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A story-telling approach to the introduction of the multiplicative structure at kindergarten[1]

Abstract: We present a design study based on a story-telling approach, to introduce multiplicative thinking at Kindergarten with an algebraic perspective. Starting from some theoretical assumptions about the use of narration in Math Education and about the psychological roots of multiplication, we build a suitable narrative context in order to promote children's actions consistent with such roots. We analyse the development of this path and its management, emphasizing the special role played by the dialectics between actions upon objects and representations.

Introduction

Storytelling continues to arise the interest of practitioners in different research fields, from psychologists (starting from pioneering studies, like e.g. Bettelheim, 1976) to anthropologists (since Propp, 1958, to Gottschall, 2012), to the effect that it is widely recognized as a universal expression of human culture (everyone everywhere enjoys stories) and therefore as a crucial item in human cognitive development. In this sense a story is not just some more or

[1]This paper is an extended version of (Mellone, Spadea & Tortora, 2012).

less casual entertainment, rather it reflects a basic powerful form in which we make sense of the world and of our experience[2].

More recently, the value of storytelling is being affirmed also within the domain of education. Generally speaking, stories can be seen as controlled contexts for understanding and managing even complex aspects of life, where they help to identify meaning threads in several domains and environments. Storytelling can also be used to pursue educational goals in specific disciplines (see for example Egan, 1986). As Liljedhal and Zazkis underline, "*the value of the story to teaching is precisely its power to engage the students' emotions and also, connectedly, their imaginations in the material of the curriculum*" (Liljedhal & Zazkis, 2009, p. 3). In particular, telling a story can be a tool to make sense within mathematics, for example to 'see' underlying mathematical structures: in a story context we can offer pupils an authentic modelling experience by leading and supporting them in recognizing the mathematical structures as embedded in *actions* and *facts* and embodied in human ways of managing them.

The idea of *action* at the roots of abstract thinking, in particular when the construction of arithmetical structures is involved, emerges from different perspectives. Some recent studies coming from neurosciences have pointed out how "abstract thinking" is built using the same circuits that we use to perform actions[3] (Gallese & Lakoff, 2005). Moreover, as the discover of mirror neurons has shown, the same brain circuits are excited both when we ourselves make an action or when we see another agent making that action or even when that action is only evoked, as it happens in a narrative context. This new and at the same time old awareness induces to deepen the study of cognitive strategies recognisable as roots of mathematical structures and procedures, and to design learning environments to drive their evolution. Our research group has been working for several years in this field, in particular designing and developing prototypes of long-term paths for kindergarten and primary schools, aimed at promoting pupils' arithmetical competencies arising from actions and from reflection upon them (see e.g. De Blasio *et al.* 2008; Iannece *et al.* 2009 and 2010; Mellone & Grasso, 2008; Mellone & Pezzia, 2008). Of course, we are aware of the basic difference between actions upon objects and mathematical operations, but we think that identifying actions and reflecting upon them is crucial in order to exploit them as roots for the related mathematical ope-

[2]For a totally different perspective about this, see (Strawson, 2004).

[3]Recently Stanislas Dehaene has proposed a fascinating theory, called "neuronal recycling", in which he advances the powerful hypothesis that cultural inventions invade evolutionarily older brain circuits, inheriting, in so doing, many of their structural constraints (Dehaene, 2007).

rations which in turn will constitute the true base for the whole disciplinary structure.

In this paper we present a design study realized by our university team in collaboration with an expert teacher (one of the authors): a path is developed to introduce the multiplicative structure at a Kindergarten level. A suitable narrative context was created in order to induce actions consistent with the conceptual roots of multiplication, identified according to some theoretical assumptions. The combined use of the narration and of dramatizations and representations of some crucial parts of the story, has the goal of identifying actions at the roots of arithmetic. Along the didactic path, we also recognize some examples of semiotic mediation, in the sense of (Bartolini Bussi & Mariotti, 2008), as pivotal turning points in the development of pupils' representations and mathematical knowledge. One of the specific aims of this design study is also to recognize some features of narratives and of the practice of storytelling that can enable mathematical meanings to emerge among the several ones intertwined in a narrative context.

We present the theoretical perspective used for this design study in the next section, after which we briefly clarify the methodological equipment that has informed our experimentation; then, in the widest section we describe and analyse the main parts of the experimental path, and finally we draw some conclusive remarks from our experience.

