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Algorithmisation in teaching mathematics

Abstract: The algorithm is a concept that is often neglected in teaching mathematics. It is worth changing it because the development in mathematics occurs inter alia when general problem-solving patterns are discovered. Exploring mathematics by students can be supported by algorithms. Many objectives of mathematical education can be achieved at every level of education with multiple teaching benefits by applying the elements of algorithmisation. A teacher who is aware of these benefits can, for example, use algorithms to develop students' logical and mathematical thinking and to revise the material or to develop students' habit of validating the results.

1 Algorithmic and conceptual approach to mathematics

Through analysing various methods of solving tasks, proving theorems and ways of presenting definitions and theorems, we can see that school mathematics has twofold nature: conceptual and algorithmic. We cannot, however, make a division into conceptual and algorithmic mathematics because these two aspects constantly permeate each other and they are equally important. In order to analyse conceptual elements, we need computational methods. On the other hand, algorithms treated separately from concepts are only automatic patterns for calculations.

The development in mathematics shows that these two aspects are closely related to each other. Since time immemorial, mathematicians have done numerous calculations while conducting research. They define basic notions after they notice certain relations and then they analyse the mechanisms behind a given problem. This stage often leads to the formulation of certain hypotheses. When they are able to deductively prove the new theorem, calculations are no longer needed. It often happened that the lack of appropriate computational methods hampered the calculations or even made the problem impossible to solve. Therefore, scientists have always made efforts to discover algorithms for solving problems. A good example illustrating this direction in the development of mathematics is the history of algebra. From the third century to the late nineteenth century it was mainly preoccupied with algebraic equations and the ways of solving them. At first mathematicians tried to solve linear equations in one unknown. Their research led to the discovery of zero and negative numbers. The formulas for solving second degree equations were known in ancient times. Later on, mathematicians discovered algorithms for solving equations of the third and fourth degree. Throughout the years scientists were trying to discover general methods of solving algebraic equations of the nth degree. But it was not until 1799 that Paolo Ruffini showed that for the equations of the fifth degree such methods do not exist, although his proof was not considered valid at that time. Few years later Niels Henrik Abel showed precisely that equations of more than the fourth degree cannot be solved by means of four basic operations (addition, subtraction, multiplication and division) and roots. However, further research showed that roots of some certain specific polynomials can be found. Evariste Gaolis, a mathematician who lived in the first half of the nineteenth century, specified which equations can be solved that way. In order to solve the problem, he created a new theory, which constitutes the basis for modern-day algebra. Galois introduced such terms as: group, normal subgroup and field, and his works gave impetus to the development of semigroup or quasigroup theory and ring theory inter alia. Therefore, the search for general methods for solving more and more complicated equations was one of the engines of development.

The above-mentioned history illustrates perfectly mathematicians' way of working, for whom discovering new concepts and relations is usually connected with striving to discover more general ways of solving different problems. Algorithms are created in such a way that they can resolve a very large class of tasks using just one and the same system of operations. Thus, they provide technical means to deepen the conceptual knowledge of mathematics.

Taking into account both sides of mathematics, teachers should emphasize the role of algorithmic elements in mathematics and appreciate the importance of its conceptual elements at the same time. Students learning a new definition or a theorem must realize the functional and operational sense of this information, i.e. they need to know what they can do with it, what operations are possible and what calculations they can perform. If this does not happen, the information will be useless. In addition to the knowledge of basic algorithms and ability to use them correctly, a broader understanding of the algorithm is necessary, which allows the student to produce an abstract object (concept) which is the result of numerical and algebraic calculation. A teacher who in his work consciously emphasizes both conceptual and algorithmic approach to mathematics allows students to learn this field of knowledge comprehensively. This encourages the development of students' mathematical thinking because theoretical teaching together with functional teaching enable intuitive and formal understanding of new problems.

2 Educational benefits of teaching with the use of algorithmisation

The use of algorithms in teaching mathematics can bring a lot of educational benefits. In order to see the advantages of this method better, it is worth recalling what an algorithm is and what features it should have.

In the workbook for second-grade students of junior high schools the following statement can be found: "The algorithm is an accurate scheme or description of the solution for a problem expressed with operations that a performer understands and is able to carry out". This definition includes the basic features of an algorithm. One of them is the elementariness of operations. It means that each operation appearing in the scheme is controlled by the student. This feature is relative and depends on the skills of the student at a given level of education because some operations that are not elementary at a certain stage may become elementary in the further course of study.

