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Anna Pietrucha, Bożena Maj-Tatsis
University of Rzeszow
Poland

Traits of geometrical language used by future mathematics teachers

Abstract: The paper presents a research on how future mathematics teachers use mathematical language and how they transmit and communicate their reasoning. The aim of the research was to elaborate on the characteristics of geometrical language used by students during transforming verbal information into figural one and vice versa. The results show the difficulty in expressing one's thoughts by formal language. Meanings can be easily distorted because of the use of colloquial language that causes lots of ambiguities. The conclusions driven from relevant research depict the role that intuition plays in expressing mathematical thoughts. Moreover, it is not as easy as it may appear to transform verbal information into figural one, and figural into verbal one. However, teaching and developing these competences should be an integral part of mathematics education.

Introduction

Representations, models or images of geometrical concepts can be found in many aspects of our reality. Geometry is a domain of mathematics which students get familiar with at the beginning of their education. However, Ben-Chaim, Lappan and Houang (1989) in their research have shown that students in various educational levels encounter problems with communicating geometrical information, especially the spatial one.

Key words: language and communication, geometrical language, pre-service mathematics teachers.

A number of studies have focused on the analysis of the visualisation processes in two and three-dimensional shapes in all educational levels, including teachers (Clements, 1999; Gutiérrez, 1996; Malara, 1998). The particular aspects of the translation from the visual to verbal representations of geometrical figures was studied by Tatsis (2007) who has found that students rely on everyday language for their descriptions and moreover they seem to have a difficulty in identifying some topological and geometrical features of the given figures.

Bearing in mind the above findings, we designed a research in order to examine how future mathematics teachers use mathematical language and how they communicate their reasoning in the context of an educational game that requires the description and the drawing of geometrical figures.

Theoretical background

Our theoretical considerations stem from a variety of fields, including linguistics, psychology and communicational studies in mathematics education. This was useful for the purpose of our study, in which the geometrical objects are examined as discursive constructs and as representations of "ideal" geometrical concepts.

Linguistic background

In the paper the results of the analysis of the language used by future mathematic teachers are presented. It is worth to describe the way in which language itself is defined and which factors cause so many semiotic and grammar mistakes. According to Saussure (1983) language (la langue) can be divided into "la langue" (a homogenous, abstract system of signs existing in social reality, independent of an individual) and "la parole" (an act of speech, tangible, real, specific for an individual). Saussure (1983) states that because of this "sign duality" it is possible to talk about unreal objects, including geometrical ones. Each sign or word expresses a mental image by the means of a sound image, not the sound itself. Chomsky (1957) while pondering over phenomena of language duality, made a division of language into competence (theoretical background of language) and performance (actual use of language). Competences exist in speakers' minds and constitute a linguistic knowledge that is very often unconscious; nevertheless it is the factor that makes possible the formulation of correct sentences and understanding others' utterances. Performance, according to Chomsky, is competence in practice. Chomsky (1957) realizes that there are a lot of factors that have an influence on the final form of speech. Among these factors he points stress, tiredness or the situation in which the speech was produced. Hence sentences that are not correct may be not the result of the lack of knowledge (competence errors) but in fact may be caused by some external factors (performance errors). Language itself is a human intrinsic ability. Chomsky (1981) states that there is an area in the brain that is responsible for the production of speech and the process of acquiring language. This area is active only in childhood which explains why it is so difficult to learn new language after one's puberty. Each person is born with the intrinsic ability to acquire any language, not a specific one. The language used by an infant's family and mates is a factor that decides which language the infant will acquire. The language that we are exposed to since birth decides which language will become in future our mother-tongue. Mathematical language cannot be treated as English, Polish, or Spanish. In fact it is a "jargon", a style that is being used in a particular environment. Each jargon has its own characteristic vocabulary, connotations and ways of communicating meanings (Condillac, 2014). Jargon is not a kind of a mother-tongue for a child; it has to be taught and learnt. Using a "jargon" and learning how to express ideas using a "jargon" is a significant activity. The issue of great importance is making pupils aware of the fact how crucial it is to learn the "mathematics jargon" and creating an environment in which pupils can learn it in the most natural way. A teacher is the person who is responsible for providing such an environment. Therefore it is crucial for the teacher to use the "mathematics jargon" properly in the first place.

