ANNALS OF THE POLISH MATHEMATICAL SOCIETY
5TH SERIES: DIDACTICA MATHEMATICAE 37 (2015)

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Problem Posing as a Learnable Activity

Abstract: Compared with problem solving, students' ability to pose problems in general, and its connection to mathematical creativity in particular, has received less attention. The current study aims at investigating how instructing students in posing problems influences mathematical creativity, as expressed in (1) their ability to pose problems, and (2) their ability to solve problems. To investigate these issues, pre- and post-tests on solving and posing problems were given to two groups of mathematically advanced students. The findings supported our initial assumption that systematic teaching of posing problems can lead to some improvements in students' ability to modify a given problem, and indirectly improve their ability to produce more creative solutions of the given problem. Differences between the two groups may relate to the students' mathematical background.

Introduction

Both problem-solving and problem-posing abilities are considered to be indicators of mathematical creativity: "Problem posing, along with problem solving, is central to the discipline of mathematics and the nature of mathematical thinking" (Silver, 1997, p. 75). The connection between the ability to solve problems and creative mathematical thinking has been widely investigated (Karp, & Leikin, 2011; Leikin, & Berman, 2010). Recently, students' ability to *pose* problems in general and its connection to mathematical creativity in particular, has received increasing attention (for example, Singer, Ellerton, & Cai, eds., 2013).

Key words: creative mathematical thinking; what-if-not strategy; problem posing; problem solving; fluency, flexibility and originality

Silver (1997) claimed that when professional mathematicians do mathematics, it involves being creative with mathematics; but on the other hand, creativity has little to do with school mathematics. "It is argued that inquiry-oriented mathematics instruction which includes problem-solving and problem-posing tasks and activities can assist students to develop more creative approaches to mathematics." (p. 75). In their book on "The Art of Problem Posing", Brown and Walter (1983) suggest that in order to achieve this goal, teachers' ability to pose problems should become an important asset in their everyday practice. They claim that the ability of problem posing can be developed systematically through the "What-If-Not?" teaching strategy presented in their book.

Inspired by Brown and Walter's ideas, we designed and implemented a short learning unit for school mathematics with two different groups of students. The aim of the current study is to investigate how instructing students in posing problems influences their mathematical creativity, as expressed in (1) their ability to pose problems, and (2) their ability to solve problems.

Theoretical background

Research into the field of creativity in general, and mathematical creativity in particular, has become a growing arena for innovative research – as evidenced, for example, by two recently published special issues (Leikin & Pitta-Pantazi, eds., 2013; Singer, Ellerton, & Cai, eds., 2013). Interestingly, these research efforts did not yield one acceptable definition of creative mathematical thinking in general, or of creative mathematical thinking in the context of school mathematics, in particular (Mann, 2006). Nevertheless, there are some accepted characteristics for creative mathematical thinking. For example, Guilford (1967) linked creative thinking with divergent thinking (or production). Divergent thinking involves the creative generation of multiple answers to a particular problem (in contrast to convergent thinking, which aims at a single, correct solution).

Torrance (1974) identified creativity in general by specifying three components: *fluency* refers to the number of ideas generated in response to a prompt; *flexibility* is assessed by the number of apparent shifts in approaches taken when generating responses to a prompt and *novelty* considers the originality of the ideas generated in response to a prompt. Balka (1974) adapted from Torrance (1974) the notions of fluency, flexibility, and novelty as measures for general creativity. He applied these measures to the domain of mathematics by asking subjects to pose mathematical problems that could be answered on the basis of information provided in a set of stories about real-life situations.

In his analysis of students' responses, he measured fluency by assessing the number of problems posed or questions generated, flexibility by counting the number of different categories of problems generated, and originality by assessing the rarity of a response, as compared to the set of all responses. Getzels and Jackson (1962) assessed the creativity of their subjects according to the complexity of the procedures needed (i. e., the number and type of arithmetic operations used), in order to obtain a solution to their posed problems. Thus, fluency, flexibility, and novelty are considered core dimensions of creativity (e.g., Presmeg, 1981).