Each algorithm should be unambiguous, effective and general. Unambiguity means that it should precisely define the sequence of operations leading to the result. Therefore, a student who has mastered basic operations is able to get to the solution of a complicated task by doing the step-by-step activities planned in the scheme. The effectiveness of an algorithm guarantees that the resulting outcome is the correct solution of the task after a finite number of steps, whereas the generality condition means that an algorithm should comprise the whole class of tasks by working on parameters, the specification of which defines a given task. Of course, an algorithm always works in the same way for the same initial data. The feature of unambiguity imposes this.

Regardless of how an algorithm is presented (a verbal description, a list of steps, a block diagram or a programming language), its features impose specific working methods, which results in multiple teaching benefits.

The main advantage of working according to an algorithm is the certainty that it leads to the correct result. Knowing the algorithm exempts from having to search for the solution, which makes the operation more efficient. Thus, the use of algorithms improves calculations and allows automatism. In school a student is forced to learn basic arithmetic and algebra algorithms. It is the prerequisite for further development of mathematical knowledge at every level. Students become familiar with written numerical algorithms very early since they only need to have the knowledge of the multiplication table and the ability to add natural numbers to be able to perform addition or multiplication of multi-digit numbers correctly. However, the introduction of too many ready-made algorithms may not be beneficial. It may cause difficulties in remembering them, which results in confusion between the schemes. Moreover, teaching algorithmic procedures without considering the conceptual and comprehensive approach to the problem may result in a situation in which, in the student's awareness, mathematics will be just a set of isolated computational methods. For this reason, the algorithmic teaching has to be preceded by the conceptual solution to the problem.

Analysing algorithms is beneficial for students too. It allows them to notice the precision and simplicity of a logical sequence of operations leading to the solution through its unambiguity. A student considering the consecutive steps of the scheme can deepen understanding of the problem and notice fixed and variable elements of an algorithm (distinguish between the parameters and the procedure), and focus not on the result but on the method. In this way, the student is looking for the general idea of the logical argument, which may lead to the perception of the algorithm in a broader perspective. A paricularly important advantage of algorithmisation is the ability to create nonverbal forms of presenting relationships and procedures. The graphical representation of the operation increases the chances of a comprehensive understanding of the method.

Teaching mathematics can be done through constructing algorithms by students. A number of mathematical issues can be expressed in the form of an algorithm. This is confirmed in works of Z. Krygowska and T. Rams among others, whereas A. Engel even claims that: "One cannot wholly understand something until one cannot explain it to the computer, i.e. express it in the form of an algorithm". Z Krygowska writes: "It is the most important is that students constructed algorithms as the solutions for the problems by themselves. It can be achieved at any level because every level of contact with mathematics manifests its operative character, and specific categories of mathematical thinking need to be trained at every level of education". Creating an algorithm on one's own forces the student to do a great number of mental activities. He has to plan the operations clearly and explicitly and formulate this plan very precisely as well. This involves multilevel reorganization and reconsideration of the knowledge gained so far. The student must consider which of the operations are basic, which of them will serve as input parameters of the algorithm and what possible solutions to the problem can be. The search for a general method of the procedure which explicitly leads to the correct result forces the student to do some precise, logical thinking. Creating a functional plan requires formal reasoning, which is the main way of thinking in mathematics. Intuitive reasoning, of course, may be helpful in the initial stage of the search for the structure of the solution, but every idea has to be formally verified. Therefore, by creating algorithms the student gets used to the formalization of some of mathematical issues.

Creating algorithms also develops reflective thinking and enables self-control. Having proposed the scheme, the students execute it by selecting a variety of initial parameters. They try to choose the examples independently and often enthusiastically so as to be able to go through each possible "path" of an algorithm. Therefore, the efforts to make a wide selection of examples become the student's goal.

The above-mentioned benefits are reflected in the goals of mathematics education defined both in Polish and European documents. The analysis of the current Polish mathematics core curriculum in accordance with the regulation of the Minister of National Education shows that at every level of education teachers can achieve many of the objectives of mathematics education inter alia through algorithms. Some expressions appearing in those objectives are directly related to the use of schemes (such as the use of algorithms), whereas the other point to the skills that are fundamental in creating and understanding algorithms.