Communication of mathematical thinking processes

Pirie (1998) states that mathematic knowledge can be transmitted through the use of different means of communication, namely everyday language, mathematical verbal language, symbolic language, visual representations, unstated assumptions and quasi-mathematical language. Turnau (1990) claims that communication in mathematics is performed by both natural and mathematical language. Tatsis (2007) states that regardless of our belief on a separation between formal and common language, we still cannot ignore the fact that they are inseparably connected with each other. Common language is one of the ways to communicate mathematical content. Kanes (1998) shows that it is still a controversial issue whether this inseparability deforms mathematical meaning. Nevertheless, every mathematical problem may be perceived as a linguistic message. Czajkowska (2014) writes that effective communication of mathematical thoughts by the teacher or by the author of the problem is possible when the student is able to correctly decode the content of the problem and to interpret the information included in it. In mathematics education it is very important to familiarize students with proper mathematical language; this is possible through communicating and transforming one's thoughts (Niss, 2003).

Object and its representation

Pieron (1957) defines the notion of an object as a general term, a class of items having a number of common properties. The notion expresses the idea, some ideal representation of the class of objects based on their mutual properties. Object's representation is according to Pieron a sensory depiction of an object or a notion. In the process of abstract thinking representations present the object's general meaning, its actual, tangible depiction closely connected with the common characteristics of the class of objects. As geometry deals with an imaginary world, objects such as a point, a line, a square or a triangle do not exist in the real world. They are only ideals. They are notions that cannot be presented on paper. Representations that are objects' reflections are presented by the means of words, symbols, schemata or images. These objects have abstract nature but simultaneously they have also intrinsic figural nature (Fischbein, 1993). Geometrical objects do not have their physical substitutes. Points (zero-dimensional objects), lines, segments (one-dimensional objects), figures, areas (two-dimensional objects) do not have their equivalent models in reality. Objects that can be met in everyday life are three-dimensional. Fischbein (1993) claims that constructions and geometrical figures are general representations; in fact they are never copies of particular, real objects. When the square is being referred to, it is drawn in order to check some properties. The image of the object itself is not identified with the object; it refers to some shape that has the common characteristics of an infinite class of objects. Following Fischbein, it is worth noticing that the figures' properties are somehow inbuilt. They are imposed by the object's definition that belongs to the already created axiomatic system (for example Euclidian Geometry). This fact demonstrates the abstract nature of geometrical objects. Hence a square drawn on a piece of paper by a student is a confined shape controlled by its definition (even though it may be inspired by a real object). Fishbein concludes that a geometrical figure can be described as having some abstract properties; nevertheless it is still not only a notion; it is a visual image.

Competences connected with mathematical knowledge

Saenz-Ludlow (2006) describes competences that are indispensably connected with mathematic knowledge. These are:

- representations that enable communication;
- simultaneous usage of different semiotic systems;
- identifying objects variously represented without identifying the object with its representation;
- transformations of an object within and outside the systems;
- constructing and interpreting meaning transmitted through signs and symbols.

Bishop (1980, 1983) states that it is not possible to create a unique definition of geometrical competences. Focusing on educational processes, he suggests discussing about two different abilities. The first is the capability of interpreting figural information, whereas the second is the ability of visual transformations. Bishop understands visual representations in a wide sense, even in situations where there is no visual stimulus. Visual transformations can be understood not only as the ability of performing some mental manipulations with geometrical objects but also as the ability of describing images of these objects as well as transformations of information: from verbal into visual and vice versa. Bishop (1986) and Presmeg (1986) notice that mathematical objects, which are subjects of various studies, are almost always being identified with the objects that have some physical and visual properties. The connection between geometrical concepts and physical objects or images makes the relation between geometry and visualization even more complex than it may be perceived. This complexity led us to design our research with a strong focus on the interplay between verbal and visual aspects of geometrical objects.

Methodology

The research was conducted among twelve future mathematics teachers during the "Quality Class" organized at the CIEAEM conference in Barcelona in 2011. The participants were female and male at the age between 21 and 25. They came from different European countries but all of them were able to speak English very well. The research was conducted in English. The participants were students of Mathematics Education at different levels: bachelor, master and PhD. They already had some teaching experience.

