthrough in the personas further life.

The course of the fact of this meeting results in the fact that the creative person begins to concentrate his life on a chosen problem, material or experience (Szmidt, 2012, p. 77).

The authors suggest (Walters, Gardner, 1986, as cited in K. J. Szmidt, 2012) that in the case of the greatest talents, crystallizing experiments are inevitable, and most importantly, they happen more often in the case of musicians and mathematicians.

These views were an inspiration in establishing the purpose of my research. However, I decided that it is worth considering what other factors could have influenced the development of mathematical aptitudes or whether it was just a stroke of luck (M. Shermer 2018, p. 2) or other factors. In the development of the research program, I found the publication of Charlotte Bühler (Bühler, 1999) and the methodological concept of conducting research into the course of human life really helpful.

Speaking about aspects of human life according to Ch. Bühler opens the perspective of three problem areas that I will distinguish in the overall course of human life. These three problem groups are: first, the course of human life as a biological process, as the development and destruction of the body and its functions; secondly, the course of life as individual behaviour and personal experience, studied on the basis of biographical data and subjective experience; third, the course of life in its objective results, its impact on others, its production and its historical role in the broadest sense of the word. (Bühler, 1999, p. 34).

Milestones in my understanding are: "determined by me (or indicated by the interested) key events and moments in their history of reaching the highest status and recognition in the field of mathematics". For example, these are important experiences in a person's life that played a crucial role in choosing mathematics as a career, or confirmed the belief that mathematics is the right choice.

The choice of a group of outstanding mathematicians was determined by the fact that I was already relatively familiar with the problems of mathematical aptitudes and also in what is not conducive to developing talents. I therefore considered it beneficial to consider the course of life of mathematicians in terms of what favours the development of talents.

I decided that it will be easier to organize the education process so as not to miss anything, when we know what foreshadows successes. I also wanted to know in what period of life these milestones appear, whether they are individual or perhaps characteristic for a given period of time.

Another determinant of my research program was the conviction that it is worth considering the lines of life and milestones of outstanding mathematicians living in a certain time span. I decided that it would help to separate the age groups. I assumed that I would consider the fate of eminent mathematicians who had been active scientifically from 1930 to the present. In connection with this I have determined that I will analyze the lines of life and milestones that have had an impact:

- on the development of mathematical talents of outstanding mathematicians in times of war (World War I and II) and right after the war,
- on the fate of universally recognized mathematicians who have developed their mathematical aptitudes since the 1960s and achieved their greatest successes in the previous century,
- on persons currently pursuing their PhD's and those who obtained a PhD degree in the field of mathematics, and their school education and studies fell in the last years of the previous century and the first decade of the new century,
- on the fate of young people who have won in mathematical Olympiads over the last ten years. Their school education took place at that age.

I dealt with the analysis of the course of life and the determination of the milestones of the outstanding mathematicians of the above-mentioned groups for about the last 80 years.

2 Research program

In this research, I dealt with a section of pedagogical reality: getting to know the favorable factors that affect the career of outstanding mathematicians. The literature of the subject did not facilitate the choice of the theory on the basis of which one could build reasonable hypotheses – I did not find research related to

the development of mathematical aptitudes from the side of favorable factors. Ch. Bühler – a researcher of fundamental importance to my work – used the hypothetical-deductive order in her theories of the course of life.

At this point, I would like to emphasize the fact that the monograph "The Run of Human Life" by Ch. Bühler, who was the inspiration, the core of my dissertation contains a biographical method which has not been found anywhere else and has never been described in textbooks. An exception is Teresa Rzepa (Rzepa 2005, 2011) who did not carry out research using this method, but mentions it several times and emphasizes its methodological value.

In this situation, I have chosen an inductive research strategy, which is a way of reaching new scientific assertions and checking them. It involves the generalization of unitary empirical facts according to the principle "observe and generalize the results of observations – first observation, then theory". That is why I have formulated goals and distinguished research tasks from them, which are precisely defined by the selected fragment of pedagogical reality. Next, I have chosen research methods, ensuring that each of them provides information about the variables studied (Such, 1969, p. 140). I realized the goals in the order given, because each of the following results from the previous one.

With all this in mind, I formulated goals and the following research tasks:

1. A research goal: Establishing beneficial events that have appeared in the minds of outstanding mathematicians and significantly influenced their scientific achievements. By setting these events, I will follow the directions of Ch. Bühler (1999).

   Within the first objective, I distinguish four research tasks. Each concerns one age group: the first – the late outstanding mathematicians, the second – living, outstanding mathematicians, the third – outstanding PhD students and PhDs, and the fourth – math Olympians. In accomplishing this goal, I used the biographical method. As for the first research task, the main source of information is the analysis of text, diaries, biographies, and extended interviews. However, as far as the second, third and fourth research tasks are concerned, they concern living people with whom I could personally talk, so the main source of information is narrative interviews (they were recorded on a digital medium, then transcribed and analyzed) supplemented by publicly available information from other sources.

Here is the research task carried out within the first objective:

- Research task (1.1) What milestones, which were noticed on the development of mathematical aptitudes, were preserved in the minds of people whose scientific successes fall into post-war times until the end of the eighties of the last century?
- Research task (1.2) What milestones have been recorded in the minds of people whose scientific successes start in the 1960s and have achieved the greatest successes in mathematics in the previous century? They are scholars with whom I could still conduct extended interviews to determine who or what had a significant impact on shaping their minds towards mathematics.
- Research task (1.3) What milestones have been recorded in the minds of people whose school education and studies were in the last years of the previous century and the first decade of the new century? They are outstanding PhD students and young PhDs of mathematical faculties.
- Research task (1.4) What milestones have been recorded in the minds of people who have won mathematical olympics in the last ten years, ie which school education took place at this age?