Creative mathematical thinking for mathematicians can be thought of as "the ability to produce original work that significantly extends the body of knowledge" or "opens up avenues of new questions for other mathematicians" (Liljedahl & Sriraman, 2006, p. 18). However in the context of school mathematics, one can hardly expect this level of creativity in students' contributions. In school mathematics, the idea of relative mathematical creativity is more in order. Leikin (2009) suggests that mathematical creativity in school should be evaluated with reference to students' previous experiences and to the performance of other students with a similar educational history.

The issue of relativity needs to be considered also with regard to the nature of the involved mathematical problems. Schoenfeld (1989) defined a school mathematical problem as "a task (a) in which the student is interested and engaged and for which he wishes to obtain a resolution; and (b) for which the student does not have a readily accessible means by which to achieve that resolution" (pp. 87–88). That is, the same task may function as a mathematical problem for one student and as an algorithmic problem for another student. Leikin (2009) claims that an explicit request to solve a problem in multiple ways can be used both to elicit students' creative mathematical thinking and to evaluate it.

Several researchers employed different methods to assess creative mathematical thinking. Recent studies have used fluency, flexibility, and novelty as a means to assess the level of students' creativity at various grade levels, while solving problems. Kattou et al. (2013) studied problem solving at the elementary school level, and indicated possible links between mathematical creativity and their mathematical ability. A confirmatory factor analysis also suggested that mathematical creativity is a subcomponent of mathematical ability. Tabach and Friedlander (2013) studied fourth to ninth grade students and found that at the elementary school level, the number of solution methods and creativity scores increased with age. At the middle-grade level, the solution methods produced by the eighth graders seemed to narrow almost exclusively to algebraic strategies, and to increase again at the ninth-grade level. Leikin and

Lev (2013) conducted a large-scale study that explored the relation between mathematical creativity and the mathematical ability of high-school students. They found that in addition to dependence on mathematical ability, levels of creativity also depend on the mathematical tasks at hand. Pitta-Pantazi, Sophocleous, and Christou (2013) investigated the relationship between the creative process in mathematical tasks and spatial, object, and verbal cognitive styles of a group of 96 prospective primary school teachers. The results of a multiple regression analysis employed in this study indicated that visual cognitive styles (spatial and object imagery) were statistically significant predictors of participants' creative abilities in mathematics, but verbal cognitive style did not predict these abilities.

As indicated in the introductory section, in addition to problem solving, problem posing can be another means to promote and evaluate creativity. Mathematical problem posing was defined by Stoyanova and Ellerton (1996) as the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and from these situations formulate meaningful (i.e., non-trivial) mathematical problems. They categorized three types of problem-posing situations: *free,* when students are asked to generate a problem from a given, contrived, or naturalistic situation; *semi-structured*, when students are given an open situation and are invited to explore the structure of that situation, and to complete it by applying knowledge, skills, concepts, and relationships from their previous mathematical experiences; and *structured,* when problem-posing activities are based on a specific problem. (For examples of the three types of problem-posing situations, see Van Harpen & Presmeg, 2013.)

However, studies that used fluency, flexibility, and novelty as a means to assess problem posing are less common. Voica and Singer (2013) asked fourth to sixth graders with above-average mathematical abilities to modify a given problem (i.e., a structured problem-posing situation). They found evidence of links between problem-posing abilities and cognitive flexibility. They also reported that students' capacity to generate coherent and consistent problems in the context of problem modification may indicate the existence of a strategy for generalizations, which seems to be specific to mathematical creativity. Others studied problem-posing activities by other means. Cai et al. (2013) used problem posing in a structured type situation to measure the effect of the middle-school curriculum on students' levels of achievement in high school. Bonotto (2013) employed a problem-posing situation of a semi-structured type to investigate connections between teaching methods and student activity. Singer and Voica (2013) investigated the process of problem solving in order to drive a theoretical framework for designing problem-posing tasks to promote students' learning.