The participants of the study were divided into three groups of four. All groups were working separately and they could not communicate with each other. Each group had its leader who was the first solver of the task. The leaders (S1) received the original figure shown in Figure 1 (Marczak, 2011).

Their task was to provide a written description of the figure on a piece of paper. They were instructed to write down everything that should be said if they would be communicating by phone. The second participant of every group (S2) received the description done by the leader. Her/his task was to draw the figure according to the received description. Then, the third participants (S3) who received the figure made by their predecessors described it in written form. The last students from each group (S4) had to draw the final figure according to the information given on the paper by their predecessors. Only the leaders saw the original figure. There were no time restrictions. In case of any doubts, the participants could ask the researcher about technical aspects of their task. At the end, each participant's worksheet was collected. The whole research lasted 60 minutes.

The figure to be described was chosen because it consists of a mixture of basic geometrical figures, but at the same time it is complex enough (see Figure 1).

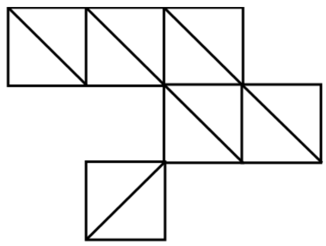

Figure 1. The original figure given to the leaders of every group.

The choice of the figure was also based on its possibility to be described in more than one ways. Nevertheless, we did not expect the participants to create an "ideal" description, since we were aware of the various interpretations that might come out. The main aim of the research was rather to examine how future mathematics teachers use mathematical language and how they transmit and communicate their reasoning. Particularly, we were interested in:

1. examining traits of geometrical language used by prospective mathematics teachers.
2. examining the ability of using precise mathematical language to express one's thinking processes.
3. investigating the abilities of coding and decoding visual and verbal information (expressing and reading).
4. distinguishing some traits of colloquial language used during transmitting visual and verbal information in geometrical contexts.

For that purpose, firstly the results of every group's work were analyzed and then they were compared with each other. Our analysis can be characterized as qualitative and descriptive (Ritchie & Lewis, 2003) and our data consisted of written descriptions and drawings.

The results of the research

The results of two groups which participated in the research are presented below. Firstly the data of particular participants are presented, followed by an analysis of their work.

The results of group 1

The results of the work of participants of the first group are presented below: a description made by the first student (leader – S1), a figure drawn according to this description made by the second student (S2), a description of the figure drawn by the second student made by the third one (S3) and the final drawing created by the fourth student (S4).

The leader of group 1 described the original figure (Figure 1) as following:

> S1: "Draw 3 squares in one row. They are all connected to each other. Then draw another two squares under the right square from the first row (first square of the second row have one common side with the right square from the first row). Then draw one more square that touches the left bottom vertex of the left square from the second row. This square is parallel to the squares from first and second rows. Each square apart from the last drawn have a diagonal sketched form left top vertex to the right bottom one. The last square has the diagonal going from top right vertex to the left bottom one."

The drawing made by the second student (S2) based on the description of S1 is presented below:

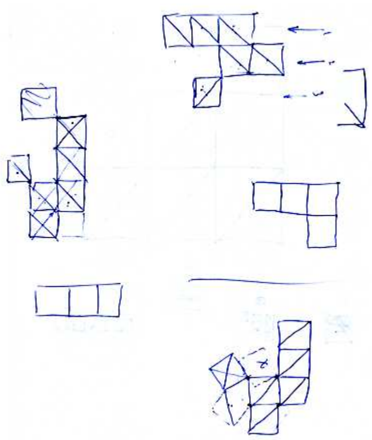

Figure 2. The drawings made by student S2 (group 1).

Student S3 wrote the following description based on Figure 2:

> S3: "Draw 3 squares in a row with each diagonal from right high to left low. Draw 3 squares again in a row at the left hand side of the first row in such a way that the two higher squares of this row connect with the two lower squares of the first row. In this row the two lower squares have the same diagonal sketched, the other square has only the other diagonal sketched.
> You see a square with two squares above and two squares down.
> The above squares get a number and a letter:
> left one: L1;
> right one: R2."