2. Research goal: Analysis and organization of information collected as a result of the implementation of the four research tasks listed above. This will allow me to set the course of life, career and milestones among the four groups of respondents.

   I used Charlotte Bühler's research recommendations for plotting the way of life – the timeline and the embedding of facts on it, influencing life's decisions and enabling success in a career in science.

3. Research goal: Analysis of collected and ordered information about life lines and the context of events; identification of the mathematical milestones (specific or common in the period from 1930 to 2016) that affect the development.

   So it is a synthesis of my research achievements. The result is a picture of life paths gifted by outstanding mathematicians. Realizing the third goal, I again used the suggestion of Ch. Bühler and I used a graphical representation of the events that make up the life line, with an indication of what positively influenced the development of mathematical aptitudes. Thus, I presented the visualization of the line of life separately: the late outstanding mathematicians, outstanding mathematicians who are

alive, outstanding PhD students and PhDs of mathematical faculties, mathematical Olympians.

4. Research goal: Determining the period of life – the time in which the studied persons – outstanding mathematicians from four groups – started their scientific activity. This realization is the culmination of my research. I hope that the results I have obtained will be a contribution to knowledge about the factors that influence the development of mathematical aptitudes.

   It also seems important to the future mathematical careers. The education of gifted people and its practical problems should be dealt with in special pedagogy. After all, anything that does not fit into the broadly understood norm belongs to the issues of education of people with special educational needs. Undoubtedly, gifted children and young people have such needs. Meanwhile, special pedagogy deals mainly with the education of people who have educational problems because of their limitations. I intend to formulate conclusions that will supplement this state of knowledge about the need to support talented people.

   The milestones that I have found can undoubtedly help to organize the education of younger and older students. Thus, contribute to ensuring that people with mathematical skills are given adequate support and opportunity to develop their strengths.

3 Selection of the subjects and the method of their selection

Initially, I narrowed down the research group to Polish scholars. However, with the advice of authorities in the field of mathematics, I also included two professors born in Slovakia, living in the Czech Republic and one professor from Ukraine, acknowledging that their life experiences, due to the similarity of social background and historical context, can be considered similar. I limit my research to mathematicians who lived and made their scientific activity in the years 1930-2016.

Due to the necessity of anonymity in research, I do not provide accurate data of all subjects, although I have detailed information. The names of those mathematicians who gave their consent were put in the article.

Every respondent who wanted to remain anonymous was marked with one of the letters of the Greek alphabet.

The studied group is homogeneous. All its members are mathematicians. Most were born in Poland, apart from the three above-mentioned cases. All respondents (apart from the Olympians and one PhD student) have taken part in scientific internships in research centers in the West. The subjects are men – no one has pointed to any woman.

I collected about 25 interviews, from which 18 were used in my work. This was due to the completion of the trial (when in the indications began to appear the same surnames given by subsequent competent judges, I decided to close the research group at this point) and disturbing variables that obliged me to give up those that did not fit into my research assumptions. One of the professors refused to take part in the study – in the second part of my research – at the stage of authorizing the results.

In this part I will describe how I selected people for research. Regarding the late mathematicians, I did not have much trouble. I assumed that those are the people mentioned in the History of Polish Science and the Chronicle of Polish Science. Those people have already been recognized as outstanding by competent judges.

Trouble began when I proceeded to select people who belong to the second group, that is, living outstanding professors of mathematics. Because I am a teacher, it was very difficult for me to do it. Therefore, I used the advice of the Promoter to use the chain of subsequent recommendations and apply the method of competent judges in a specific manner.

It proceeded as follows: The Promoter pointed me to the first eminent professor of mathematics of international fame. I asked him for an interview and obtained his consent. When finishing the interview, I asked him to point me to the next outstanding mathematician. I used the given address, I interviewed and again at the end of the interview I asked to recommend me an outstanding mathematician. Each of the eminent mathematicians whom I asked for suggested another outstanding personality. Thanks to this, each of the respondents was rated as outstanding by the professor of mathematics who were recommended to me. I also used this snowball method, as the reviewer pointed out, when selecting outstanding PhD students and PhDs.

I would add that all the scholars with whom I conducted interviews showed modesty and stated that they are not outstanding mathematicians. Each of them showed me a person who in his opinion is an outstanding mathematician, enabling me to contact him. Thanks to this, each of the mathematicians studied was rated as outstanding by the professor of mathematics who recommended me. In a similar fashion, I chose outstanding PhD students.

Almost all prominent mathematicians are somehow connected to universities and easily pointed to the other scientists who in their opinion can be

considered outstanding. On the other hand, I distinguished outstanding Olympians through the Polish magazine "Perspektywy", (2014) in which rankings of the best Olympic schools are created. Then I reached out to people who were highly ranked in the Olympic rankings.

4 Justification of applied research procedures

In my research, I applied a biographical approach, and the methods are: text analysis (journals, diaries, extended interviews, autobiographies) and narrative interviews. As I have already pointed out, I used Charlotta Bühler's clues. (Bühler, 1999) for ordering and interpreting the information.

"The course of human life„ is a precursory work, inaugurating a change in thinking about the psychological development of a man. (Bühler, 1999, p. 15). Ch. Bühler embraced man and human life in a comprehensive way.

Teresa Rzepa in the introduction to the Polish edition of the monograph "The course of human life" emphasizes an important, fundamental aspect of Ch. Bühler: "... choice is an attribute of individual experience, understood as the totality of experienced events. At the same time, it is the choice that triggers the possibility of change, development, self-fulfilment, transcendence and transgression." (Bühler, 1999, p. 5). According to Ch. B̈hler only in the early stages of development, human behavior is conditioned by external factors.