In the current study, we employed an intervention based on both problem posing and problem-solving activities and considered its effect on student creativity as measured by its three components of flexibility, fluency, and novelty.

Methods

Research questions

The current study is guided by the following two connected questions: Can a short-term, focused instruction in problem-posing based on the "What-if-Not?" strategy increase sixth, and seventh-grade students' creativity, as expressed in (1) their ability to pose problems, and (2) their ability to solve problems?

Since the "What-If-Not?" strategy employed in this study involves identifying and modifying the underlying structure of a given problem, we expected a stronger influence on students' measure of flexibility – as compared with the measures of the other two components of creativity.

Participants

The participants in the study consisted of two groups of students who were identified by their teachers as having a higher mathematical ability within their cohort of eighty students. One of the groups consisted of 13 seventh-grade students. In their regular classes, these students participated in a first-year beginning algebra course, and throughout the month of this study, they were introduced to the topic of linear equations. Thus, at this stage, the students' proficiency in formal symbolic algebra was quite limited. The second group of students consisted of 22 sixth-graders. In their regular classes, these students participated in the last year of an elementary mathematics course that was mainly focused on decimal numbers, ratios, proportions, and percent.

Research design

The study was conducted during five 90-minute-long lessons – including a 90-minute pre-test and a 45-minute post-test administered during the first and the last lessons, respectively. The lessons were video-recorded, and the students' work was collected. Chronologically, the seventh graders were the first to participate in the course. Owing to the second group of sixth graders' different mathematical background, several adaptations were made especially for them. Next, we describe the intervention conducted with the first group, and the modifications made for the second group.

Pre and Post-tests

Pre-test

During Lesson 1, a multiple solution problem (Leikin, 2009) was administered to students, with an explicit request to provide as many solution strategies as possible (Figure 1). Next, the students were asked to modify the given problem and create new problems, based on the problem they had just solved. To ensure that the students were creating their new problems in a responsible manner, they were also required to provide a solution for each of their posed problems (see also Voica & Singer, 2013).

Jonathan was born when his mother was 28 years old.
After several years, Jonathan noticed that his age is one third of his mother's age.
How old were Jonathan and his mother at that stage?

1. *Please solve the problem in many different ways.*
2. *Create other problems about relations among ages.*
   Give a solution for each of your new problems.
   Note: We recommend going beyond changes in the numbers given in the original problem.

Figure 1: The Grade 7 pre-test problem*.

* In Grade 6, the numbers were slightly different – the mother's age was 29, and Jonathan's age was half that of his mother.

Post-test

The second half of the fifth lesson was devoted to the post-test (Figure 2). In

Two seventh-grade classes had 10 and 38 students, respectively.
After the same number of students was added to both classes, the number of students in the second class was three times as large as in the first class.
How many students were added to each class?

1. *Please solve the problem in many different ways.*
2. *Create other problems about students and classes.*
   Give a solution for each of your new problems.
   Note: We recommend going beyond changes in the numbers given in the original problem.

Figure 2: The post-test problem for both groups.

order to enable a pre-post comparison of students' work, the pre- and post-test problems were similar in their underlying mathematical structure, but they were set in two different contexts. In both tests the students were required to follow the same activity sequence: first, solve the given problem in several ways, and then pose their own related problems and provide one solution for each.

As noted before, the pre and post-test problems had the same mathematical structure (or attributes, according to Brown & Walter, 1983):

- two initial quantities are given
- the same quantity is added to both
- the ratio between the two changed quantities is given
- the solver is required to find the quantity that was added.

Note that although the two problems have a common underlying structure, the contexts of the two problems may slightly influence the students' potential to pose new problems: the context of the post-test problem allows one to add or remove students, or to move several students from one class to another, whereas the context of age in the pre-test problem allows only the addition of the same number.