1 Theoretical background

In the last decades many studies have been developed about the cognitive roots of arithmetical structures. Without pretending to be exhaustive, we can distinguish two trends: to look for a correspondence between a given arithmetical operation (or arithmetical structure, i.e. the operation, its inverse and their properties) and an action scheme, as in Piaget or in (Davydov, 1992); or to classify the different situations in which the use of the operations is needed (e.g. Vergnaud, 1983; Greer, 1992; Steffe & Cobb 1998). The second kind of studies seems very useful especially for detecting cognitive problems that might underlie a given recurrent mistake, whereas, in our opinion, the first approach is more fruitful for planning class activities, particularly when arithmetic is addressed since the very beginning in an algebraic perspective: to be more precise, we use the term *algebraic*, as in (Iannece *et al.*, 2010), when a special attention is devoted to the structures which underlie numerical operations and relations.

In particular, we refer to Davydov's suggestion that, rather than viewing a direct correspondence between each action and each mathematical operation, makes the whole multiplicative structure depend on a specific *psychological* need. In his vision, indeed, the psychological root of multiplication is identified in the change of measure unit, when some magnitude has to be measured: "*If the magnitude of an object is depicted by A, the small unit of count by a, the large unit by b, then the system of operation, carried out by determining the numerical value of A* ***indirectly*** *through a, can be expressed by the following formula:* $\frac{A}{a} = \frac{A}{b} \times \frac{b}{a}$" (Davydov, 1992, p. 11, see Fig. 1). This idea is referred both to continuous and discrete magnitudes, indeed in the author's approach the experience of measuring continuous quantities is viewed as preliminary to the introduction of numbers (Davydov, 1982). In such perspective, counting itself may be conceived as the particular measuring process of sets of discrete objects. Therefore, Davydov suggests that managing continuous quantities should precede the introduction of natural numbers since early education. Of course, the problem is not that of assigning a didactic priority to the discrete or the continuous case, but that of designing and realizing effective strategies of early cultural mediation, in order to create solid links between the discrete and continuous aspects of numbers, as we argue in (Iannece *et al.*, 2009). However, in this paper, we use Davydov's idea about the root of multiplication as change of measure unit in a context of discrete magnitudes (Fig. 2).

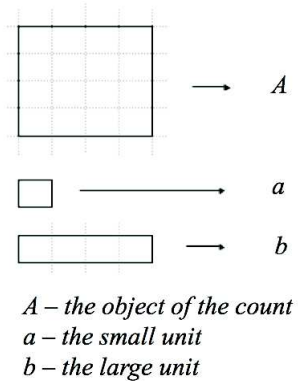

Figure 1.

According to Davydov's studies, we think that children can explore the arithmetical structures in an algebraic perspective since kindergarten, by exploiting their cognitive strategies and using their languages. At this level the graphic representation plays a special role since it can be viewed both as a perceptual counterpart of paradigmatic/structural aspects and as a cognitive support toward generalization (Stetsenko, 1995). According to Vygotsky (1978), sign and meaning arise together, so the integrated use of graphic, verbal and symbolic representations lets the concepts as well as the expressive tools develop. The functional role of drawing in children's cognitive and emotional development and its interplay with other communication tools have been explored in the socio-

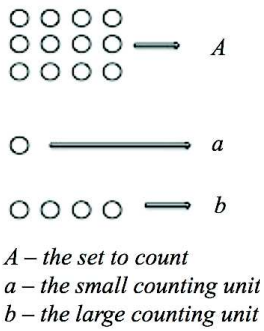

Figure 2.

cultural perspective. In particular it has been observed that "*young children do not radically differentiate between drawings and writing* [...] *At least part of this confusion must be due to the fact that children view both drawing and writing primarily as ways of communicating with others*" (Stetsenko, 1995, p. 150). In other words the intertwined development of drawing and of written and oral language in early childhood can be related to children's need to gradually grasp adults' means of communication. In this study we will show how this knot can be exploited and driven toward "paradigmatic" aspects of language, in particular by promoting the *array* (see Fig. 2) as an effective representation of the multiplicative structure.