"Along with acquiring proficiency in using the ability to make choices (conditioned by past experiences), the basic determinant of human behavior is internalized. From this developmental moment, the individual can overcome his own past, "project" into the future, defining it by formulating and achieving the goals set. In this way, he or she assumes the responsibility for their own life." (Bühler, 1999, p. 17).

While constructing research, I applied an inductive research strategy, which is a way of reaching new scientific assertions and checking them. It involves the generalization of unitary empirical facts according to the principle "observe and generalize the results of observations – first observation, then theory".

At this point, we want to refer to the methodology of grounded theory (Konecki, 2000). My research fits into the trend of this theory. The creators of the grounded theory methodology treat "building the theory as a process, it is not a verification of previously constructed hypotheses based on later collected data. Data collection, building hypotheses and their verification are not clearly separated in time, as is the case in traditional surveys, but are procedures that mutually interweave many times during the long process of generating the theory." (Konecki, 2000, p. 27).

That was also true for my research. At the beginning, I did not make hypotheses that would later be verified during the study. I started with observing a given slice of reality, gathering as much data as possible about it. In the case of my research, it was a detailed analysis of documents and narrative interviews. Then I described what I observed.

I used different kinds of comparisons (for example, I compared a group of the late outstanding mathematicians with living outstanding mathematicians, mathematical PhD students with Olympians in the field of mathematics), I was looking for common points and differences.

I prepared the theoretical categories derived from direct research data, while retaining the appropriate coding pattern: "(causal conditions – the phenomenon [central category] – context-intervening conditions – actions interactive strategies and techniques – consequences]" (Konecki 2000, p. 48).

Characteristics of the subjects based on the collected information.

The study included 24 people, divided into 4 groups:

- the late outstanding professors of mathematics,
- living outstanding professors of mathematics,
- outstanding PhD students and PhD's of mathematical faculties
- winners of mathematical Olympiads.

The Act of 19.08.1997 on the Protection of Personal Data (Journal of Laws, 1977) obliges in the case of such works to anonymize data. The reservation does not concern only the first group – data of these people are in public archives and collections of state libraries. We can become familiar with them.

I limited my search to outstanding mathematicians living in Poland or countries whose history after the Second World War was similar in terms of organization of education and scientific advancement. (Adamczyk 1996; Bielec 2008; Cipro 1963; Motyka 2013; Śliwa 2002; Walczak 1999).

5 The late eminent professors of mathematics

I analyzed the fate of the late eminent professors of mathematics and decided that this group includes:

a) Stefan Banach (1892-1945),

b) Hugo Steinhaus (1887-1972),

c) Stanisław Mazur (1905-1981),

d) Andrzej Pelczar (1937-2010),

e) Andrzej Lasota (1932-2006).

All subjects in this group are connected by:

- they have demonstrated mathematical aptitude from an early age,
- they created their works during and after the Second World War,
- everyone in their memories (interviews, journals, autobiographies) mentions and clearly defines events that were related to the development of their own mathematical aptitudes.

Two people did not formally complete their studies (they did not receive diplomas). Three of the respondents have – besides mathematics – other education (philosophy, philology, physics). All respondents went abroad as part of a scholarship, research internship, and studies.

I would like to remind you that I have selected the respondents through the help of competent judges. There are nine people in the group of living, outstanding professors of mathematics.

All people in this group:

- they showed mathematical aptitude quite early,
- all have full, formal mathematics education (university diploma),
- all of them have a similar career (all have academic degrees: PhD/ postdoctoral degree./ full professor.
- their success in the field of mathematics falls within the period 1960-2010.

Five of the nine discussed here held a foreign research internship, one defended a PhD dissertation abroad (USA), four of them lectured as visiting professors. One of the respondents started a year earlier to attend school.

6 Outstanding Ph.D. students and Ph.D of mathematical faculties

I have also chosen outstanding PhD students thanks to the competent judges method. Initially, I went to the head of PhD studies at the University of Warsaw. I was directed to a few PhD students who, in his opinion, had significant scientific achievements. In this way, I got contact information of some of the outstanding PhD students and PhDs who agreed to take part in the interview. Then each of them pointed to more people who, in their opinion, belong to the

outstanding ones. In this way, every outstanding PhD student who gave me contact to the next, became a competent judge.

There are five people in this group.

All subjects:

- showed mathematical talent quite early,
- took part in school competitions, national and foreign Olympiads,
- have completed their formal MA studies in mathematics departments,
- had scientific successes that fell between 2010 and 2016.

Two PhD students studied abroad (England, Norway), one took a research internship abroad (Pennsylvania State University), one – completed the first year of mathematics studies while still in high school.

7 Winners of mathematical Olympiads

I selected the laureates of mathematical Olympics on the basis of the ranking published in the Polish magazine "Perspektywy" monthly (2014). Then I made contact with the institution they represented, and through it – with the Olympian himself. There are four people in this group.

All subjects:

- showed mathematical aptitude quite early,
- took part in school competitions, national and foreign Olympiads with above-average results,
- had success in the field of mathematics in the period from 2010 to the present.

Most of the respondents participated in mathematical circles and workshops.

8 Lines of life of outstanding mathematicians with the indication of milestones that determined their scientific successes

Ch. Bühler recommends that the life course and the milestones highlighted in it to be presented graphically. Therefore, I arranged the information gathered in the form of an oblong chart, marking on it a line symbolizing the course of life, divided into years. I have marked all the milestones on it in the whole course of their life that I managed to find. I marked them with colors:

- red: events, people, facts to which the examined persons returned (e.g. during an interview) stressing that if it were not for them, they would not be who they are today,
- blue: material forms of recognition (scholarships, awards) – important for a career in science, but not so much that all outstanding mathematicians point them out as events directing their mind towards mathematics,
- green color: acquired professional promotion grades, titles, successes in learning, functions performed at the university or other institutions and organizations, competitions won,
- yellow: the most important publications of the mathematicians studied.