Using the above text student S4 draw the final figure (Figure 3).

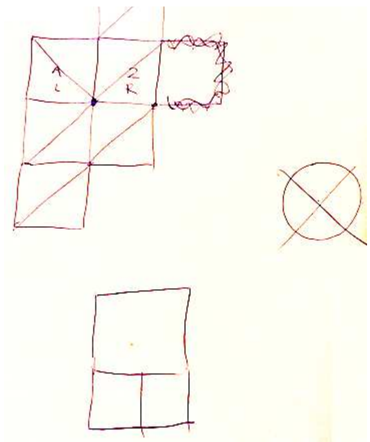

Figure 3. The drawing made by student S4 (group 1).

Analysis of group 1 work

Analysis of S1 description

In S1's text some difficulties in describing the orientation of the figure can be noticed. The student used some words such as "row", "touches", "connected" which qualify to the everyday language and do not belong to the "mathematics jargon" (Condillac, 2014). The word "row" is used in order to express how the squares are located, while the word "touches" expresses the notion of the common vertex of two squares. The phrase "the square is parallel to..." is also not correct mathematically. We can assume that the participant wanted to say that the sides of the squares are parallel but the thought was not properly communicated. We can also notice an attempt to define how the squares are located in relation to the squares that are in the neighborhood. The student uses the phrase “under the right square” which cannot be used in the context of Euclidian geometry that has no orientation hence there is neither left nor right side. The concept of neighborhood was also expressed through the use of colloquial language. The student described the squares as “connected to each other”. In fact there are only few expressions taken from formal mathematical language and they are very basic ones (square, side, vertex, diagonal). The phrase “common side” is used to state the mutual location of two squares which is a proper geometrical expression.

Analysis of S2 drawing

On S2’s paper there were more than one drawings, a fact that shows that it was difficult to decode the received information. The two initial figures have an opposite orientation compared to the final one. The biggest problem aroused while drawing the square which “touches the left bottom vertex of the left square from the second row” and is “parallel to the squares from first and second rows”. The student firstly drew the described square as if it is located in the same row in which the two previous squares are placed. After that he probably realized his mistake and started drawing the second version of the figure. This time the described square appeared below the previously drawn squares and it has a common vertex with the square above. But still the information that the sides of this last square should be parallel with the sides of previously drawn squares has not been used. Only the last figure (see the bottom right drawing in Figure 2), which is according to its creator the correct one, accounts for all the given information. Next to the final figure the students spotted three arrows. They are to state the orientation of the figure and to indicate each “row” of the figure. An interesting fact is that all of the squares are congruent, even though there was no information to suggest it. While analyzing the figure we can notice that for the student it was very difficult to draw the diagonals in a correct way. In the first figure there are squares having both diagonals drawn. The second figure depicts all squares with one diagonal except for the one located in the “third row”. The final figure presents squares with one diagonal each. Despite of the difficulties in drawing the proper figure, the result of the work done by student S2 is a figure congruent to the original one. Therefore, the third member of the group receives the same figure as the original one.