Intervention

Brown and Walter's (1983) principles provided a basis for designing three and a half lessons on problem posing. The lessons were conducted on a weekly basis, during regular school hours by the second author as the teacher of the two groups. (The first author was also present at all lessons.) Each lesson followed the same sequence of activities: the teacher presented a problem, and asked the students to solve it. After about ten minutes of student work and the teacher's monitoring and providing occasional help, the class reconvened and several students presented on the board and explained their solution methods. The class was required to compare and decide whether some of the presented methods were different or equivalent to each other. At the next stage, the focus shifted to problem posing, and the lesson followed the same sequence of student work (this time, modifying the initially given problem) followed by class discussion (presenting and analyzing some of the student-posed problems).

The first given problem was the same for the two groups, whereas the other problems given to the sixth-grade group were less complex as compared with those given to their older peers. We describe here the sequence of problems given in Grade 7.

Lesson 1.

The students were required to solve a very simple initial problem (Figure 3). After discussing its solution, they were asked to modify the initial problem. Two modified problems (see Figure 4) were presented on the board by their authors, and their structure was discussed and analyzed.

Four girls bought a present costing 28 Shekels, and shared the cost equally among themselves. *How much did each of them pay?*

Figure 3: The initial problem given in Lesson 1.

The following discussion focused on the suggested modifications to the initial problem. The group concluded that, as compared to the initial problem, the modified problem suggested by the first student had the same context but a different mathematical structure, whereas the second problem was based on a changed context, but had the same underlying structure.

a. Several girls bought a present costing 132 Shekels. Each of them paid 12 Shekels. *How many girls bought the present?* *b. Six people robbed a bank. They took 6,000,000 Shekels and shared it equally among themselves. How much did each of them get?*

Figure 4: Problems posed by students in Lesson 1.

Next, other students suggested their modified problems, and the group was required to identify the modified parts of the original problem.

Lesson 2.

In this lesson, the students were given a more challenging initial problem (Figure 5). The solution of the given problem allowed students to understand the pricing strategy. Next, students were asked to create similar problems. At first they tried to modify the rule for calculating the children's fare. Then both students and the teacher suggested problems based on modifying other characteristics of the initial problem – for example, given the total amount paid by one adult and one child, find several possibilities for the price of each ticket. The presentation of each modified problem was followed by a discussion of its characteristics. Finally, the students were asked to write another modified problem and solve it.

On a train station in India the following sign explained the pricing policy regarding children's train fare:
Children over five and under twelve years of age will be charged half of the adult fares and this will be rounded up to the next higher multiple of 5 Rs [Rupees].
Children 12 years of age and above will be charged the full fare.

Find the corresponding children's fare:

Adult	*Child*
20 Rs	
24 Rs	
15 Rs	
242 Rs	

Figure 5: The initial problem given in Lesson 2.

Lesson 3.

A list of all the modified problems submitted by the students at the end of Lesson 2 was handed out and discussed at the beginning of Lesson 3. The discussion focused on two of these problems (see Figure 6) that were selected and presented by the teacher.

a. *The Levi family (two adults, a four-year-old boy, a 10-year-old girl, and a 13-year-old boy) had a total sum of 72 Rs for their train trip. Will they be able to buy tickets for all the family, given that:*
The price of an adult ticket is 25 Rs, a ticket for a child 12 years old and above costs a quarter of an adult's fare, rounded up to the nearest multiple of 7, a ticket for a child under 12 years old is half the price of an older child rounded off to the nearest multiple of 4, and a ticket for a child under 5 years old is a fifth of an adult's fare.
Do they have enough money?

b. *A child's fare is half that of an adult, rounded up to the nearest multiple of five. The father noticed that his ticket is 1.5 times more expensive than that of his son. How much did the father pay for the two tickets?*

Figure 6: Problems posed and submitted by students at the end of Lesson 2.

The teacher asked the students to solve the two problems, and decide which problem is simpler and why. The group concluded that even though the conditions of the first problem are more complex, its solution is a straightforward arithmetical one. On the other hand, the conditions of the second problem are relatively simple, but its solution is more complex, since it requires reverse thinking and possibly an algebraic solution.