All charts were built in the Office Timeline program. For the purposes of this article, I present only one chart I have drawn from each group. Each created biography (apart from dead biograms, eminent professors of mathematics) I based mainly on information obtained from the respondents themselves, although I also supported online sources, which if necessary were corrected by the respondents themselves.

For the purposes of this article, I chose one person from each group. I showed them the course of life on the timeline and the whole description – the characteristics. I started with a group of the late professors, then living, outstanding PhD students, PhD's and Olympians.

9 Hugo Dyonizy Steinhaus (Waliszewski et al., 1988, Dawidowicz, 1981, Steinhaus, 2011, Duda, 2014, born on 14. 01. 1887 in Jasło, died on February 25, 1972 in Wrocław)

A Polish mathematician of Jewish descent, professor at the University of Jan Kazimierz, co-founder of the Lviv school of mathematics, co-founder and editor of the magazine Mathematica, an aphorist. He was the author of many works in the field of game theory, functional analysis, topology, set theory, trigonometric series, orthogonal series, the theory of real functions and applications and popularization of mathematics.

There is a certain pattern on the chart: the most milestones in the lifetime of Hugh Steinhaus are in his childhood and adolescence. The most productive time was his junior high school education. The first milestone considered to be so by Steinhaus himself was his father arousing curiosity about the world, by

taking him to different places. Another is the physics book Thursday Thursdays, which H. Steinhaus received during illness. He also read "The Adventures of Tom Sawyer" in German, not knowing the language well yet. He and his colleagues created a secret library and read very different books. Nietzsche heavily influenced him.

In his memoirs he also stated that he played chess (with Dziunia, the seven-year daughter of a father's friend, and a five-year-old sister – Irena, whom Steinhaus liked very much, and which also had a huge impact on his development and intellectual efficiency). In his memoirs, H. Steinhaus also mentioned his grandfather, who wanted to become a scholar himself and effectively discouraged him from military service. After graduating from high school in Jasło, Steinhaus struggled with the choice of studies.

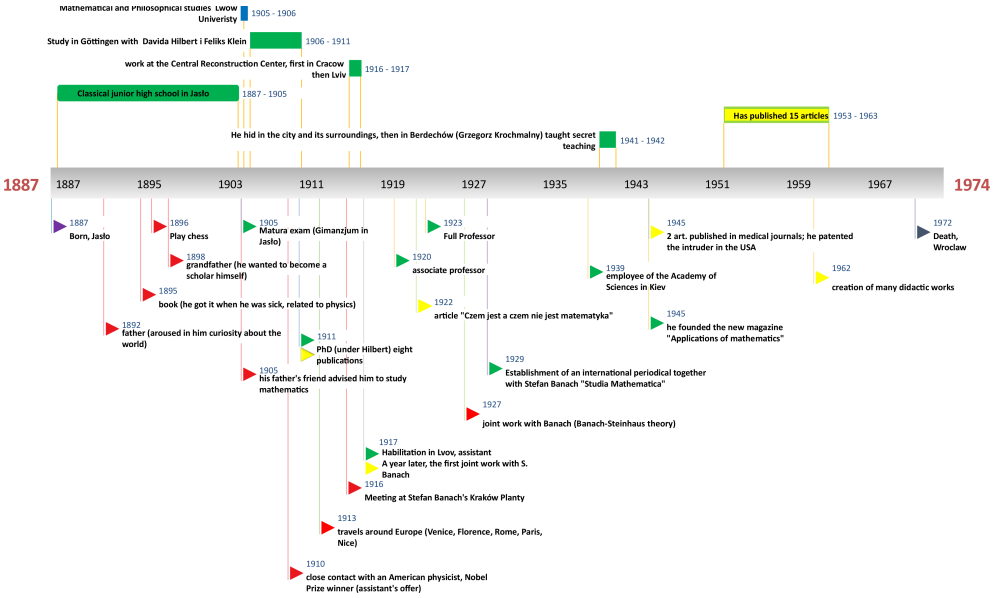
His father's colleague, a young engineer Ludwik Silberman, advised him to study mathematics. The decision to carry out such studies was another milestone that influenced the choice of a specific, mathematical path. H. Steinhaus, in his memories, also puts considerable emphasis on close contact with the American physicist Albert A. Michelson, Nobel laureate, from whom he received an offer of an assistantship in Chicago.

The next milestone is the event with signs of a smile of fate "the meeting of Steinhaus with Banach on Krakow's Planty Park". Steinhaus himself regarded Banach as"his greatest mathematical discovery".

In the memoirs of H. Steinhaus, I also established milestones related to acquiring formal education and various functions that he fulfilled: a PhD obtained under the direction of Professor David Hilbert and a postdoctoral degree in Lviv, where he had served as an assistant for a year. In his memoirs, he also assigns a significant role to his numerous trips around Europe (Venice, Florence, Rome, Paris, Nice).

Also, the acquisition of an extraordinary professorship and a few years later ordinary professorship is a milestone in the life of H. Steinhaus. Very important were also numerous scientific papers, including articles with the participation of Stefan Banach. H. Steinhaus also published two articles in medical journals, as well as a patented US introvisor (device for spatial X-ray localization of inaccessible objects, patented in the USA). He worked at the Academy of Sciences in Kiev. These are all milestones in his biography that he managed to identify.