This study moves from our research interest on graphical representations to explore in addition the role of tales to build mathematical meanings. Moreover, following the semiotic mediation framework developed by Bartolini Bussi and Mariotti (2008) that put a task at the centre of the didactic cycle, we insert this task in a narrative context. As we will see, this allows to carefully follow the developments, lead by the teacher, of signs/tools proposed by children. Following the *semiotic mediation* model proposed by Vygotsky, the signs can work both as tools (artifacts) aimed at external (operational as well as communicative) activities and as signs on the psychological level. In this direction the progressive and intentional introduction of artifacts (in our case the array) into educational activities can play a crucial role if teachers use them as instruments for *semiotic mediation*: "*In particular, the teacher may guide the evolution towards what is recognizable as mathematics. In our view, that corresponds to the process of relating personal senses (Leont'ev, 1964/1976, p. 244 ff.) and mathematical meanings, or of relating spontaneous concepts and scientific concepts (Vygotsky, 1934/1990, p. 286 ff.)*" (Bartolini Bussi & Mariotti, 2008, p. 754). The tool can be viewed as such in two ways: firstly for the pupils it is a tool for accomplishing the given task, secondly it becomes a tool for the teacher for helping students to extract mathematical meanings from the situated context of the task. Moreover, the authors define a particular class of signs, the *pivot signs,* which can be used by the teacher to foster the evolution of the situated signs and meanings towards cultural (i.e. mathematical) signs and meanings. Indeed, the crucial feature of these signs is their polysemy: they can refer both to meanings developed within the specific task and to more abstract mathematical meanings.

As we have argued in the introduction, the idea of inserting the core task of our design study in a narrative frame is linked to the awareness that children's imagination is one of the most powerful and energetic learning tool (Egan, 1986). Children are particularly sensitive to contexts where human intentionality can be recognized, and use this key to interpret and give meaning

(Donaldson, 1978). To understand people's reasons and feelings is also what Jerome Bruner identifies as the specificity of the "narrative" thought, in contrast with the "paradigmatic" or "logical-scientific" one. Narrative thought is characterized as an open space of possibilities that frames the temporal sequence of events and the choices taken time after time, while the paradigmatic one is linked with the formal constraints (logical, physical, and so on), which rule the space of possibilities opened by the tale. The complementarity of the two kinds of thoughts is put in evidence in several Math Education contexts, as in problem solving activities (see e. g. Mellone & Grasso, 2008). About this, Zan (2011) observes how a word problem is both perceived as a description of a 'human' situation, and analysed for its paradigmatic features with the goal of solving a question. For this reason the mathematical information in a word problem has to be consistent with the narrated story and vice versa, in order to get resonance between the narrative thought and the paradigmatic one. Otherwise the risk is to produce a "'*narrative rupture' in the text of the problem, i.e. the question and the information needed for the solution are not consistent from the point of view of the narrated story*" (Zan, 2011, p. 341).

As we will show in the sequel, we have tried to take this need into account in building our own tale, by describing characters who are moved by understandable feelings and goals, and by linking such feelings and goals with the mathematical questions. Also the teacher's management of the activity has been careful in connecting and balancing the human and the paradigmatic aspects of the story.

Indeed, one of the crucial differences between a real problem solving situation and a school word problem is that the latter is *hetero-posed*, that is who poses the problem is someone (the teacher or the textbook) different from who has to solve it (Zan, 2011). Therefore any word problem has to be expressed by a (generally, written) text, to communicate to the solver *what* he/she has to solve, through an explicit request. On one hand, this is an advantage, in that the students too must become able to use a language adequate enough to express their argumentations and solutions; but on the other hand, any hetero-posed problem suffers from some lack of authenticity. On the contrary, in real problem solving situations where problems arise directly from who has to solve them, meaningfulness and authenticity are guaranteed, but since verbalization is not necessarily requested, the development of language and of paradigmatic thought could be inhibited.

Well, what we do think, is that problems arising in suitable storytelling contexts yield the advantages of both real life and word problems. They are *hetero-posed*, nevertheless they have the typical features of human activities, intentions and feelings. By the *identification process* naturally trigged in a

story listening (Bettelheim, 1976), children are emotionally involved and can feel problems arising in a story telling context as authentic, memorable and meaningful. As a consequence, also the mathematical meanings embedded in a story context can easily emerge and be impressed in pupils' minds.