The general requirements presented in Polish current core curriculum which can be fulfilled with the help of the discussed method are listed below. At primary level (classes 4-6), they are present within all the types of acquired skills, namely:

1. Accounting performance: The student knows and uses the algorithms of written calculations and is able to use these skills in practical situations.
2. Using and creating information: The student interprets and processes text, numerical and graphical information...
3. Mathematical modeling: The student chooses the appropriate mathematical model of a simple situation, (...) processes the text of the task into an arithmetic calculation...
4. Understanding and creating strategies: The student carries out simple reasoning consisting of a small number of steps, determines the order of the operations (and thus calculations) that lead to the solution of

the problem, is able to draw conclusions from the information given in various forms.

The learning objectives at higher levels of education can be analysed in a similar way. For each level, statements indicating its link with algorithmisation can be found (e.g. the student builds a mathematical model of a given situation taking into account limitations and reservations, creates a chain of arguments and justifies their correctness).

It is worth examining more specific objectives of teaching mathematics contained in the discussed document. Through working with algorithms one can achieve the objectives associated with intellectual and personality development. These include such objectives as:

- the development of the habit of logical and correct reasoning and practical application of the principles of logic,
- the development of critical and creative thinking and the ability to draw conclusions or to make and verify hypotheses,
- shaping the ability to use patterns and letter symbols in everyday situations,
- the development of the ability to use computer techniques to solve various mathematical problems,
- the development of the ability to lead a substantive discussion which aims at finding the optimal solution,
- the development of the habit of verifying the results and possible correction of errors,
- getting used to careful and solid work,
- teaching to present solutions to the problems and tasks clearly.

Algorithmisation also fits well with the description of mathematical competences presented in the "European Framework of Reference". Mathematical skills that every European should have are primarily concerned with the use of mathematics in everyday life. European objectives include very practical skills, such as: the ability to apply basic mathematical principles and processes in everyday situations at home or at work or the ability to track and evaluate the sequence of arguments, however the acquisition of these skills can be supported by the work with algorithms.

Algorithmisation cannot be treated merely as a tool for achieving specific demands but rather as one of many methods for improving knowledge, acquiring new skills and good habits.

3 The elements of algorithmisation appearing in Polish textbooks

The analysis of Polish mathematics textbooks shows that students deal with the elements of algorithmisation at every level of education. This approach is not very popular – it varies from one textbook to another but it is not totally absent. The function that algorithms fulfil is also seen variously. Children learn only few algorithms which are only the description of consecutive steps that lead to the solution of a problem. More frequently, they are used as a method to acquire other skills. I am going to present some of them along with their description.

3.1 Algorithms as action patterns

The basic algorithms that children deal with at school are the algorithms of written operations (addition, subtraction, multiplication, division). Students learn them in the fourth grade of primary school. It is explicitly stated in many textbooks that they are algorithms. The operations are performed on natural, rational numbers and on natural numbers ending in zeros. They are usually explained with an example, sometimes illustrated step-by-step, as in the example from the textbook "Matematyka z plusem" for fourth-grade students.

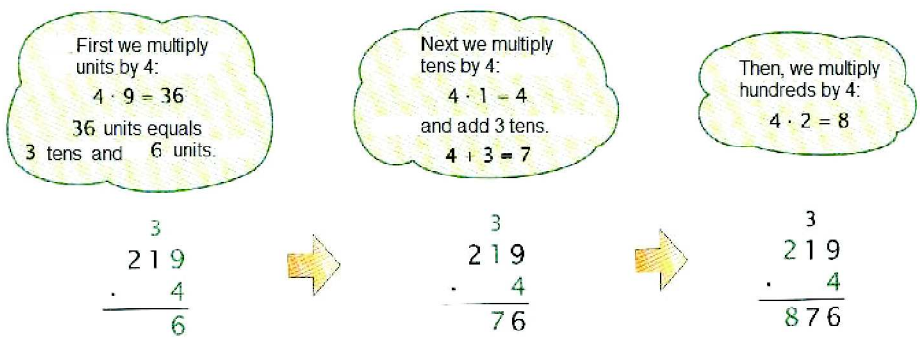


Figure 1.