10 Timeline – Hugo Steinhaus



11 Andrzej Schinzel (Mączyński, 2012, born on 05/04/1937, Sandomierz)

He deals with the theory of numbers. He is a member of the Polish Academy of Sciences. He graduated from the University of Warsaw in 1958. He received his PhD under the supervision of prof. Wacław Sierpiński in 1960. He has been a postdoctoral researcher since 1962, an associate professor since 1967, an ordinary member of the Polish Academy of Sciences since 1979, and a member of the Polish Academy of Sciences since 1979. He is a member of the Mathematical Institute of the Polish Academy of Sciences. In 1998, he was awarded a honorary PhD of the University of Caen, and in 2012 he was honored with the same title by the University of Caen. Adam Mickiewicz University in Poznań and the University of Stefan Cardinal Wyszynski in Warsaw.

He has published over 200 works and deals with many areas of number theory: analytic number theory, algebraic number theory, diophantine equations, number geometry, especially algebraic and arithmetic aspects of polynomials.

For almost forty years he was the publisher of “Acta Arithmetica”. His PhD students included Henryk Iwaniec, Maciej Zakarczemny, Rolf Wasen and Iskander Aliev – outstanding mathematical figures. In 1992 he received the Medal of Stefan Banach.

Decorated with the Knight's Cross, the Officer's Cross, the Commander's Cross of the Order of Polonia Restituta, the Pro Ecclesia et Pontifice Cross, the Medal of the National Education Committee and the Medal for Merit for the Archdiocese of Warsaw. He is a member and active protector of Archikonfraternia Literacka in Warsaw.

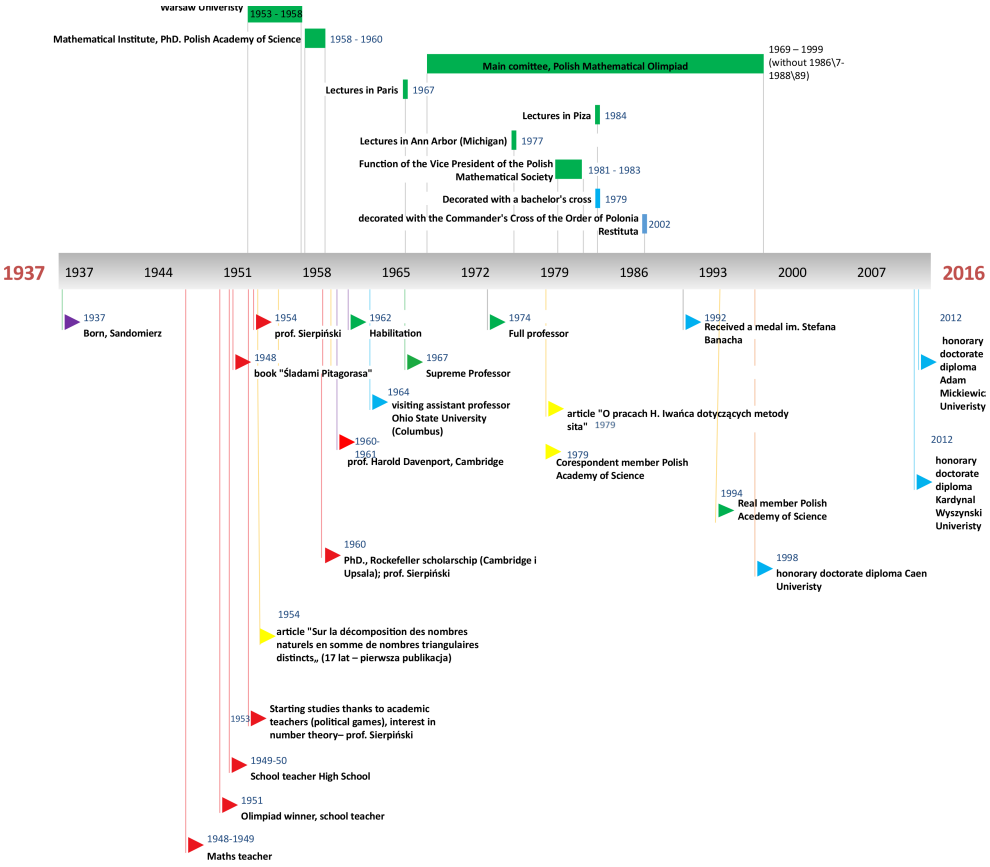
For almost 30 years he was involved in the mathematical Olympiad – in this many times he was the chairman of the main Olympiad committee. Then, for seven years, he was responsible for the mathematics-Austrian-Polish competition. It seems that most of the milestones important for the professor's mathematical career can be dated to his childhood and early youth.

The first milestone he pointed out was the teacher in elementary school. He was very kind to young Andrzej and devoted much time and attention to him. Another – was to win a mathematical Olympiad in the second grade of high school. The then teacher of mathematics then released the future professor from the obligation to attend maths classes until graduation. The demanding highlight is that Andrzej Schinzel wrote his first scientific work in 1954, at the age of seventeen. Together with Professor Sierpinski, he published seven scientific papers. The next important event, which was recorded in the memory of the Professor, is the support of Kazimierz Kuratowski in the recruitment process. Thanks to the intervention of Professor Kuratowski, despite previous failures, Andrzej Schnizel got to study.

Another milestone in the scholar's biography of Professor Schinzchel was a meeting with Professor Wacław Sierpiński. It was he who drew the professor's attention to number theory – Professor Sierpiński conducted a seminar, available to first-year students, which was quite exceptional. Professor Sierpiński also enabled Professor Schinzl to go abroad – to obtain a Rockefeller scholarship (after a PhD in 1960). Professor Schinzel spent some time in Cambridge, England (Professor Harold Davenport played a significant role here) and in Upsala.

There are books read by the professor that are engraved in his mind e.g. *In the footsteps of Pythagors, Manuals of Kazimierz Kuratowski and Wacław Sierpinski*. Professor Andrzej Schinzel wrote his first scientific work in 1954, at the age of seventeen. Together with Professor Sierpinski, he published seven scientific papers.

