ANNALS OF THE POLISH MATHEMATICAL SOCIETY
5TH SERIES: DIDACTICA MATHEMATICAE 37 (2015)

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Introducing additive compare problems – traditional vs. constructivist approach

Abstract: In the article I have presented a proposal to amend the existing methodology of teaching to solve additive compare word problems by introducing a curriculum based on the constructivist approach.

When the sets are not equal and one set has more elements than the other, a surplus of is visible and one can ask a natural question: How many more?

The results of the constructivist introduction to teaching additive compare problems proved to be optimistic. Almost all pupils succeeded in simple situations, answering the question: How many more?, although they had not studied these issues at school. This suggests that the pupils beginning school education already possess some extracurricular personal knowledge on the subject.

1 Introduction

Additive compare problems[1] have appeared in several Polish nationwide surveys of the learning outcomes in the elementary school: in the test in Grade 6 organized by the Central Examination Board, in the Study of Basic Skills in Grade 3 (Dąbrowski, 2007, 2009), and in the "Improved School" study organized by GWO, Gdańsk Educational Publisher (Demby, 2011).

It is well known that compare problems are specifically difficult (Nesher, Greeno & Riley, 1982; Kinitsch & Greeno, 1985; Valentin & Chap Sam, 2004).

[1] In this paper we restrict ourselves to compare problems involving only one arithmetic operation. Complex problems involving multiple operations are not be discussed here.

Sources of these difficulties may be of two origins: difficulties that are inherently rooted in the nature of these problems[2] and those which are caused by inadequate teaching methods.

The main objective of this paper[3] is to investigate the following hypotheses:

(1) An important factor in unsatisfactory learning outcomes of these problems are the traditional teaching methods.

(2) The outcomes may be significantly improved by using methods based on a constructivist approach.

2 Background: traditional vs. constructivist approach to compare problems

The term "traditional methods" means here the standard methods concerning compare problems as described in Polish publications for teachers and used in textbooks for children. Their key features include:

(α) It is expository teaching, taking as a starting point a paradigmatic example of a compare problem, sometimes accompanied by an appropriate illustration. The teacher shows how a typical phrase such as "is greater by 3" or "is smaller by 3" should be converted to the corresponding arithmetic operation and how to solve the problem and formulate a response.

(β) Teaching begins with the additive types of tasks (i.e. the tasks defined in the curricula as "so many more", e.g. Jim has 5, Bill has 3 more than Jim, how many does Bill have?. or "so many less"), regarded as the easiest type serving as an introduction of terminology. Then the teacher moves to the major types of problems ending with questions "How many more?", "How many less?". These two types are regarded as much more difficult than the two preliminary types and are explained by inverting the given relation and the corresponding arithmetic operations.

(γ) Teaching is implemented in blocks of several hours. One day the teacher starts a topic new for the students: the additive compare problems, and on the following days one-operation problems are being solved, according to the increasing degree of difficulty[4].

[2] Those difficulties have been described in (Mrożek, 2010).

[3] Results presented here were part of the PhD thesis presented at the Pedagogical University in Cracow on 9 May 2013. The supervisor was Prof. Zbigniew Semadeni.

[4] Sometimes, such a block is divided into two parts: first the types of tasks containing

The term "constructivism" has several interpretations, based on different philosophical foundations[5]. We do not accept its extreme interpretations and adopt an approach derived from Piaget's genetic epistemology and moderate social constructivism in Vygotsky's tradition, as in Sfard (1998, p. 498) and Nunes (2004, p. 58).

The main point of constructivist approach is that mathematical knowledge cannot be directly transferred to the student by the act of teaching, but it must be actively constructed by the student in his/her mind through suitable activities performed by himself/herself (usually directed by the teacher), thinking of the results and discussing them with others. The construction of knowledge is based on gathering experience, comparing the results, gradually noticing some regularities and checking them — this statement, formulated for older students (Demby, 2000, p. 70), applies (with suitable modifications) also to the case considered here.

An essential element of the constructivist approach is the creation of meanings by the subject in the learning process. In our case this applies to compare problems.

According to Aebli (1982, p. 25), the teacher should not wait until the child's mind "captures a copy" of the demonstrated paradigmatic example, but should make the student actively deal with the given data. Mathematical concepts cannot be just explained to the child (even with pertinent examples).