Next, a new initial problem was presented and solved (Figure 7). Note that at this stage, the students had no formal knowledge of multiplying binomials. The students produced two-solution strategies. The first strategy employed trial and error, and the second involved drawing an x by x square and an $(x+4)$ by $(x+1)$ rectangle, and writing the equation $(x+1)(x+4) = x^2+34$ without being able to solve it.

The two pairs of parallel sides of a square were lengthened by 4 cm and by 1 cm, respectively. As a result, the area of the newly obtained rectangle was larger by 34 cm^2 *than that of the original square.* *What was the area of the initial square?*

Figure 7: The initial problem given in Lesson 3.

Again, students were asked to write a modified problem and solve it. Then, some of the suggested problems were discussed, and other modified problems were collected and discussed during the first half of the next lesson.

Throughout the intervention, the following problem-posing principles were considered and discussed – first implicitly, and then at a later stage, explicitly.

- A posed problem should include all the necessary data – including the unchanged characteristics of the initial problem.
- Changing a problem's numerical data, but leaving its structure unchanged will not be considered a modified problem.
- Creating "fancy" imaginative changes in the context, but leaving its structure unchanged will not be considered a modified problem.
- Modified problems should be analyzed and compared in view of their solution. Usually, a problem that requires a solution based on numerical computations or "direct reasoning" will be considered less complex than one that requires "reverse reasoning" or an algebraic solution.
- The omission or the addition of data to a problem may lead to a new multiple-solution problem – a worthwhile modification strategy in itself.

Data Analysis

In order to examine possible changes in students' posed problems, we present first a case study of a student's performance on the two tests. In addition, students' creativity in *problem pos*ing was analyzed quantitatively by the following three group averaged measures.

1. Fluency – measured by the average number of modified problems posed per-student.

2. Flexibility – measured by the average number of modified problems having a different mathematical structure, as compared to the initially given problem posed per-student

3. Novelty – measured by the average number of unconventional modifications of the initially given problem posed per-student.

Similarly, students' creativity in *problem solving* was analyzed according to the following three measures (each of them averaged for the size of the group):

1. Fluency – measured by the average number of solutions of the initially given problem produced per student.

2. Flexibility – measured by the average number of different solution methods of the initially given problem produced per-student.

3. Novelty – measured by the average number of unconventional solutions (as opposed to the more commonly used solution methods) of the initially given problem produced per-student.

In addition to these measures of creativity, we also analyzed the students' strategies employed in the solutions of the two tests and related them to one of the following categories: random trial, methodical trial, calculation, quasi-symbolic, symbolic, visual – number line, visual –graphical, mixed symbolic and trial, mixed reasoning and trial, and verbal reasoning.

Findings

In this section we compare the pre and post findings regarding (1) modified problems posed by the students when given a problem, and (2) the students' solutions of the given problem. In both tests and throughout the teaching sequence, the aspect of problem solving preceded the aspect of problem posing. However, since this study focused on students' ability to pose problems, the findings are presented in reverse order.

Problems posing

As an illustration of pre-post differences in problem-posing creativity, we present first the pre- and post-test problems posed by a seventh-grade student (Fig. 8). In this particular case, the context of both pre-test problems was modified by adding a person, but their mathematical structure and context remained unchanged. Moreover, due to some missing information (the mother's age at Jonathan's birth) the first pre-test problem cannot be solved.

a. Pre-test

(1) *Jonathan's little brother was born when his age was three times less than that of his mother two years later. Fifteen years had passed. What was Jonathan's age when his brother was born?*

(2) *Jonathan and his twin brother Moses were born when their mother was 28 years old. Jonathan married when his mother was twice his age. Five years later, his son was born and his brother got married. When did Moses marry?*

b. Post-test

(3) *25 students participate in the art class and 11 students in the drama class. After a while, the same number of students left both classes. Yossi noticed that the number of students who left each class was half the average number of students in both classes plus the sum of digits of the initial number of students in the drama class. How many students were left from each class? Which class ceased to exist?*

(4) *Two circles have a radius of 10 and 3. The lengths of both radiuses are increased by the same number, and as a result, the area of the first circle is 102 times larger than that of the second circle. What was the change in each radius?*

Figure 8: The pre-test (a) and post-test problems (b) posed by one student.