Analysis of S3 description

The description made by student S3 has the form of instructions which are to be followed in order to draw a correct figure. All sentences are in the imperative and contain a description of the actions that are to be taken in order to draw a figure congruent to the described figure. There are some phrases taken from common language: "at the left hand", "connect", "the same diagonal", "other diagonal" and "row". The description suggests that it was difficult for the student to state the reciprocal relationship between the squares as well as the concept of neighborhood. The expressions used are supposed to refer to the reader's intuitions connected with that concept. Student S3 did not use formal mathematical language; he replaced it with everyday one which is, as he claimed, an easier one. It is obvious that common language is more natural in use and seems to be easier to be understood, although it cannot be used in order to express mathematical, in particular geometrical, thoughts as it is not precise and unambiguous enough. This can be clearly seen while analyzing the fourth student's attempt to draw a figure according to this description. The only formal expressions used by the third student are the names of basic geometrical figures such as "square" or "diagonal". The participant chose to not use the term "vertex" while talking about it. He used the phrase "from right high to left low". In this expression the subject that is being described did not appear. Student S3 referred to the reader's intuition, this time the one connected with the notion of vertex. For that student the notion of the vertex is so obvious that it can be taken for granted. Similarly to the first description, this one also contains the notion of "row" which is used in order to facilitate the imagining of how the squares are located in the figure. The student wrote also about left and right side which is not proper as Euclidian geometry has no orientation. After the first three sentences, the information placed was repeated. It was the second attempt of describing the same things, maybe in order to make it more clear or readable. Then there is an attempt of marking some squares: "left square L1, right one R2". This part of the description has not been completed. It can be the result of the fact that the describer realized that this part of description is not understandable or is in some way improper. Still this part of the description was not deleted. It seems that the decision about the usefulness and correctness of that information is left to the reader. The reader in fact received a description that consisted of two parts connected to each other, but one of them unfinished. The third student wrote that in the figure there were six squares located in two "rows" whereas in the figure that he was describing there were six squares which are located in three "rows". There is no information about the congruence of the squares. That fact has been taken for granted. A square is a regular figure, thus it is obvious that if there are more than one squares in the figure, all of them, according to the student, have to be congruent.

Analysis of S4 drawing

There are three figures on student S4's worksheet (Figure 3). The one that is at the top seems to correspond to the first part of the description. There are six squares in it but they are situated in two columns instead of two rows. Apparently, the use of the notion of rows that was to facilitate the drawing of a proper figure turned out to be ineffective; the student confused the notions of columns and rows. In the worksheet, the trials of decoding and using information from the second part of the received description can be seen. Two of the squares were named L1 and R2. Then student S4 probably realized that the figure is not correct, hence the next figure is created. There is a square in it, and according to the information, there are "two squares down", but this square lacks the "two squares above". The most puzzling is the third figure, the one that is located on the right side of the worksheet. There is a circle divided into four congruent cuttings or being erased. This is how this student decided to draw a circle. There is a small possibility that the definition of a circle has been mistaken with that of a square. In fact there are three figures drawn by student S4, but the one that should be perceived as the final one has not been pointed. This may be merely because the participant has forgotten to give the information about which figure is the final one. However, the hypothesis that the participant himself did not know which figure should be the right one is probable.

The results of group 3

The results of the work of group 3 participants are presented below: a description made by the first student (leader – S1), a figure drawn according to this description made by the second student (S2), a description of the figure drawn by S2 written by the third student (S3) and the final drawing created by the fourth student (S4).

The leader of group 3 described the original figure (Figure 1) as following:

S1:

1. "Make an "imaginary" rectangle of a base with three little squares and high with four little squares about 2 cm side each.
2. You only draw the line of the square situated on the row-column like in a matrix : $1-2, 2-1, 2-2, 3-1, 3-3, 4-1$.

3. Then you draw a diagonal to these six squares:
   - to the five squares of the first and second column you draw the diagonal from down-left to up-right;
   - to the only square located in the third column you draw the diagonal from up-left to down right."

The drawing made by the second student (S2) based on the description of S1 is presented below:

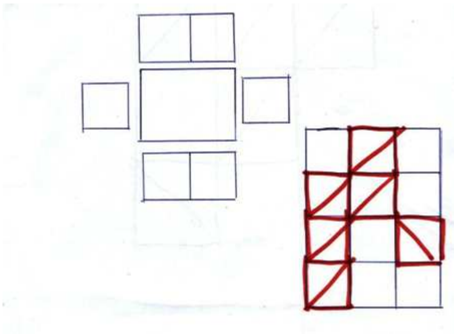

Figure 4. The drawings made by student S2 (group 3).

Student S3 wrote the following description based on Figure 4:

S3: "There are 3 rectangles in a column – without any break. The first one and the third one are divided into 3 squares. The second rectangle is a bit higher. At both sides (left and right) of this rectangle there are squares placed (with breaks). These squares have the same area as the squares I described above (talking about rectangles divided into squares)".

Using the above text student S4 draw the final figure (Figure 5).