Many authors have emphasized the damaging impact of excessive expository teaching, oriented on techniques of performing certain calculations and solving certain types of problems in a schematic way. Negative effects manifest themselves at tasks in which the subject should go beyond the known schematic patterns. Concepts should be created in the mind of the child in a relatively long process of transition from activities performed on concrete objects to activities performed in mind. An important element of such learning may be the emergence of a cognitive conflict, due to which, in a new problem situation, the student has motivation to think about it, particularly in the case of the natural learning and acquiring knowledge based on common sense. Expository teaching may block natural processes of cognition. *First, the student should intuitively understand the meaning of a term or action (and possibly*

the phrases "so many more" and "so many less" are introduced, then the topic changes and typical equations with windows appear, and only later the teacher returns to the substantially more difficult types of tasks ending with the questions "How many more?" and "How many less?".

[5]The term "constructivism" is understood here as it is used in psychology and mathematics education. Apart from the name, this has little in common with constructivism in philosophy of mathematics, related to Brouwer's intuitionism, considered, e.g., by Davis & Hersh (1994).

express it in his/her own way), and then this should be a basis for a symbolic representation.

The main point of this paper is the following: At the end of preschool and the beginning of schooling children are usually given tasks in which there are two sets A and B of objects (movable or pictures on a sheet of paper) and the child is asked which set has more elements. He/she is supposed to match successive items one to one. When this is done, the child can see that either all elements of both sets are included in those pairs or one or more elements of one of the sets are left; in the latter case the child points out the set having more elements.

At this moment the activity should be extended by asking an additional question: How many items are left single and do not match any item from the other set? When there are more elements in one set than in the other, the surplus is visible and the child can answer the natural successive question: "How many more?" Activities of this sort should be a mild way of introducing additive compare problems.

We will use the term ***surplus*** in a technical sense, explaining its meaning with an example of a task, e.g., *On the table there are 6 saucers and 4 cups. How many more saucers than cups are there?* In such a situation we say that we have a surplus of 2 saucers. The surplus is neither a number (because it consists of saucers) nor a set (because it is not known which two saucers are meant); thus, it is neither the difference of numbers nor the difference of sets. It is a certain abstract concept, intuitively clear. Its formalization (as an equivalence class) is possible, but it would be artificial in this context and would obscure the concept. In the case of continuous quantities (e.g. sticks with the lengths of 25 cm and 28 cm) the surplus is not a part of a stick but an abstract quantity 3 cm. Determining the surplus by a student (in case of cardinal numbers or e.g., comparing lengths) should the starting point of the long process of learning how to solve compare problems (a suggestion of such an approach is in Semadeni, 1986, p. 335). Children should learn the meaning of "how many more/less" in the context of small surplus in concrete cases, first without any need of subtracting numbers.

Expository teaching, as in (α) above, should be replaced by individual activities with concrete objects. Instead, as in (β), starting from the simplest introductory types "so many more" of tasks, the types "How many more?" "How many less?" should appear at the beginning, in easy situations. Finally the arrangement (γ), i.e. whole learning blocks devoted to compare problems, should be replaced by extending the learning process of these topic to some years and weaving them in the current arithmetical material.

3 Design and method

The main objective of the study was to study the conjecture that a constructivist approach should give better results. The method was based on the observation of spontaneous reactions of students to two kinds of questions concerning manipulatives and objects shown in a pictures:

a) *Which is more?* — it is a type of question that the students had already met,

b) *How many more?*

Part b) concerns students' *personal knowledge* (in the sense of Klus-Stańska, 2005, pp. 119–122), acquired outside the classroom.

The following research questions guided and centered the study:

— How do students answer questions of the type: *Which is more?* and *How many more?* What are the types of their arguments?

— In what ways do students determine the surplus in tasks ending with a question of the type: *How many more?* How often and in what way do students count elements of the given sets and compare the numbers? How frequently and in what way do students match the elements of two sets? How often and in what way do students spontaneously subtract the numbers while solving compare problems? What factors determine the children's choice of a way of finding the surplus?

— How often do students answer correctly the question: *By how many is this number greater than that number?* without any reference to anything concrete? What are the types of their arguments?

The main part of the study was carried out among the students at the beginning of Grade 1. The interviews were carried out in November 2010 with each student separately and lasted for about 30 minutes. Each interview was recorded on dictaphone. This part consisted of two stages, conducted immediately one after the other:

I. Tasks with manipulatives. First, I put real cups and saucers in front of the child (Task 1) and asked the questions described below. When the child responded to the questions about the cups and saucers, I put two types of candy in front of him/her (Task 2) and asked similar questions.