A pre-post comparison of the posed problems indicates that the two post-test problems are of a better quality: the mathematical structure of the first modified problem was created by introducing new mathematical terms (the average and sum of digits), the mathematical context of the second problem was created by changing it to geometrical figures, and this time the problems provide all the required information. In addition, as compared to the pre-test problems, the structure of both problems requires a more sophisticated solution based on reverse thinking.

Next, we present a quantitative analysis of the data collected from the two groups. Note that the limited size of the two groups did not allow an elaborate statistical analysis.

Table 1 summarizes the test data regarding per-student average measures of creativity in *posing problems*. A pre and post-test comparison indicates similar trends for the two groups. More specifically, in comparison with the pre-test.

- A slight increase in fluency (the average number of posed problems) was found for both groups.
- As compared with the other two measures, the increase in flexibility was the largest for both groups – i.e., on the average, more problems having a different structure were posed.
- A slight increase in novelty measures was found for both groups.

Criteria	Problems per student grade 6		Problems per student grade 7	
	Pre-test (18 students, 39 problems)	Post-test (22 students, 65 problems)	Pre-test (12 students, 28 problems)	Post-test (13 students, 32 problems)
Fluency	2.16	2.95	2.33	2.46
Flexibility	1.34	2.32	1.58	2.15
Novelty	1.11	1.23	0.25	0.46

Table 1: Pre- and post-test per-student average* measures of creativity in problem posing.

*The fluctuations in group size are due to students missing in test-days.

Problem solving

Next, we present a quantitative analysis of the problem-solving data collected from the two groups. Table 2 summarizes the pre-post data regarding per-student average measures of creativity in solving problems. The data presented in Table 2 indicate a slightly different picture for each group: slight increase in flexibility measures and a notable increase in novelty were detected for sixth graders, whereas the data of the seventh graders showed a slight increase in measures of fluency and stability in measures of flexibility and novelty.

Criteria	Methods per student in Grade 6		Methods per student in Grade 7	
	Pre-test (18 students, 39 solutions)	Post-test (22 students, 46 solutions)	Pre-test (12 students, 24 solutions)	Post-test (13 students, 35 solutions)
Fluency	2.05	2.09	2.00	2.70
Flexibility	0.37	0.45	0.58	0.54
Novelty	0.21	0.41	0.33	0.31

Table 2: A comparison of pre-post per-student* creativity measures of problem-solving.

*The fluctuations in group size are due to students missing in test-days.

We also analyzed the students' problem-solving strategies employed in the two tests. Table 3 presents the categorization of each group's pre and post-test problem-solving strategies. The data indicate that in each test the students employed various solution strategies. We noted the following trends of the more frequently used categories of solution methods (see the shaded cells in Table 3):

- For both groups and at both stages, systematic trial was a dominant strategy, and it was employed by more than half of the sixth graders.
- At the post-test stage, the symbolic solution became the most frequently used method among the seventh-graders, and it replaced the most widely employed method of verbal reasoning at the pre-test stage.
- The variety of solutions increased from five to seven methods for the sixth-graders, whereas it remained stable at seven methods for the seventh-graders.

	Grade 6 students		Grade 7 students	
	Pre test	Post test	Pre test	Post test
Trial & error	–	9	9	6
Systematic trial	39	56	31	37
Numerical calculations	26	9	–	–
Quasi-symbolic	–	4	4	–
Verbal reasoning	20	13	35	11
Symbolic	–	–	17	40
Visual one axis	10	7	–	–
Visual Area/Graph	5	2	–	3
Mixed trial & error/symbolic	–	–	–	3
Mixed trial & error/reasoning	–	–	4	–

Table 3: Pre and post-distributions of categories of problem-solving strategies (in percent)*.