2 Methodology

The experience we are going to analyse comes from a wider research project carried out for several years in Naples by some researchers in Math Education and a group of Kindergarten, Primary School and Lower Secondary School teachers. This group has been working at building and validating prototypes of long-term paths for the teaching/learning of arithmetical structures in an algebraic frame. A common feature of these activities is the assumption of a Vygotskian perspective about learning, in particular on the role of signs in the semiotic mediation process. The research group has been working for several years about the use of the *array* as support for multiplicative thinking. In this study we explore the possibility of using such representation with 4-5 year-old children. Therefore, in order to introduce multiplicative thinking in an algebraic perspective, we have built, in collaboration with a kindergarten teacher-researcher, a path that starts with the telling of a story. However, our goal was not just to validate in a class activity a path packed in advance, but rather to be able, starting from an initial plan, to repeatedly modify the path itself, according to classroom events and interactions, following in this way the typical design study methodology (Cobb *et al.*, 2003). Indeed our study owns the typical ecologic and engineering features of the design experiment.

In Italy since 1991 official guidelines for kindergarten suggest to organize educational activities around "Fields of Experience", where children are offered a set of experiences within a bounded domain, in order to be actively involved in the process of knowledge building. Within any field of experience children give sense to their own actions, make discoveries and develop linguistic, procedural, etc., skills. The particular activity we are going to describe can be framed in the section "*learning about the world*" (MIUR, 2012) and concerns several knowledge domains. It took place in the third year of kindergarten. During the previous years the children had experienced some facts about the number line and the proportionality relation, by working with rhythms and patterns.

The class was composed of 18 (4-5 year-old) children and the designed path lasted three months, intertwined with other activities. Such a long time has been judged useful to let children feel involved in the rich story and apply many reading lenses (not only the mathematical one) to deeply understand

the different meanings of the narration. The didactic implementation of the path was cared and managed by the class teacher, while a researcher collected the empirical data. These include children's drawings, transcripts from class discussions, photos, audio and video recording.

Moreover, in order to further reinforce the *identification process* naturally triggered by any activity of story telling, we have asked the children to dramatize parts of the story. We have already explored this opportunity of letting children directly experience the actions described in a narration, see e.g. (Pezzia *et al.*, 2008). In our view the dramatization, arranged along the development of the story, represents a crucial step of the process by which the children, supported by the adults, starting from actions, and using specific paradigmatic tools, in our case the *array*, experiment how the mathematical structures gradually and quite naturally emerge.

In the next section we will illustrate the main parts of the activity by selecting some excerpts from it. Along the educational path, we will analyse children's representations (language, drawings, gestures), in particular pointing on the polysemy features discussed in the theoretical framework, in order to identify some notable turning points of their cognitive development. We will also observe the supporting role exercised by the teacher, and will follow the design and the redesigns of the activity (according to the design method paradigm).

3 The tale of the gluttonous king and the didactic path

The story by which the path starts has been invented to provide a narrative context for a situation where a change of measure unit occurs. The story tells the adventures of a king's servant who has to do several trips through a tangled wood in order to reach a bakery and to buy cakes for the royal family, composed of four members. The cakes are carried 'two at a time' (first change of the measure unit in comparison with the usual way to count 'one at a time") since the oven takes only two cakes out each time, one chocolate and one strawberry cake, and each royal member wants to taste both. At the end of the story the teacher asks children to help the servant to pay the bill, knowing the total amount of the cakes bought (here, notice the care for consistency between narrative and paradigmatic aspects). Of course, the teacher tells the story emphasizing the crucial point she wants to focus on, namely the relationship between quantities of cakes and trips; but, as usual for her, the story is also enriched by every sort of details, concerning the different characters and the sequence of events; moreover, the verbal language

is accompanied by the gestural one, so quite naturally children express their wish for a mime show that would dramatize the story.

It is just Carnival time, and the teacher decides to organize THE GLUTTONOUS KING party: each child has to build his own crown adorned by several kinds of cakes (cut and pasted from newspapers or drawn by the children themselves, Fig. 3-4). Then the teacher, in order to support an *identification process* of the children with the character of the king, let them, wearing their own king dresses, be involved in a *pantomime*, played by reproducing and even inventing gluttonous behaviours. In this way the children understand motivation, intentions and actions of the principal character.

Figure 3.

Figure 4.

