

Elżbieta Mrożek
Institute of Mathematics
University of Gdańsk
Poland

Semantic and syntactic analysis of the additive and multiplicative compare word problems¹

The additive and multiplicative compare word problems are specifically difficult. The sources of these difficulties may be of two types:

- *difficulties inherent in the nature of these tasks,*
- *difficulties arising from inadequate teaching methods.*

The inherent difficulties, among other things, lie in the idea that the words used in these tasks are static, and the information provided to a student is relational. Namely, in the tasks there are some relationships between specific cardinal numbers or quantities given, and the pupil is supposed to conclude what arithmetical operations are needed to achieve the result. According to cognitive psychology, such reasoning is significantly more difficult for children from tasks where something happens, some actions take place, and these actions can be naturally reflected in the arithmetical operations. This is particularly evident in tasks requiring inversion of operations. When the reciprocal can be brought to the reversal of these actions (done in the pupil's mind or through a simulation on specific objects), the child is able to understand the

¹This paper is a modified version of the author's lecture presented on 9th of May 2013 at the Pedagogical University of Cracow during her doctoral defense. Professor Zbigniew Semadeni (University of Warsaw) was the supervisor of the PhD thesis. The thesis was evaluated by referees: Professor Marianna Ciosek (Pedagogical University of Krakow) and Professor Ewa Swoboda (University of Rzeszow).

way of achieving the result. However, in the case of compare word problems, such a possibility does not exist and it is necessary to draw conclusions directly from verbally given, static data.

The source of difficulty in understanding compare problems may also be the teaching methods applied to teach to solve such tasks. The methodology traditionally and currently used in Polish schools is characterized by the following features²:

- It is the teaching of giving — a teacher shows a way in which a common phrase of comparing should be transformed into a proper arithmetic operation.
- Teaching begins with the tasks with phrases “so much more”, “o much less”, and then proceeds to the types of tasks ending with questions: “how many more?”, “how many less?” through the formal reversal of the given relations and corresponding arithmetical operations.
- Traditional teaching is carried out in blocks of several hours. One day a teacher starts a whole new subject and during the next several lessons children solve tasks in accordance with the increasing degree of difficulty.

In the thesis I have presented a proposal to amend the existing methodology of teaching to solve compare word problems by introducing a curriculum based on the constructivist approach. This change consists in the fact that at the beginning of schooling (i.e., about two years before a systematic teaching of the additive compare problem) the pupils get familiar with the compare word problems on manipulated specific objects and simple numbers.

Additive compare problem should be derived from the pupils’ activities associated with one-to-one correspondence assigning on the occasion of comparing the sets of numbers at the beginning of schooling. When the sets are not equal and one set has more elements than the other, a surplus of³ is visible and one can ask a natural question: How many more? Determining surpluses in subsequent situations (including, among others, comparing lengths) is the starting point of a long process of shaping concepts connected with additive compare problem⁴.

²The traditional methodology can be described on the basis of methodological guidelines contained in Polish educational publications in the last several decades and the analysis of textbooks published in the last several years.

³In the dissertation I have introduced the term *surplus*. For example, let us consider the task: On the table there are 6 plates and 4 cups. How many more plates than cups are there? The surplus is neither a number nor a set, it is also neither a difference in the numbers, or a difference in the sets. This is some abstract concept, intuitively clear.

⁴The suggestion of such an approach is found in: Semadeni Z. ed., *Nauczanie Początkowe Matematyki*, Volume 3 , WSiP , Warsaw , 1986, 335

In the idea of the constructivist approach to the multiplicative compare problem, it derives from the pupils' activities, referring to measuring length by repeatedly applying a device (e.g. a stick) to a measured object. Apart from the typical questions such as: "How many times have you replaced the stick to get the length of the measured object?" or "Give the length of the object using measuring sticks" a teacher can just ask additional questions such as: "How many times longer is the object than the stick?". Another way of development are tasks such as: "How many times a block can be fitted in the other?" and the classic tasks of the containing leading to the dividing cardinal numbers, as well as more abstract questions like "How many times 3 is in 18?" and "How many times one segment fits in the second one?".

The main objective of the dissertation was to examine the following hypotheses:

- (α) An important factor in the unsatisfactory results of learning compare word problems is the invalid traditional methodology of teaching such tasks.
- (β) The learning results can be significantly improved by moving away from traditional teaching methods and replacing them with a methodology based on the constructivist approach. There should be a greater emphasis on semantic aspects of the tasks, on the construction of meanings in the pupils' minds, and a reduced emphasis on syntactic aspects (although there should be a balance between the two).

In connection with the hypotheses, I have conducted the following analyses:

- (1) Testing the mastery of additive and multiplicative compare word problems for pupils entering in Class IV (traditionally taught). There were about 70 thousand pupils, of whom about 800 works have undergone a specific analysis⁵.
- (2) Examination of pupils entering in Class I, concerning the introduction into additive compare problems directly from comparing the sets of specific objects. The aim of this study was to examine the relevance of the constructivist approach to teaching additive compare problem through observing pupils' spontaneous reactions to the two questions in the context of the objects to manipulate and objects shown in the figure: *What is more?* and *How many more?* Individual interviews were attended by 23 students of one Class 1 of primary school.