*Shaded cells indicate relatively high percentages.

Discussion

Brown and Walters (1983) claimed that in mathematics, problem solving and problem posing are intimately connected, and consequently based their problem-posing strategy on this connection. The present study followed the same path, and investigated students' ability to solve problems as well. The questions that guided the current study were: Can short-term, focused instruction in modifying given problems using the "What-if-Not?" problem-posing strategy increase sixth-grade, and seventh-grade students' abilities (1) in creative problem posing, and (2) in creative problem solving?

The scope of this study was quite limited both in the number of participating students and in the duration of the instructional intervention. However, we can conclude that students can be exposed successfully at a relatively young age to tasks that require explicitly to pose problems. Our findings and experience throughout the intervention supported our initial assumption that explicit and focused teaching of problem posing can improve some aspects of students' creativity in modifying a given problem, and indirectly, improve their creativity in solving problems.

The instructional intervention was based on the "What-If-Not?" problem-posing strategy and consisted of identifying first the underlying structure of a given problem, and then modifying one or more of its components. As a result, we aimed at improving mainly the students' flexibility in posing new problems. The collected data showed that, as compared with the pre-test, the post-test *problems posed* by the students were more varied in their mathematical structure and context, whereas the other components of creative mathematical thinking, namely, fluency and novelty, improved slightly. Thus, in terms of creative problem posing, we found that the measures of student flexibility were influenced the most by the intervention.

A comparison of our pre-post test data on *problem solving* indicated that students' creativity measures were almost stable. This may be due to the short duration of the intervention, and/or to the focus of the intervention on modifying problems, rather than on their solution. The seventh-graders' relative increase in problem-solving fluency can be attributed to an expansion of their repertoire of problem-solving strategies as a result of their beginning to learn algebra during the period of this study – as compared with their sixth-grade peers who could employ at this stage only arithmetical methods. These findings are also coherent with our previous study (Tabach & Friedlander 2013), in which we found that acquiring algebraic tools can be detrimental to students' creativity in problem solving. Similarly, Levav-Waynberg and Leikin's (2012) study on multiple solution problems found only a partial influence on students' creativity in problem solving. Their results indicated a significant change in fluency and flexibility measures, but no significant differences with regard to novelty measures.

In view of these findings, we recommend: (1) to include problem posing as a standard requirement in mathematics courses, and (2) to conduct further research with regard to the potential of posing problems to facilitate and enhance mathematical creativity.

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Tworzenie zadań jako aktywność, której można się nauczyć

Streszczenie

Umiejętność budowania problemów (a w szczególności związek takiej umiejętności z kreatywnością matematyczną) zajmuje mniej uwagi badaczy niż zagadnienie umiejętności rozwiązywania zadań. W przedstawionym studium głównym problemem badawczym było przeanalizowanie w jaki sposób kształcenie uczniów w tworzeniu problemów wpływa na ich matematyczną kreatywność wyrażoną jako: (1) ich umiejętność tworzenia zadań, (2) ich umiejętność rozwiązywania zadań. Aby zbadać to zagadnienie, w grupie uzdolnionych uczniów zostały przeprowadzone dwa testy: wstępny, a następnie oraz po serii zajęć dotyczących rozwiązywania i tworzenia zadań test kontrolny. Wyniki testów potwierdzają nasze wstępne przypuszczenie, że systematyczne prowadzenie zajęć dotyczących stawiania problemów i tworzenia zadań może doprowadzić do pewnej poprawy uczniowskich umiejętności w modyfikowaniu zadanego zagadnienia. Może też pośrednio wpływać na poprawę umiejętności w tworzeniu bardziej kreatywnych rozwiązań. Różnice między dwiema obserwowanymi grupami mogą być związane z matematycznymi kompetencjami badanych uczniów.