The first phase of the work is devoted to reflect upon the meaning of the words: the children are invited to repeat the story and to discuss about the situation and the characters, and then the teacher suggests a proto-analysis of the text by means of the classification of the words into noun-words, quality-words and action-words, like in the following examples, taken from phrases used by the children.

Noun-word: *Gluttonous king.*

Quality-words: "he is *fat* because he eats a lot of cakes", "if he doesn't eat them he gets *nervous*".

Action-words: "he always *wants* sweet things, like my little brother", "he's *got angry* and *doesn't want to speak* to anybody".

As it is well argued by Egan (1986), story-telling/listening is in its own nature a rich and inter-disciplinary activity that advances the intelligence of a child in a lot of directions. This phase of the path, seemingly concerning only the linguistic area, is useful also for our goal of leading children to live mathematics as part of human activity. Indeed, in our view emphasis is on any kind of human meaning: as we claimed in the first section, the narrative thought can support the paradigmatic one via the listening and the dramatization of a story, until a symbolic representational level.

Figure 5.

In the second phase the teacher asks children to represent the story by a drawing, in order to identify the things that impressed more the children, and to orient her didactic mediation toward both her own goals and children's needs. The children draw only the passages of the story that turn out to be more meaningful or simpler to be represented. The "paradigmatic" aspects are left apart, certainly also because the previous work about the characters has favoured the narrative thought (see e. g. Fig. 5).

The teacher decides to resort again to a bodily work, as a premise for reflecting on actions, and also to force more "paradigmatic" representations useful for catching the mathematical meanings of the story. After all, if we recognize that actions do lie at the roots of comprehension, then we have to act. A motor activity is organized to reproduce the path covered by the servant from the castle to the bakery: six traffic cones and a cloth tunnel represent the wood, so a gymkhana has to be travelled to reach the bakery (the class kitchenette), that contains two tiles as the cakes (Fig. 6-7-8). Each child performs his/her own servant's path in order to interiorize the trip as a meaningful experience. This means to carry a plate, to reach the bakery and to buy two, one chocolate and one strawberry, cakes, as many times as needed to satisfy all family members.

Figure 6.

Figure 7.

Figure 8.

Such a complex staging corresponds to the exigence of carefully distinguishing the three "dimensions" involved: the "things", i. e. the cakes, the "times", namely the trips, and the "things-at-a-time", that is the constant number of cakes brought during each trip across the wood. Finally, the children are invited to represent by drawings the trips made and the cakes taken each time. This time, all the children try to answer the numerical question: nobody feels inadequate, everybody is involved. This enhances children's self-esteem and confirms the effectiveness of the teaching methodology employed, which includes a careful balance between the needs that all pupils live successful experiences and that nontrivial disciplinary contents are addressed.

In children's drawings a greater attention to the paradigmatic aspects of the story arises, maybe supported by the motor activity and, in particular, by the iteration of trips. In all the drawings we can "see" the multiplicative structure expressed by the grouping: the cakes are linked to the trips and drawn as rhythms of repeated plates (Fig. 9), some children represent the trips as lines or in some cases as half-circles (Fig. 10), perhaps recalling the shape of the cloth tunnel (compare Fig. 7). Only two children (one of which less than 5 years old) use a person-marker (Fig. 12), while only Maria Giovanna outlines a sort of array (Fig. 11). We recognize in Maria Giovanna's drawing a *pivot sign*, where the array arrangement "two times four" of the cakes represented by circles fits well with the particular situation of the story, but is potentially suitable for any other "two times four" multiplicative situation, and of course the insight of the array arrangement can be as well extended to other numerical multiplications. Ivana traces also the numerals 2 and 4, although as simple drawing ornaments (Fig. 12).

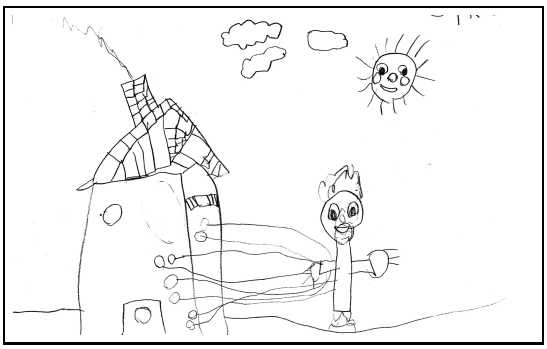

Figure 9.