12 Timeline – Andrzej Schinzel



13 B (Born 1998, Cracow)

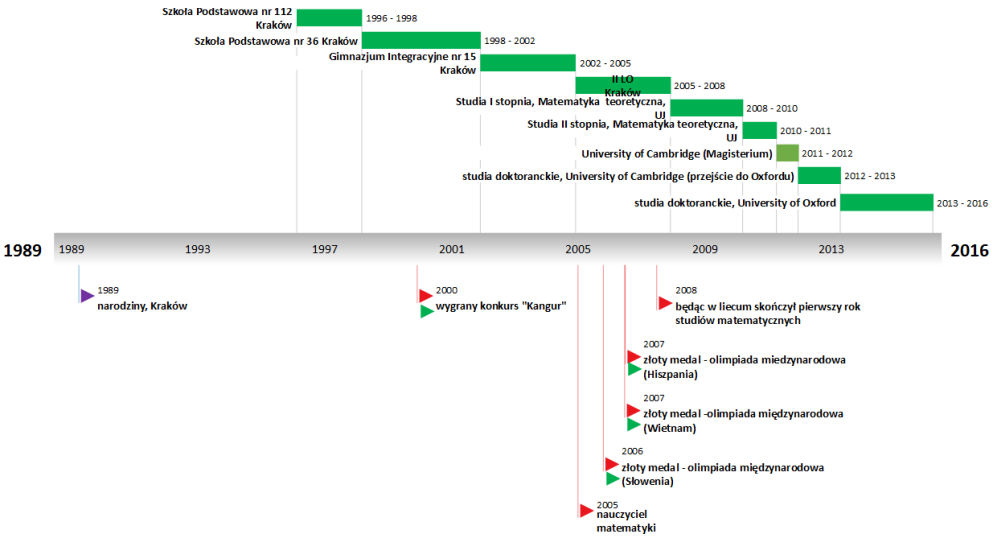
Three-time gold medalist of international mathematics Olympiads in Slovenia (2006), Vietnam (2007) and Spain (2008). Graduate of the Secondary School Jan III Sobieski in Kraków, Cambridge University and Oxford University student. In an interview, a young mathematician said that "he was basically born with love for numbers" – milestones in his biography, so you can start looking in childhood.

For a milestone in the development of interests and a mathematical career, the respondent considers winning the mathematical competition "Kangaroo" still in primary school (fourth grade). He admits that good results in competitions motivated him to continue working. A teacher in high school recognised his talent, nurtured it and tried to reduce his school workload. For example he

didn't have to test other subjects so that he could focus solely on mathematics. The subject completed the first year of university mathematics while still in high school. The above-mentioned teacher even found another teacher and mentor for him, after the subject went to university.

At the time when I was interviewing, the young researcher already had the date of defense of the PhD dissertation (2016). As it turns out, however, he did not tie his further career with work at the university. When I asked him about the reason, he explained that the financial situation prompted him to make such a decision.

14 Timeline – B



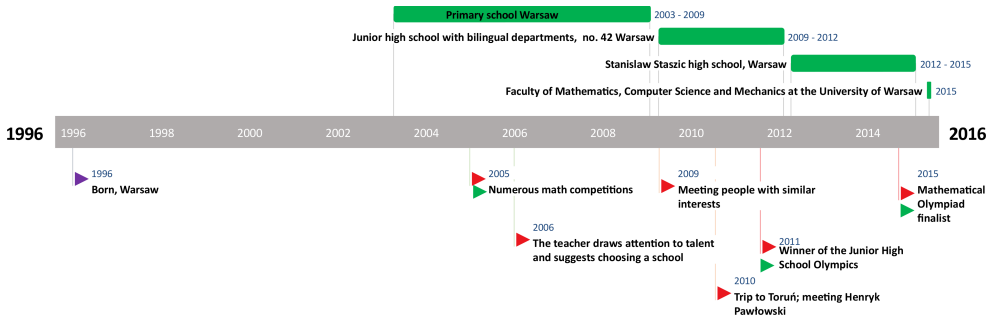
15 H (Born 1996, Warsaw)

A young, outstanding Olympian attended a small, social elementary school and junior high school in Warsaw. Then he graduated from the XIV High School of Stanisław Staszic in Warsaw. He is currently a student at the University of Warsaw. Among the milestones pointed out by him in the development of his mathematical talents, interests and important for the future career, the researcher mentions numerous mathematical competitions. There were many of them in his student career and all motivated him to continue his work. Today he remembers that he liked to take part in them. In the second place, as a strong motivation for development, he mentions the support of his parents.

He believes that it was thanks to his parents that he could develop freely, because "They always answered my questions".

His parents provided him with unfettered development in every field, including mathematics. He also received support from the teachers in the development of his interest in mathematics.

16 Timeline – H



17 Milestones determining the development of talents and scientific achievements of mathematically talented youth and outstanding mathematicians

At this point I will focus on describing the milestones that I managed to establish, analyzing their impact on the lives of the late outstanding professors of mathematics, outstanding eminent professors of mathematics, outstanding PhD students and Olympians in the field of mathematics.