II. Tasks with pictures. Students received a handout containing seven drawings (numbered 3–9) adopted from textbooks for Grade 1 written by Z. Semadeni.

III. **An additional part of the study** was conducted with the same students in June 2012 at the end of Grade 2. Children were given a single task (Task 10) concerning just numbers, without referring to anything concrete.

During each task the students were asked **two crucial, consecutive questions.** The preliminary question ***Which is more?*** was followed by the main question: ***How many more?*** In addition I asked: *Why? How do you know?*

Questions of the type: *How many less?* were not asked, as generally such tasks are easier than those with *How many more*, presumably because involved subtraction may appear in conflict with the word *more* (Mrożek, 2010).

4 Participants

All the 23 students of the same Grade 1 class (20 six-year-olds and 3 seven-year-olds) were interviewed. In this school I taught mathematics to children in grades from four to six and also taught the computer classes to the studied group. Students of this public school achieve results above average in the Pomeranian voivodeship. These facts should be taken into account when analyzing the results.

I wanted to see students' natural reactions to questions of the type: *How many more?* In particular, I wanted to find out whether and in what way the students spontaneously (i.e., without learning it at school) react to such questions. Prior the interviews their regular lessons included tasks containing the phrases: *Which is more?*, they wrote down the numbers from 1 to 10 and learned addition (including the plus symbol). However, they had not learned subtraction (in particular, they had not written down the minus symbol) and were not given any tasks with phrases: *How many more?, How many less?* in the classroom.

5 Tools and procedure

Stage I – tasks involving manipulatives

Task 1. *"Cups and saucers".* Six cups and 4 saucers were placed on a stand as shown in the picture below.

First question: *Today I brought cups and saucers. Can you answer my question: which is more, cups or saucers?* After the answer, the second question was: *Can you tell me: how many more?*

Task 2. *"Candies"*. I gave candies (19 and 23) to students in two ways: Task 2a given to 20 students and Task 2b given to 3 students. I wanted to see if the change of the arrangement would influence the way of solving the task.

Task 2a. *"Candies: stack + stack"*. All candies were the same size, but wrapped in papers two of different colours, arranged like in the picture below.

First question: *I brought two kinds of candies: the cuckoos and Zozole candies. Can you tell me which are more?* After the answer, the second question was: *Can you tell me: how many more?*

Task 2b. *"Candies: stack + row"*. Candies were laid as in the picture below. Questions were the same as in Task 2a.

Stage II – tasks with pictures

The students first received a sheet of A4 paper on which pictures for the Tasks 3–5 were placed. When the students had answered the questions, they received another sheet of A4 paper with illustrations for the Tasks 6–9.

Task 3 was supposed to provide information whether the presentation of the task data in the form of a ready-made match affects the way of solving the task (children and balloons are connected with strings and similarly children and dogs are linked).

Task 3. *"Children, balloons, dogs"*

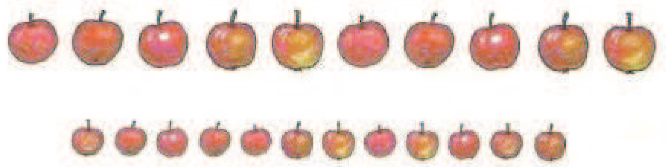
Questions: *Which is more: the balloons or the children?*, then: *Which is more: the children or the dogs?* and: *Which is more: the balloons or the dogs?*

Tasks 4–9 were expected to deal with the problem solving strategies and included questions: *Which is more?*, then: *How many more?*

Task 4. *"Plates and cups"* (as shown)

Task 5. *"Eggs and saucers"*

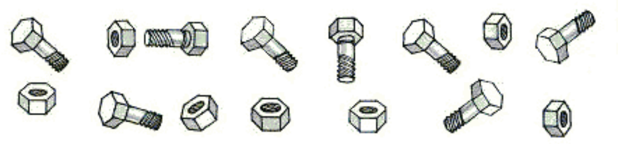
Task 6. *"Big and small apples"*



Task 7. *"Pears and plums"*

Task 8. *"Walnuts and hazelnuts"*

Task 9. *"Screws and screw-caps"*



Stage III – tasks for compare problems concerning abstract numbers (without concrete objects)