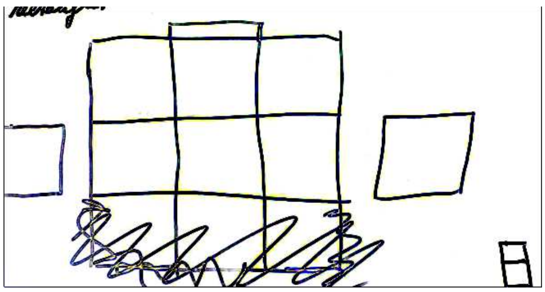

Figure 5. The drawing made by student S4 (group 3).

Analysis of group 3 work

Analysis of S1 description

An original way of describing the figure has been applied by the leader of group 3. Algebraic knowledge about matrixes has been applied here. The student S1 in his description creates an imaginary system of coordinates in which the figure is placed. This system of coordinates takes the form of a rectangle having the side of a square as a unit, while its area consists of the sum of the areas of the unit squares. The reader is supposed to decode the figure through a reference to this system of coordinates. The unit squares that are to be sketched are listed by reference to matrixes notation. The way the notation is to be read is clearly explained by the student: firstly the row and secondly the column is coded. Concerning matrixes and algebraic language, expressions such as row and column are used in a proper, formal way, and they cease to be merely everyday language phrases. At the very end, the information about which diagonal should be marked is given. Reference to the algebraic notation can be spotted here; nevertheless the student has difficulty in saying which diagonal is to be drawn. The notion of the vertex has not been used, although the student refers to it by the expression "draw a diagonal from left bottom to right top" and then "from left top to right bottom". In the description made by the leader of group 3 geometrical notions have been connected with the algebraic ones resulting in a brief and accurate description of the figure.

Analysis of S2 drawing

There are two trials of interpreting the description made by the leader (Figure 4). The final one is bolded. In the first trial the information on how to create an imaginary system of coordinates has not been adequately decoded. The student S2 drew a rectangle and some squares separately in its neighborhood. The number of the squares, however, does not correspond to the number of the squares that the leader of the group has described. The student probably noticed his mistake and realized that the rectangle that has been drawn should include squares that are supposed to indicate the length of the sides of the figure. Hence, the second drawing is created. Firstly, an imaginary system of coordinates is sketched, then the squares are marked in the drawing. Eventually, the diagonals also appeared. The final picture corresponds to the original one that was described by the leader of the group.

Analysis of S3 description

The description of student S3 is very concise. There are many common language expressions in it: “column”, “rectangle without breaks”, etc. These expressions constitute his attempts to describe the relative position of the figures. Then, according to the given order, the reader is expected to divide one of the rectangles “into three squares”. Unfortunately, it is not clear how the operation of division is defined. The student appeals to the reader’s intuitions related to the notion of union of sets. Unfortunately, the expressions used by him have nothing in common with geometrical language. Interestingly, there is no information about the diagonals that are to be drawn. This may be the result of oversight or confusion but it is also possible that, for the particular student, the diagonals do not constitute a part of the figure and they are not its integral parts. There is also information about “the squares with breaks”. What are these breaks? They may be understood at least in three possible ways. First hypothesis is that these squares do not have any common part with the drawn rectangles. Second, one may imply that the squares are supposed to be drawn using a dotted line, and finally, according to a third interpretation, these figures are open polygons. It is also difficult to not spot that the given information does not correspond to the picture that is being described. Most probably it is a result of the fact that the student S3 has mixed the description of the final (bolded) figure with the second one that is visible in his predecessor worksheet. Furthermore, mathematical language has not been used. The only formal expressions used are the names of figures: square and rectangle.

Analysis of S4 drawing

The student S4 received a description that is not convergent to the original figure. At the beginning, three rectangles that are “divided into squares” are drawn. Afterwards, one of the rectangles has been deleted. According to the student S3’s description, there should be three rectangles in the picture but only two of them should include squares. The information that one of them should be “a bit higher” has not been decoded. Student S4 drew two convergent rectangles with a longer side being the common side of both figures. The third rectangle is added in the drawing in a way that its longer side is common with the side of the central square included in the firstly drawn rectangle. Its shorter side is in fact shorter than the shorter sides of the remaining rectangles. The drawing is finished by adding two squares, each of them located “at one side of the second rectangle with breaks”. Strangely, the squares have been drawn in a way that their sides are parallel to the appropriate sides of the rectangular, despite the fact that the relevant information has not been given. In the student's intuition two ways of orientating the picture have been employed: perpendicular and level.