Figure 10.

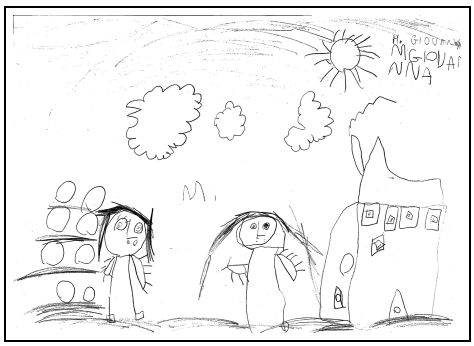

Figure 11.

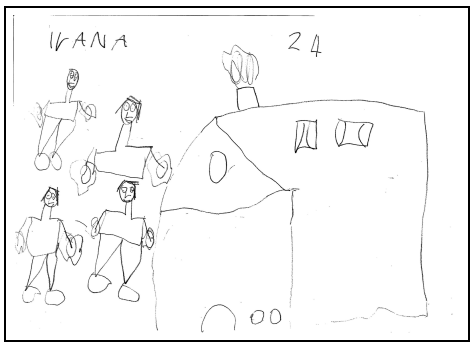

Figure 12.

We have already observed above how fuzzy is the boundary between drawing and writing at this age (Stetsenko, 1995), both abilities being linked to children's attempts to appropriate adults' means of communication.

The day after the teacher orchestrates a mathematical discussion about the different representations. This is a crucial part of the teaching mediation based on children's reflections upon their own and their fellows' behaviour. After that the drawings are distributed to the pupils, their comments rapidly focus on the effectiveness of the representations in order to select and then to share the best symbols. Everyone illustrates the way he/she has represented trips and quantities, then everyone is invited to redraw his/her symbols on the blackboard. In this way all symbols are under the eyes of everybody, and thus, after an analysis and a comparison of their features, the children choose the most effective among them. (On the blackboard of Fig. 13 we can recognize the symbols used in the drawings of Fig. 9-10-11). The half-circle is selected as the best representative of a trip, against teacher's expectation, who hoped children would have chosen, since this phase, the array as a powerful sign to represent at the same time trips and quantities of cakes. At this time the teacher decides not to intervene in the choice, even though she has clear the need to work on the *pivot sign* produced by Maria Giovanna. The next meeting between the teacher and the research group is so devoted to understand why the children didn't choose the array, although it appears in one of the initial representations (Fig. 11), and why the collective discussion and the teacher's guide couldn't induce this choice: maybe the two dimensions "trips" and "cakes-at-

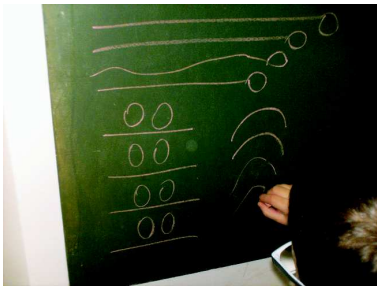

Figure 13.

a-trip" are not so meaningful till that moment, to deserve a special attention and a distinctive mark. Therefore, according to a Vygotskian approach, we decide to introduce an artifact to better represent the two dimensions of the multiplicative structure: a rectangular tray divided in two-times-four boxes, into which the children can arrange the cakes during the dramatization. The teacher tells a further part of the tale of the Gluttonous King, in which the number of cakes for each trip is inverted with the number of trips, in order to suggest a two-dimensional representation, as well as to evoke a new change of measure unit (from two-at-a-time to four-at-a-time):[4]

> The Gluttonous King wants to organize a party for his family, where everybody will get a chocolate cake and a strawberry cake. Knowing that the baker has now a larger oven where four cakes at a time can be cooked, what has the servant to do?

Maria Giovanna: He must cut the cakes into small pieces.

Teacher: But the King doesn't like small pieces, since he is gluttonous!

Martina: Otherwise they need a still larger oven.

Mattia: No, the servant must make several trips, carrying two cakes for each trip.

Teacher: Look, I have prepared a tray for arranging the cakes. So, what has the servant to do?

Sara: He must go to the bakery, buy the cakes, and put them on the tray.