Task 10. *"Abstract numbers"*. I asked students at the end of Grade 2 (several months after Tasks 1-9): *If we have the numbers 6 and 9, which is greater?* And then: *By how many more?*

6 Types of students' argumentation

In the students' reaction to the preliminary question: ***What is more?*** the following types of arguments were distinguished:

I) `Counting and comparing numbers` — the student counted the elements of each set, uttered numbers and compared them immediately. For example, in Task 1 a student counted cups (there were 6) and saucers (there were 4) and then said: *There are more cups because there are 6 of them and 4 saucers.*

II) `Compare by matching` — the student combined the elements of one set with those of the second set into pairs and inferred which set was larger, commenting, for example, in Task 5: *There are more eggs because this egg has no pair.*

III) `Compare by counting and matching` — the student counted the elements of each of the sets, uttered both numbers (without comparing them) and immediately started matching. For example, in Task 4: a student first counted the cups and saucers, said: *There are 8 cups and 7 saucers*, did not say which is more, and after that he joined the cups and saucers into pairs and said: *There are more cups because these two do not have pair.*

This type of mixed arguments was used only by those students who solved Task 2 of stack+row type (i.e., candies of one type were aligned and those of a different type were stacked).

IV) `Visual estimation` — the student neither counted the elements nor matched them, but the answer based (as it seems) on an overall visual inspection of the sizes of the sets. Typical phrases: *I see that there are more cuckoos* or *Because when I see that there are so many of these, and those hmmmm, I guess, there is a bigger pile here.*

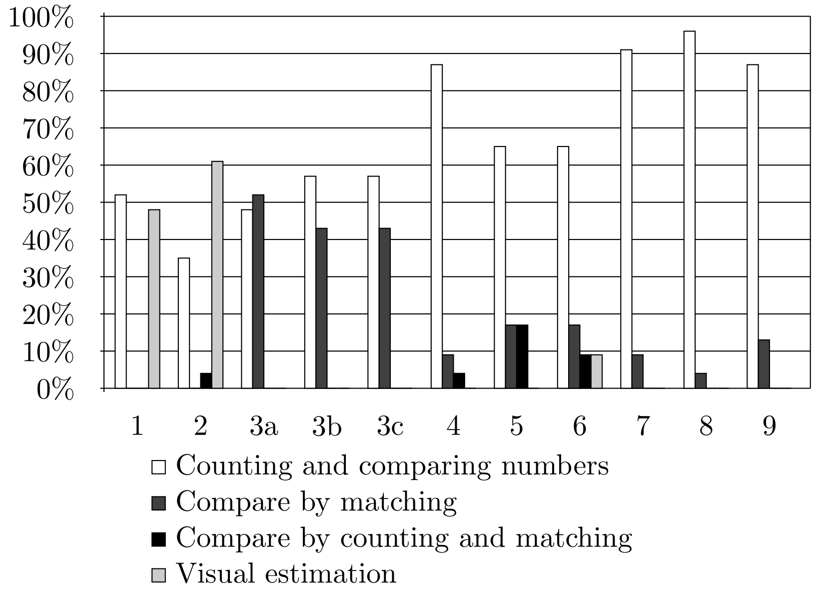


Diagram 1: Distribution of the types of argumentation to the question: *Which is more?* in Tasks 1-9.

The most common type (on average, 65% of respondents) was: `Counting and comparing numbers.` This type prevailed in Tasks 4—9 (on average 80% of respondents), while in Tasks 1-–3 it was less frequent (on average 50% of respondents).

The `Visual estimation` and `Compare by matching` were noted on average in 15%–20% of respondents. The type: `Compare by counting and matching` was observed on average in 5% of respondents.

In Task 1 the prevailing types were: `Visual estimation` (slightly more than 50% of respondents) and `Counting and comparing numbers` (slightly less than 50%). In Task 2: `Visual estimation` (60%) and `Counting and comparing numbers` (35%).

In the tasks presented with pictures (Tasks 3-–9), the dominant type (from 50 to 95%) was: `Counting and comparing numbers`. This type occurred less often in Task 3 (50%) where the students used arguments such as: `Compare by matching` equally often.