Milestones, as I pointed out earlier, are treated as: "determined by me (or indicated by the interested) key events and moments in their history of reaching the highest dignity and recognition in the field of mathematics." For example, these are important experiences in a person's life that played a huge role in choosing mathematics as a direction for further development, or confirmed in the belief that mathematics is the right choice. When discussing categories, I keep order according to the periods of my life – according to the concept of Ch. Bühler. And so I distinguish the following categories of milestones:

- persons supporting the development of mathematical aptitudes: people from the immediate family, friends of the house, teachers from primary, secondary and academic school; scientific authorities,
- successes in school and out-of-school mathematics, PhD studies, scientific achievements in the field of mathematics. The perception of these successes by other people was the motivator for choosing the field of study (mathematics),
- books, textbooks that influenced the development of mathematical interests, for example, the *Footprints of Pythagors*, textbooks by Kazimierz Kuratowski, Wacław Sierpiński, Richard Courant, Stefan Banach,
- school and academic conditions for developing mathematical aptitudes: inspirational maths lessons, lectures in college, general high school environment, colleges, PhD seminars, an opportunity to show your skills, applause – encouragement to continue working,
- random events, cases affecting the development of interest in mathematics: the possibility of correcting teacher's mistake, mistakenly given to students a test for the senior class, which gave the opportunity to reveal the talents and above-average knowledge of one of the respondents,
- trips and scholarships abroad, extending the possibilities: first experiences in study trips, annual scholarships, lectures at the invitation,
- other: mathematical summer schools, trips to math camps, chess games, interests, internal, spontaneous motivations, so strong that they led to deepening knowledge of mathematics.

These milestones can be divided into those that result from objective facts and those that appeared in the lives of people very unexpectedly, in an informal way and yet had a decisive impact on the decision of the respondents. I will explain that in more detail in the conclusions from the research.

The milestones that I mentioned are widely known. What is attractive concerns time, place, location on the lines of life.

Meaning and time of the mentioned milestones in shaping careers in the line of life: that is, about the formation of scientific careers.

In this part of the paper I focus on a detailed interpretation of the existing regularities in the lines of life – the biographical axes of the time of the examined people. I analyzed the fate of extremely gifted mathematicians who lived approximately over the last eighty years.

All four groups are characterized by certain regularities: Significant people and stimulation of talents in all groups, significant people were important as

milestones. I have observed a very clear influence of people coming from the immediate surroundings of the subject (i.e. family, friends, etc.) and teachers, academic lecturers, great scientific authorities – masters. All the respondents pointed to a person or a few people during their lifetime (especially from the immediate environment), which had a direct relationship with directing their mind towards mathematics. It is very important to see the mathematical aptitudes in early childhood (Gruszczyk-Kolczyńska, 2012, p. 50).

According to the Gruszczyk-Kolczyńska study: "mathematically gifted children can be counted as every fifth five-year-old, every fourth six-year-old and only every eighth seven-year-old" (Gruszczyk-Kolczyńska, 2012, p. 50). These studies also confirm the importance of people from the closest environment to stimulate mathematical aptitudes.

As we can see, an adult can contribute a lot to developing a child's abilities. So he should give him enough time and attention. There is the problem of the fastest possible detection of the mathematical aptitude requirements, as well as adequate support in development, as an adult can effectively suppress this development. Congenital inheritance is the basis for the development of talents.

They are also of a general nature and can develop in various directions under the influence of education and education during specific activities of the child. The effect of developing and nurturing an innate gift in a given direction is the formation of special talents in a given field (Gruszczyk-Kolczyńska, 2012, p. 23).

18 Meaning and time of the mentioned milestones in shaping careers in the line of life: that is, about the formation of scientific careers

In this part of the paper I focus on a detailed interpretation of the existing regularities in the lines of life – the biographical axes of the time of the examined people. I analyzed the fate of extremely gifted mathematicians who lived approximately over the last eighty years.

18.1 Significant people and stimulation of talents

In all groups, significant people were important as milestones. I have observed a very clear influence of people coming from the immediate surroundings of the subject (i.e. family, friends, etc.) and teachers, academic lecturers, great scientific authorities – masters. All the respondents pointed to a person or

a few people during their lifetime (especially from the immediate environment), which had a direct relationship with directing their mind towards mathematics.

It is very important to see the mathematical aptitudes in early childhood (Gruszczyk-Kolczyńska, 2012, p. 50). According to the Gruszczyk-Kolczyńska study: "mathematically gifted children can be counted as every fifth five-year-old, every fourth six-year-old and only every eighth seven-year-old" (Gruszczyk-Kolczyńska, 2012, p. 50). These studies also confirm the importance of people from the closest environment to stimulate mathematical aptitudes.

As we can see, an adult can contribute a lot to developing a child's abilities. So he should give him enough time and attention. There is the problem of the fastest possible detection of the mathematical aptitude requirements, as well as adequate support in development, as an adult can effectively suppress this development. Congenital inheritance is the basis for the development of talents. They are also of a general nature and can develop in various directions under the influence of education and education during specific activities of the child. The effect of developing and nurturing an innate gift in a given direction is the formation of special talents in a given field (Gruszczyk-Kolczyńska, 2012, p. 23).

18.2 Competitions and prizes – supporting the development of talents

One of the methods is the diagnosis of talents and the second is the extraction of talented children by mathematical school and inter-school competitions, mathematical olympiads. High effort is associated with achieving high results. To cope with it, the child must have developed urgency. The sense of urgency in children plays a huge role in home, pre-school and school education. According to Gruszczyk-Kolczyńska, children who have such a capacity, experience pride, satisfaction and joy when they perform the task, and above all see the sense in the activities performed, similarly to athletes.

Teachers should also spread knowledge about the nature of mathematical aptitudes. It is very important to make all teachers aware that general abilities are distinguished – in relation to intellectual capabilities (for example, measured by an intelligence test) and special abilities (conditioning success in a specific field) (Gruszczyk-Kolczyńska, 2012, p. 22).

As I have previously noticed, the second way to search for talents among children is to search for them through mathematical competitions. In the course of life of the people I studied, very often there was a category of winning mathematics competition that provoked further actions in the field of mathematics. Often the mere awareness of winning resulted in satisfaction and

willingness to act in this area.