The above transcript shows how the teacher mediation tends to justify the resort to the artifact-tray. It is also interesting to notice how she refrains from directly intervening on Mattia's difficulty, who has not caught the change of measure unit with respect to the first part of the story. Instead, she prefers to address the whole class, using a different strategy: she encourages the children to dramatize again, making the same path but using this time the special tray to carry the cakes. And when Mattia, at the end of his path, arranges the cakes grouping them by two, as in Fig. 14, the teacher stops the play and lets all the children look at the tray.

Ciro: In this way it looks like the servant has gone twice and has got two cakes each time.

Mattia [resentful]: No, I went only once [Mattia changes the cakes arrangement on the tray, putting them in a unique row].

[4]Some fragments of a similar path have been presented and discussed in (De Blasio *et al.*, 2008).

Figure 14.

Figure 15.

Obviously, what is really crucial is not the artifact-tray as itself, but the teacher's mediation that helps children to catch, from the collective discussion, the analogies and the differences between the two parts of the story. The social interaction works very well at this moment: Ciro's remark, which is in better accordance with the use of the tray, immediately produces Mattia's reaction. Teacher's suggestion to reason upon his actions and not only upon the narrated story turns out to be effective, indeed if Mattia gets angry for being misunderstood, from the other side he is ready to conform himself to the rules of the game. Finally the teacher invites a child to draw all the cakes on the blackboard, promoting in this way another step from the representation of the experience through the object-tray toward a representation through symbols. Martina goes to the blackboard and draws a first row with four circles, then she begins a second row, as in Fig. 15. Martina's way of reporting the 'mathematical story' can be described as a sort of rhythm, already implicit in some previous drawings, where the circles displayed in two rows clearly prefigure a typical array.

Figure 16.

Teacher: Let's look at Martina's drawing. What does it suggest to you?

Mattia (and others): That he's gone two times . . . and has got four cakes.

Teacher: Do you agree that now we understand what the servant has done? [She takes two equal 'two-times-four' trays and puts them close to each other, but differently oriented, see Fig. 16] What has changed?

Antonio: Now the oven is larger and cooks four cakes at a time.

Teacher: But is the number of cakes the only thing that is changed? How many times the servant comes from the bakery with his full tray to satisfy everybody?

M. Giovanna: Twice.

Chiara [pointing at the columns of the array]: One and two, one and two.

Mattia: I don't see any change!

Teacher: Are you sure? I see a difference...

Chiara [her hand traces a turning in the air]: They become equal just if we turn them.

Martina: Of course, since in this case the cakes are four and the times are two, while in the other case the trips were four and the cakes were two.

Ivana: But they are eight, anyway.

The use of the tray in the simulation has induced Martina to appreciate the value of the array in representing the performed actions. However, the teacher prefers to go back to the material representation via the tray, to promote an effective synergy between syntactical and semantic aspects of the story. This helps the children to focus on what stays and what changes between the two situations, in order to discover the commutative property of multiplication. Moreover, teacher's pressing requests of precision stimulate a refinement of children's linguistic expressions, supported by reference to the concrete experience or, as well, by representation tools like the arrays. For example, for Martina it is important to drive attention to the concrete meaning of what they did, while Ivana's statement points directly to the mathematical operation, overlooking the details of the two situations: anyway, they both obtain the same result of 8 cakes[5]. We recognize again the potentiality of *pivot signs* in Martina and Ivan's statements; of course, such observations can be extended to other multiplicative situations evoked by actions, in that, as we already noted, they refer to the commutative property of the multiplicative structure.

In the last part of the school-year the teacher has proposed many variants of the story, in which the numbers of trips and cakes varies, but with the usual care for the above discussed consistency between narrative and paradigmatic aspects. We have observed that not all the children used the array to represent the different situations. Clearly, our goal was not to impose the array, that is to train them to adopt a mechanical automatism, rather our goal was, in

[5]Similar behaviors have been observed in 3rd grade children (see Mellone & Pezzia, 2008).

Vygotskian words, to promote a "cultural" imitation, that is to drive children to reproduce a strategy by their own after having experienced its effectiveness.