Question: ***How many more?*** The children's actions were generally enactive, iconic or mixed enactive–iconic (in the sense of Semadeni, 1982). The following types of correct argumentation were distinguished:

1) `Compare by counting` — the student counted the elements of each set, uttered the numbers and answered the question. In this type several subtypes are distinguished:

   1.1) Operating on numbers without reference to the manipulatives — the student does not mention manipulatives and speaks only of numbers. Examples of arguments:

   (1.1.1) Adding up — for example, in Task 1 a student counted the saucers as follows: *2, because here are 4, and then 5 and 6.*

   (1.1.2) Argumentation by subtraction (note that that children had not been officially taught subtraction in school) — for example, in Task 2:
   T: How many more?
   S: These are 4 less.
   T: Why 4?
   S: Because here is 23, and here is 19, so we subtract 4 and get the same.
   Or in Task 1: *Because if we took away, there would be 4,* or *There are 6 here, and 4 there, so 6 minus 4 is 2, so 2 more.*

   1.2) Using both numbers and manipulatives — the computations referred to manipulatives, for example:
   in Task 1: *2, because here there are 4 cups; if we added 2, there would be 6, because 2 cups do not have saucers, because there have to be 2 more to be equal,*
   or: *2 more because there are 6 cups and 4 saucers,*
   or: *2 more, because if there were 6 cups and 6 saucers, there would be the same number,*
   or in Task 8: *If I cover these 4, there is the same number* -— the child covers 4 walnuts with his hand.

2) `Compare by matching` — the student paired elements of the sets. He/she stated that the bigger set was the one with elements remaining without pairs; then he/she counted the items that remained without pairs and gave the answer, for example, in Task 4: *One more, because this cup has this saucer, that one has that one, and so on, and this one is without a saucer.*

`Invalid argumentation` — the student gave inadequate reasons which could not be classified into any of the above types 1–2. For example, in Task 4 a student replied: *There are 8 more cups; because there are more*, and gave up the task. Such incorrect answers concerned 1-2 children who had learning difficulties (not only with mathematics).

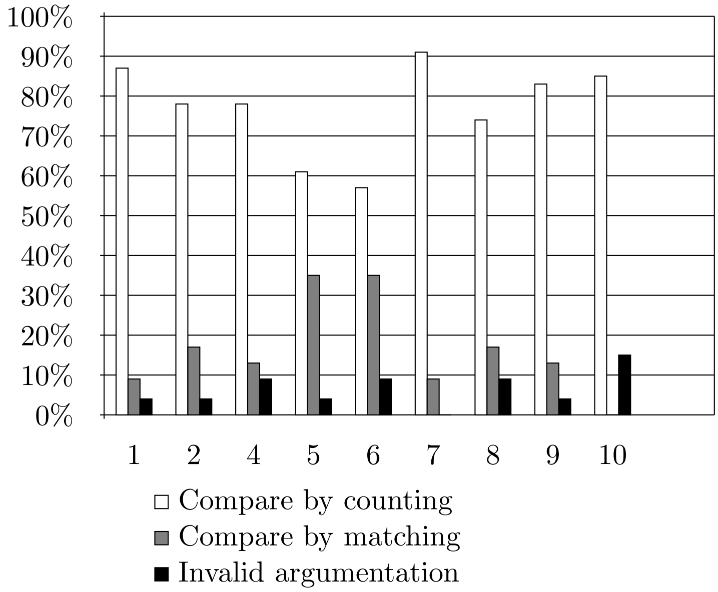


Diagram 2: Distribution of types of arguments to the question: *How many more?* in Tasks 1–10.

The most common type (on average 80% of respondents) was: `Compare by counting`. In contrast, the type: `Compare by matching` (15%) was much less frequent.

In the tasks with manipulatives (Tasks 1-2), the predominant type (80-90% depending on the task) was: `Compare by counting`. The remaining 10–20% respondents gave arguments `Compare by matching.`

In the tasks presented with pictures (Tasks 4–9), `Compare by counting` (60–90% depending on the task) was WAS the dominant type. In contrast, in Tasks 5 and 6 quite often (35%) the type: `Compare by matching` was identified —- it should be noted that in these tasks the objects were presented in the form of two rows (arranged one above the other).

In the task concerning numbers without referring to anything concrete, the only correct type was: `Compare by counting`.

7 Summary of the results

The data suggest the following answers to the research questions posed in Part 3 above.

1.How did students answer questions of the type: Which is more? What are the types of their argumentation?

The questions of the type: *Which is more?* were correctly answered by all the surveyed students. The most frequent type of argument was `Counting and comparing numbers` (approximately 60% of respondents). `Compare by matching`, `Compare by counting and matching` and `Visual estimation` were also present. It is interesting to note that many children counted the elements even in the tasks where the layout suggested matching one-to-one.