Summary of the results from all groups

None of the groups managed to present a final figure that would be convergent to the initial one. This happened in spite of the fact that in all groups the figures made on the base of the leaders' descriptions (the second students) corresponded to the original figure. The final sketches are far from accurate (see Figures 6, 7 and 8). In each of the descriptions the coexistence of everyday and mathematical language can be noticed. For the students it was easier to express their thinking processes using the language that they use in their everyday life. Nevertheless, using the colloquial language more than the formal mathematical resulted in lots of mistakes and ambiguities.

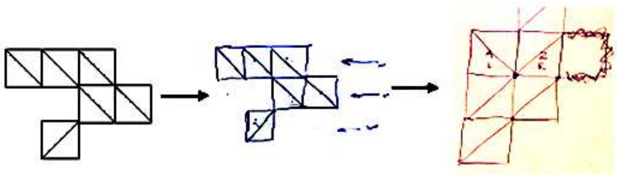

Figure 6. The evolution of group 1 work.

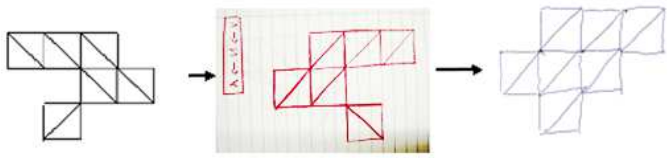

Figure 7. The evolution of group 2 work.

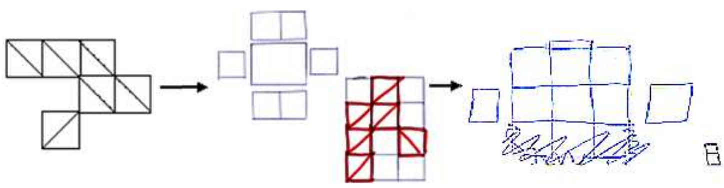

Figure 8. The evolution of group 3 work.

In the results of the research presented in this paper some repeating mistakes can be notified. Two students used an expression about a side parallel to the figure, one participant confused the notion of the square with the notion of the circle and the notion of the diagonal with the notion of the diameter. Among six descriptions in only one it is stated that all squares are congruent. Nevertheless, the information that is not clarified is decoded as it is taken for granted. Phrases such as "row", "column", "above" and "below" occur very often in the descriptions and they are used to express concepts connected with neighborhood which is a basic topological notion. The fact that colloquial expressions have been used in a formal geometrical context did not result in misunderstandings. The words "above" and "below" are used to code that the described figures have some common sides. In the vast majority of the descriptions there is an attempt of using orientation in Euclidean geometry: the students write about the left and right side and about the bottom and the top.

The majority of descriptions contain instructions in the imperative form. One of the describers (the leader of the third group) uses an interesting way of communicating how the figure should be drawn. He applies algebraic knowledge to a geometrical task. It is an original way of thinking; the student shows his creativity and understanding of mathematical language. He was the one who used the least amount of colloquial expressions.

It can be pointed out that most of the students who had to make a figure on the base of the received description had some difficulty in decoding verbal information and transmitting it into figural one. Numerous attempts of creating a corresponding to the received description figure prove this assumption.

The research has also shown that the students are not used to checking the accuracy of the performed task. The analysis of students' worksheets clearly shows that there were some mistakes that could have been avoided if verification had been made. Some students did not even mention which of their drawings should be perceived as the final one. This results in their followers receiving more than one figure, without knowing which one they should describe

Findings of the research

It may appear that mathematics students which bound their future with teaching should not have any major problems in expressing their thinking processes, especially mathematical ones. The research that was carried out has shown that they have many difficulties in properly using formal mathematical language. None of the groups had managed to succeed in the whole given task.