For this purpose, at a certain moment the teacher decided to change the experience context and to work with rhythms of sounds. The children were invited to record the sound patterns, by recognising a group of notes repeated many times. As usual, they worked representing, discussing and sharing. But this time they spontaneously chose to represent each pattern of symbols, corresponding to a sequence of repeated sounds, one below the other as in Martina's ingenious drawing (Fig. 15), instead of along a line. In this way the children built an array, putting in evidence the role of multiplication and favouring an exploration of its properties. Experiencing the effectiveness of the array in a new context has allowed children to recognize the structural analogy between two different situations. Finally, we can report that, during some further variations on the theme, children did make an autonomous use of the array. Moreover in this further development of the path we could appreciate the fact that the majority of children used typical expressions of the multiplicative jargon like "cakes-at-a-trip" and cared to specify when the numbers were used referring to cakes or to times.

4 Some conclusive remarks

The above description of the path shows that what really allowed the development of mathematical skills in our group of very young children is the recourse to many different education tools, which, thanks to a careful teacher's mediation, created a rich and fruitful didactic environment. The features and the advantages of each of these tools, like the use of storytelling, of forms of dramatization, the request to produce representations and the use of particular objects, like the array, as semiotic mediators, have already been discussed in depth in several studies, some of which have been recalled in the previous sections. In our design study the use of storytelling is the main teaching resource, but it is put in synergy with other tools. For example the dramatization played an important role in allowing children to focus on paradigmatic aspects of the tale. Indeed, the translation into direct actions of crucial situations and facts of the story made children recognize the arithmetical meanings, in particular the multiplicative ones, among the rich variety of meanings embedded in the narrative context. In this sense it is very interesting to compare the drawings made by children before the dramatization of the trips (Fig. 5) and the ones made by them after the dramatization (Fig. 9-10-11-12).

In this rich context, the particular arrangement of the cakes in a four-times-two array, proposed by the teacher by means of the didactic "trick" of the tray, was particular suitable to lead children to start using a genuinely mathematical language for multiplicative contexts, like "*cakes-at-a-trip*". Moreover, the story itself offered a valuable opportunity, providing a natural and concrete meaning to one of the crucial properties of multiplication, that is commutativity, that, on the contrary, is usually presented as a merely syntactical property. This is witnessed by Martina's observation: "*In this case the cakes are four and the times are two, while in the other case the trips were four and the cakes were two*".

However, the analysis of the path shows that there is a jump in level between the use of the array at the beginning of the experience, when the children, like Mattia, used it without exploiting its power as a structural representation of the multiplicative structure, and in the second phase, when it was used "autonomously" by Chiara in order to discover new arithmetical facts, as the commutative property of multiplication. This second way of managing the array marks the threshold toward a use of the array as a tool working on the psychological level. In this sense the adults' cultural mediation in providing the array has to be very careful, by taking into account the subtlety of this step. Therefore, we believe that what is very crucial is the role of the teacher, who has to put in resonance the different educational tools adjusting his/her actions with the contingencies and the reactions of children. The role of teacher is crucial also in recognizing the *pivot signs* produced by the children during the activity and allowing their development. It is only for this reason that it is not so surprising that five years old children can discover arithmetic facts and, at the same time, become able to control meanings and use quite refined linguistic expressions, as we can appreciate for example in Martina's above words and indeed in many other excerpts.

Finally, we believe that our study confirms Davydov's idea (1992) about the essential role played by the change of measure unit in capturing one of the core meanings of the multiplicative structure. We tried to use this suggestion in the second part of the story by inserting a change in the number of cakes taken each time. Indeed we used a narrative device by telling that the baker could cook in a larger oven four cakes at a time instead than two at a time. As we have seen, this change in the rhythm of iterated actions created a good environment in which children begun to explore and discuss some aspects of the multiplicative structure.

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Wprowadzenie struktury multyplikatywnej w przedszkolu z wykorzystaniem opowiadania

S t r e s z c z e n i e

Przedstawiamy projekt wprowadzania dzieci przedszkolnych w myślenie o własnościach mnożenia poprzez wykorzystanie historyjek. Bazując na teoretycznych założeniach dotyczących wykorzystaniu fabuły w edukacji matematycznej oraz na psychologicznych podstawach mnożenia, budujemy odpowiedni kontekst opowiadania wspierającego działania dzieci zgodne z przyjętymi podstawami. Analizujemy rozwój tej ścieżki i sposób jej modyfikowania, podkreślając szczególną rolę związków pomiędzy działaniami na obiektach i ich reprezentacjami.