2. How did students answer questions of the type: How many more? What are the types of their argumentation?

The arguments of about 90% answers were correct. Two types of valid arguments were distinguished: `Compare by counting` (approximately 75%), `Compare by matching` (approximately 20%).

In the tasks involving manipulatives (Tasks 1 and 2) the type: `Compare by counting` prevailed. In approximately 15% of the respondents the type was `Compare by matching` (slightly more often in Task 2).

In the tasks presented with pictures (Task 3–9), the most common type was: `Compare by counting`. In Tasks 5 and 6 (presented in the form of two rows arranged one above the other) 35% of responses were of the type `Compare by matching`.

3.1 How often and in what way did students count elements of the given sets and compare the numbers?

The way of determining the surplus through counting the elements of the sets and comparing the numbers turned out to be dominant (75% of respondents). In approximately 20% of respondents counting was combined with another way: on fingers (adding one by one) or by pronouncing the following numerals. Moreover, counting played also a role in the type: `Compare by matching` and in the type: `Compare by counting and matching`, where the students additionally counted the elements of both sets.

3.2 How often and in what way did students match the elements of the given sets?

Only approximately 20% of respondents paired the elements to answer compare problems, although this way the surplus was immediately conspicuous and it was enough count the elements that remained single.

3.3 How often and in what way did students subtract the numbers while solving compare problems?

While answering the questions: *How many more?* the students very rarely subtracted the numbers (which had not yet been taught). Examples:
in Task 1 a student argued: *there are 6, here are 4, so 6 minus 4 is 2, 2 more,*
in Task 8 a student argued: *there are 8 walnuts, 12 hazelnuts, so 4 more hazelnuts, because 12 minus 8 is 4.*

Traditionally in Polish schools the compare problems have been introduced to children about 8 years old after a long period of learning addition and subtraction. Problems ending with questions of the type: *How many more?* or *How many less?* have been solved by writing down the proper subtraction. Children in the study solved this problems without subtraction and without prior knowledge of the types: *so many more, so many less* (in contrast to traditional teaching).

3.4 What factors determined the children's way of finding the surplus?

Two factors turned out important: the type of the task and how the student coped with mathematics at school.

Most frequently the surplus was found by counting the elements of the sets and comparing the numbers. In contrast, in Tasks 3, 5 and 6 (because of their nature) matching was more frequent. In Task 3 matching was suggested by the fact that children in the picture were linked with strings to balloons and dogs while in Tasks 5 and 6 the elements of the two sets were arranged in two rows (one above the other), which also could affect the way of solving the task. In the other tasks the elements of the two sets were scattered.

8 Conclusion and suggestions for teaching

The results are similar to those obtained by Nesher, Greno and Relay (1982). In their study it turned out that the problems ending with the questions: *How many more?, How many less?* were easier for six-year-old children than those with the phrases *so many more, so many less.*

In addition, the results of my study are consistent with Freudenthal's idea that counting and the regularity of the decimal system is crucial to the development of the concept of number (Piaget attached too much significance to the one-to-one correspondence). The counting aspect of the numbers was very clear in the children's behaviour. It is probably due to the influence of adults and is culturally rooted. This explains why, when solving a problem with the question: *How many more?* most children counted the elements of the sets and compared the numbers. It is significant that even the students who paired objects, additionally counted the elements.

The results of the study support the hypothesis of the feasibility of teaching the compare problems already to Grade 1 students before systematic subtraction and basing this on equinumerosity. It is vital that at the beginning the quantitative question: *How many more?* should be preceded in each case by the qualitative question: *Which is more?* This question moves the child's attention to the semantic aspect of problem. Examples of exercises to use for children in kindergarten and in Grade 1 are presented by Rożek and Urbańska (2012).

The study *supports introducing compare problems early by combining them with tasks concerning equinumerosity. It is also advisable to choose the tasks so as to provoke the use of manipulatives.* This is particularly important for children who need pairing to grasp the problem. In contrast, it is definitely not advisable to require writing down mathematical operations at an early stage. Premature transition to symbolic level is difficult for children. When new concepts are being formed, it is difficult for children to absorb part of their attention for writing new symbols; this may hinder their thinking.

Szemińska (1981, p. 166) points out that it is advisable that activities aimed at comparing numbers concern sets whose size exceeds children's capabilities of easy counting.