The phenomenon of rivalry motivated the desire to win and achieve the best results. In some cases, the winning competition (related to the random participation) strengthened the willingness to develop interests. Of course, the case concerns mathematicians who had the opportunity to take part in these competitions. The status of the laureate, or winning a medal, gave a very large positive boost. The rewards received were a great payment for the effort put in, but also a motivating factor and confirming the real achievements. It should be remembered that this is a very important component of supporting the development of mathematical aptitudes.

18.3 Time to start a scientific activity

Analyzing the time axes of outstanding mathematicians over the years, I noticed that each group (starting with PhD students) begins its scientific career later and later. This involves some complications at the interface: mental development and the education system. Currently, it is impossible to complete studies before the age of 25.

Meanwhile, outstanding mental capabilities in the field of operational thinking at the formal level will end around 30 years of age. M. Spitzer wrote about it (Spitzer, 2012, p. 202): ground-breaking mathematical discoveries and Young people performed physical exercises. He gave various examples, showing that scientists in the exact sciences can make phenomenal discoveries only at a certain age. This means that only five years remain for scientific activity. This is a very short time.

The theory described above has its justification in developmental psychology and more specifically the cognitive development of man (Nęcka, Orzechowski, Szymura, 2013, pp. 429-430).

This is also shown by the results of my research, which clearly show that greater and more frequent scientific achievements were attributable to the period of teenage or early youth. In particular, the oldest group of non-surviving eminent professors can be proud of achieving significant scientific successes at an early age. Some professors themselves pointed to this fact.

Subsequent generations with the passage of years and the prolongation of formal education could not fully develop their mathematical aptitudes. Sitting in a school bench, lecturing at a university, despite all the advantages, does not give you the freedom to create, as your own scientific work. Young people today are focused on getting a formal education.

18.4 Specific attitude to scientific authorities

A very interesting phenomenon observed by me is the problem of disappearance of authorities. The analysis of the course of life of eminent mathematicians shows a tendency of a gradual decrease in the influence of authorities on scientific development. Only one of the PhD students surveyed points to their scientific master who helped him achieve some scientific goal. For many young people, however, the institution itself is much more important. Speaking of the milestones in their lives, they often pointed to foreign trips, scientific internships, and specific scientific centers that were to ensure their scientific development. One can guess that a successful scientist who in some way enabled him to develop a career was often the success of a young man.

Meanwhile, young PhD students do not mention their masters. The reason for this is, on the one hand, the lack of time and commitment of professors who are absorbed in their scientific work and have no time to help and support PhD students.

For this reason, young people may not feel scientific gratitude. On the other hand, young scientists may not recognize the value of the relationship, underestimate the time devoted to them and the opportunities for discussion, polemics and scientific development that other people create. It is puzzling, however, that even today's mathematical publications show a significant difference.

In the library of the Polish Academy of Sciences in Warsaw, I found mathematical publications from the turn of the century. The names of people are visible on them, scrupulously referred to in citations. On the title pages – extensive thanks and dedications. At the latest hearings there are no such dedications.

19 Summary of test results

My research on "Milestones in the course of life of outstanding mathematicians and mathematically talented youth" covers the period of about last eighty years. Research has shown that huge changes have taken place over the years:

- During the beginning of a scientific career: deceased Professors of mathematics began their scientific activity after the age of 20, living professors – a few years later, contemporary PhD students do not start their scientific activity until the age of 30 (they do not say anything about important milestones in their lives, publications and articles).

- The time to start a scientific activity is very important if it is considered psychologically in terms of the stages of cognitive development of the mind. Period conducive to the development of scientific activities in exact sciences is 20-30 years. If by that time a man did not undertake scientific activities in exact sciences, it is very likely that he will not take it any more.

- Changes in the master-student relationship. Already during the interviews with mathematicians, I noticed some differences in the fragments concerning the statements in which the respondents explained to me who contributed to the development of their talents. Dedications posted in mathematical works are also important, because in these dedications one can read a close relationship between the master and the student. To my surprise, PhD students and PhDs only pointed to the closest family and teachers from primary or secondary school. They focused more on institutions that enabled study visits, while mathematicians from the first group entirely referred to valuable scientific authorities. Young adults, while studying or even travelling abroad, had to meet outstanding personalities, although they did not consider it appropriate to talk about them.

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Kamienie milowe w biegu życia wybitnych matematyków i uzdolnionej matematycznie młodzieży

S t r e s z c z e n i e

W niniejszym artykule przedstawione zostały wyniki badań nad kamieniami milowymi (wydarzeniami znaczącymi, punktami krytycznymi, doświadczeniami krystalizującymi) w biegużycia wybitnych matematyków i uzdolnionej matematycznie młodzieży. Badania obejmująokres ostatnich około 80 lat. W tym czasie matematycy z czterech grup kształcili się i zdobywali naukowe osiągnięcia: – nieżyjacy wybitni profesorowie matematyki, – żyjący wybitni profesorowie matematyki, – doktoranci i doktorzy wydziałów matematycznych, – laureaci olimpiad matematycznych Zamiarem autorki było wskazanie kamieni milowych, a więc ustalonych przez autorkę (lub wskazanych przez samych zainteresowanych) kluczowych wydarzeń i momentów w ich historii dochodzenia do najwyższych godności i uznania w dziedzinie matematyki. Są to na przykłaad ważne doświadczenia w życiu człowieka, które odegrały ogromną rolę przy wyborze matematyki jako kierunku dalszego rozwoju, lub utwierdziły w przekonaniu, że matematyka jest odpowiednim wyborem.

Rozważania oparte zostały o holistyczne, humanistyczne podejście oraz metodę biograficzną w ujęciu Charlotte Bühler „Bieg Życia ludzkiego". Techniki jakie zostały wykorzystane to analiza dokumentów oraz wywiady narracyjne. Zgodnie z zaleceniami Ch. Bühler wyniki studiów zostały przedstawione w sposób graficzny na osiach czasu.

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