⁵A detailed description and main results of these studies can be found in: E. Mrozek, *Task Variables in Compare Word Problems*, *Didactica Mathematicae*, 33, pp. 73-104, 2010

- (3) The study of pupils finishing class 2, referring to introducing a pupil to multiplicative compare problems directly from quotative division of specific objects. The aim of this study was to examine the relevance of the constructivist approach to teaching multiplicative compare problem through observing pupils' spontaneous reactions to the question: *How many times more?* in the context of the objects to manipulate and objects represented in the figure. Individual interviews were attended by 23 students of one Class 2 of primary school.

The results of the constructivist introduction to teaching additive compare problems proved to be optimistic. Almost all pupils (except for a few – the ones with great learning difficulties) succeeded in simple situations, answering the question: *How many more?*, although they had not studied these issues at school. This suggests that the pupils beginning school education already possess some extracurricular personal knowledge on the subject. The tested problems clearly were in the actual level of development and were relatively easy for them.

While analyzing the results I have singled out two main types of argumentation:

- a) counting and additive compare of the received numbers – pupils often counted the surplus on the fingers (adding after one) or pronounced consecutive numerals.
- b) application of assignments – here the surplus was visible immediately after connecting elements in pairs.

The traditional method of determining the surplus – by subtracting the number mentioned in the text task – has been observed by me only later on during further testing, which related only to additive compare problems of the abstract numbers.

In the case of multiplicative compare problem, the results were also quite optimistic, which indicated that this is the right way to development. The respondents coped well with the tasks associated with quotative division and repeated laying out of a given length. However, some tasks related to multiplicative compare of quantities turned out to be more difficult.

To sum up: constructing the task meaning in the pupil's mind, shaping the ability to enter into the task situation, is difficult and time-consuming. Introducing the material in a formal way admittedly takes less time and is less absorbing both for the pupil and the teacher, as well as the author of the tasks. However, such knowledge is not grounded on intuition and can only bring positive effects in typical situations.

Semantyczna i syntaktyczna analiza zadań tekstowych na porównywanie różnicowe i ilorazowe

S t r e s z c z e n i e

Zadania tekstowe na porównywanie różnicowe i ilorazowe są specyficznie trudne. Źródła tych trudności mogą tkwić inherentnie w naturze zadań, a także wynikać z niewłaściwych metod nauczania.

W pracy doktorskiej zaprezentowałam propozycję zmiany dotychczasowych metod nauczania porównywania różnicowego i ilorazowego – poprzez wprowadzenie nauczania opartego na podejściu konstruktywistycznym. Zmiana ta polega na tym, że na początku nauki szkolnej wprowadza się dzieci w zagadnienia dotyczące porównywania różnicowego i ilorazowego na manipulowanych konkretach i łatwych liczbach.

Porównywanie różnicowe w koncepcji konstruktywistycznej powinno wywodzić się z aktywności uczniów na początku nauki szkolnej związanych z przyporządkowaniem wzajemnie jednoznacznych przy okazji porównywania liczebności zbiorów. Natomiast porównywanie ilorazowe powinno wywodzić się z czynności uczniów, nawiązujących do mierzenia długości przez wielokrotne przykładanie danej miarki (np. patyczka) do mierzonego przedmiotu.

Głównym celem pracy doktorskiej było zbadanie następujących hipotez:

- (α) Istotnym czynnikiem wpływającym na niezadowalające wyniki nauczania zadań na porównywanie różnicowe i ilorazowe jest niewłaściwa tradycyjna metodyka nauczania takich zadań.
- (β) Można istotnie poprawić wyniki nauczania odchodząc od tradycyjnej metodyki i zastępując ją metodyką opartą na podejściu konstruktywistycznym. Należy położyć większy nacisk na aspekty semantyczne zadań, na konstruowanie znaczeń w umysłach uczniów, a zmniejszając nacisk na aspekty syntaktyczne (jakkolwiek powinna być zachowana równowaga pomiędzy nimi).

W związku z tak postawionymi hipotezami, przeprowadziłam następujące badania:

- (1) Badanie opanowania porównywania różnicowego i ilorazowego przez uczniów rozpoczynających naukę w klasie IV (tradycyjnie nauczanych).
- (2) Badanie uczniów rozpoczynających naukę w klasie I, dotyczące wprowadzenia w porównywanie różnicowe wprost z porównywania liczebności zbiorów na konkretach.

- (3) Badanie uczniów kończących klasę II, dotyczące wprowadzania ucznia w porównywanie ilorazowe wprost z mieszczczenia na konkretach.

Wyniki dotyczące konstruktywistycznej propedeutyki nauczania porównywania różnicowego i ilorazowego okazały się optymistyczne. Prawie wszyscy uczniowie poradzi sobie w prostych sytuacjach z odpowiedzią na pytanie: *O ile więcej?*, pomimo że nie uczyli się tych zagadnień w szkole. Sugeruje to, że uczniowie rozpoczynający naukę w klasie I mają już pewną pozaszkolną wiedzę osobistą na ten temat. Badani uczniowie doskonale poradzi sobie również z zadaniami na porównywanie ilorazowe powiązane z mieszczieniem i wielokrotnym odkładaniem danej długości. Nieco trudniejsze okazały się dla nich zadania dotyczące porównywania ilorazowego na liczebnościach zbiorów.