Geometrical language used by the participants was not precise enough. The students – future mathematics teachers – did not pay enough attention to their language. In the descriptions made by them there were many expressions from everyday language. The results of the research have shown that the most difficult it is to express intuitions connected with neighborhood and relations in which the figures – geometrical objects – are located. The concept of neighborhood is expressed through phrases from everyday language, namely: "near", "next to", "above", "below", "on the right side", "on the left-hand" and "in a row". The definition of neighborhood functioned only on the level of intuition and it was not expressed verbally. As relevant research has shown, basic topological notions are present only on the level of intuition, they are not being discussed (e.g. Tatsis, 2007). The results show that colloquial language in communicating mathematical objects is not precise and unambiguous enough. Information coded in colloquial language can be decoded in many different ways and does not mean exactly the same from the mathematics point of view. The basic properties of square influenced the reasoning of students during their work at the task. Only one student has mentioned that all squares in the figure are congruent to each other. The rest of the students took that fact for granted, as they had probably thought that the squares depicted in one figure, as being regular figures, have to be congruent. A similar situation may be noticed during the analysis of the attempts of describing the square that does not have any common sides with the sides of the other figures. Only few students have added the information about the sides of this square being parallel to the sides of the remaining squares. The results have shown that for some students it is correct to say about a segment (1-dimensional object) being parallel to a figure (2-dimensional object). As it was already mentioned in this paper, a confusion of some basic definitions has occurred in these situations. Many students do not mention the fact of parallelism of the sides of the discussed square to the sides of the remaining figures. The intuition connected with the regularity of the square has once again played its role.

While analyzing the results of the research, it seems that intuitive thinking dominates the formal one, whereas the relevance of creating an adequate and readable description is superior to the proper use of formal language. Some of the students during their work have referred to a concept without expressing it. The situation when the students refer to a vertex saying "from right high to left low" and not clarifying the subject of the sentence is a good example of this problem. The concepts of vertex and neighborhood are being expressed non-verbally, between the lines.

According to the participants, in Euclidean geometry there can be orientation in such a way like in the real world. Thus, it was correct for them to discuss about the right and left side of the figure, its bottom and top. None of the students during the research has realized that in Euclidean geometry there is no orientation. Expressions taken from everyday language were once again used in an improper mathematical context.

The research that was conducted has shown that for the students it is not crucial to check and take a look back at the work they have just done. This resulted in many mistakes that could have been avoided. These errors concern coding and decoding both verbal and figural information. The mistakes that are analyzed in this article might be the results of absentmindedness or oversight; nevertheless, they should be spotted in the process of verifying one's own work. The research shows how important it is to stay focused on the task, even when it seems to be easy. Each description should be read at least once more in order to verify the accuracy of the description with the figure that it describes.

The research implies that it is not easy to express geometrical information through the use of proper words. Many difficulties arise when verbal information is to be transformed into figural and vice versa. Therefore, educating and developing these competences should be an integral part of the process of mathematics education, not only in terms of the students who connect their future with the profession of mathematics teacher.

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Pewne cechy języka geometrycznego przyszłych nauczycieli matematyki

S t r e s z c z e n i e

W artykule przedstawione zostały wyniki badań dotyczące umiejętności wyrażania i przekazywania myśli matematycznych przez studentów matematyki wiążących swą przyszłość z zawodem nauczyciela. Celem badań było przyjrzenie się pewnym cechom języka geometrycznego, użytego przez studentów podczas przekształcania informacji wizualnej na słowną i vice versa. Wyniki badań pokazały, jak wiele trudności sprawiło im posługiwanie się poprawnym, formalnym językiem matematycznym. Artykuł zwraca uwagę na to, jak ważne jest precyzyjne formułowanie swoich myśli za pomocą języka formalnego oraz na to, jak łatwo znaczenie może zostać zniekształcone przez użycie zwrotów z języka potocznego, które wprowadzają wiele dwuznaczności. Ważną rolę w wyrażaniu myśli matematycznych odgrywa intuicja. Badania ukazały, że nie jest łatwo informacje rysunkowe przekształcić w informacje słowne, a te słowne zakodować za pomocą rysunku. Niemniej jednak kształcenie tych umiejętności powinno być integralną częścią edukacji matematycznej każdego ucznia, nie tylko osób wiążących swą przyszłość z matematyką, w szczególności z jej nauczaniem.