It is recommended to begin teaching compare problems with tasks ending with questions of the type *How many more?* or *How many less?*, combined with tasks of the type: *so many more, so many less.*

The study has shown that most children solved tasks: *How many more?* by counting and using an intuitive meaning of surplus. Methodological procedures should concern *mainly the gradual increase of the surplus*, starting with tasks where it is small (one or two elements).

Compare problems should be then woven into various subtraction problems. More difficult examples should be *gradually* chosen so that the understanding of the need of subtracting be constructed in minds of children. Using manipulatives helps children to form their personal knowledge. Suitable procedures are then understood, not just remembered. This gives an opportunity to achieve sustainable knowledge, robust to reformulations.

In this way, with little additional time when comparing the numbers of concrete elements one can create a basis for better understanding of compare problems. When the time comes for systematic teaching of such problems and extending them to larger numbers, the teacher will rely on students' intuitions and integrate them with the adequate rules.

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Wprowadzenie w porównywanie różnicowe – podejście tradycyjne a podejście konstruktywistyczne

Streszczenie

Źródła trudności zadań na porównywanie różnicowe mogą być dwojakie: tkwiące inherentnie w naturze tych zadań, a także trudności wynikające z niewłaściwych metod nauczania.

Głównym celem tej pracy było zbadanie hipotez:

(1) Istotnym czynnikiem wpływającym na niezadowalające wyniki nauczania tych zadań jest niewłaściwa tradycyjna metodyka nauczania.

(2) Można istotnie poprawić wyniki nauczania odchodząc od tradycyjnej metodyki i zastępując ją metodyką opartą na podejściu konstruktywistycznym.

Efektem badań przedstawionych w tej pracy jest istotne wzmocnienie hipotezy o możliwości nauczania porównywania różnicowego w klasie I przed wprowadzeniem w sposób systematyczny odejmowania i oparcie tego na ćwiczeniach dotyczących równoliczności. Istotne jest tu, aby na początku pytanie ilościowe: O ile więcej? było stale poprzedzane pytaniem jakościowym: Czego jest więcej? Takie pytanie przesuwa uwagę dziecka z aspektu syntaktycznego na semantyczny.

Opisane tu wyniki badań pokazują na możliwość wprowadzenia uczniów w porównywanie różnicowe na konkretach wcześnie, przy okazji zadań dotyczących równoliczności. Co więcej, wskazane jest, by zadania były tak dobrane, by niejako same prowokowały manipulowanie na konkretach. Jest to szczególnie ważne dla uczniów, którym do zrozumienia zagadnienia potrzebne jest stosowanie przyporządkowania. Wskazane jest także, aby porównywanie różnicowe rozpoczynać od zadań zakończonych pytaniem: O ile więcej? lub O ile mniej?, a nie jak dotychczas od zadań ze zwrotami: o tyle więcej, o tyle mniej.

W pracy przedstawiam także główne strategie, które stosowali uczniowie podczas rozwiązywania zadań na porównywanie. Większość badanych dzieci zadania z pytaniem: O ile więcej? rozwiązywała przez przeliczanie i intuicyjne rozumienie sensu nadwyżki. Zabiegi metodyczne stosowane w szkole powinny zatem dotyczyć głównie stopniowego zwiększania tej nadwyżki, rozpoczynając od zadań, w których jest ona niewielka (jeden lub dwa elementy).

Zadania na porównywanie różnicowe powinny być stopniowo wplatane w zadania na odejmowanie. Należy dobierać stopniowo coraz trudniejsze przykłady, tak aby rozumienie, że trzeba odejmować, a także co trzeba odejmować, zostało wykonstruowane w umysłach dzieci. Osadzenie zadań w konkrecie i czynnościach dzieci przyczynia się do tego, że tworzy się ich wiedza osobista. Odpowiednia procedura jest zrozumiana, a nie tylko zapamiętana. Daje to możliwość osiągnięcia wiedzy trwałej, odpornej na zmiany sformułowania.

W ten sposób, używając stosunkowo niewiele więcej czasu przy porównywaniu liczebności, stworzyć można podstawę lepszego rozumienia porównywania różnicowego. Gdy przyjdzie czas na systematyczne nauczanie porównywania różnicowego i rozszerzenie tego na większe liczby, będzie można oprzeć się na trwałych intuicjach uczniów i zintegrować je z formalnymi regułami porównywania różnicowego.