Before learning the algorithm of written multiplication, students have already learned the algorithm of written addition and subtraction. While preparing to learn the algorithm of multiplication by a single-digit number, students revise the meaning of natural number multiplication. Determining the value of the product of an example 4×219, they add the 219 component four times. The purpose of such an educational step is not only determining the value of the product. Written addition, in which units, tens and hundreds are added separately, draws students closer to the mathematical basis of written multiplication, and by reflecting on the previous algorithm they recall the properties of the operations. It is now easier to realize that in order to multiply an integer by 4 one has to multiply units, tens, hundreds, etc., of a multiplied number and then add the results. This conclusion is first formulated in the textbook and then shown in an example. In this way, the student is prepared to follow the given algorithm.

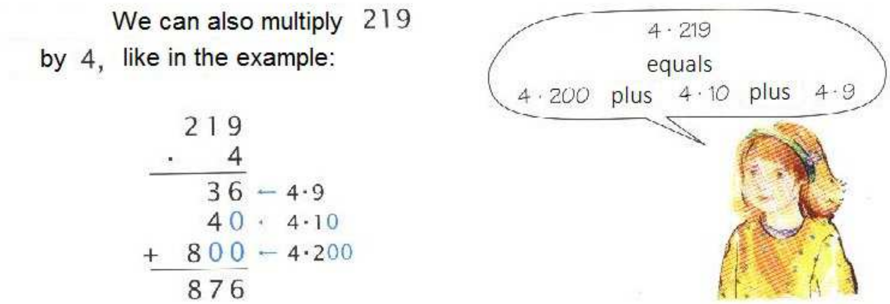


Figure 2.

The next steps to be taken are shown, the way of writing partial result is proposed, whereas with the help of the balloon's comments children can find the relationship between the performance of an algorithm and its compatibility with the properties of the multiplication of natural numbers. There are a few examples of multiplication after the presentation of the algorithm. By analysing these examples students can verify whether they understood the algorithm well. The knowledge of the algorithm develops students' automaticity in the calculations.

In middle or secondary schools students often learn the Euclidean algorithm for determining the greatest common divisor. This algorithm appears in some textbooks implicitly, whereas in some of them it is stated explicitly that it is an algorithm although only an example is given to explain how it actually works.

The algorithm for dividing polynomials is also one of the algorithms taught at school. It is often shown in many textbooks in analogy with the scheme for dividing natural numbers.

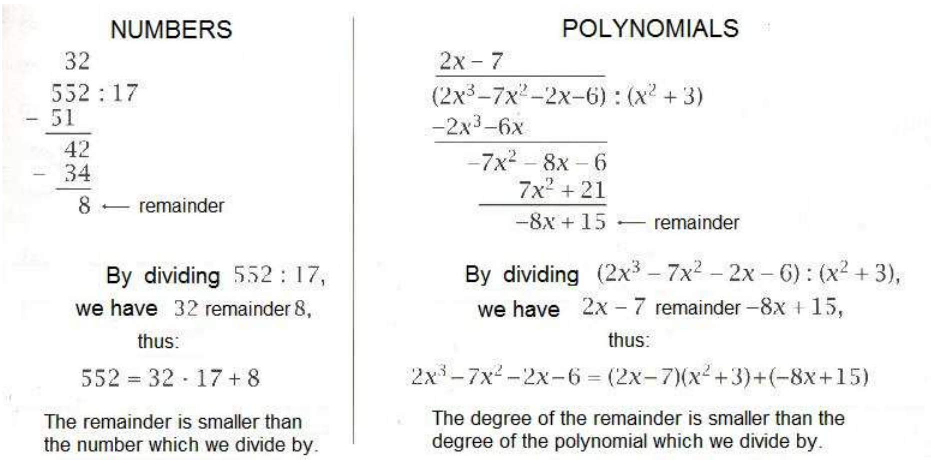


Figure 3.

Such collation allows a student to understand the general idea of dividing polynomials. When dividing polynomials, a student may refer to the corresponding steps in the algorithm for dividing natural numbers. The conclusions formulated in the example are similar to those that can be drawn from the division of natural numbers.

The examples show that although students are provided with a ready-made algorithmic procedure, in order to understand it they perform many actions that are typical actions of a mathematician. They refer back to their prior knowledge, search for analogies and learn how to rationalize both actions and the notation of mathematical procedures.

3.2 Algorithms aiming at deepening the understanding of educational content and the development of the mathematical method

Ready-made algorithms or fragments of algorithmisation which aim at acquiring a variety of mathematical skills can be found in the textbooks for all types of schools. We can distinguish the following elements of algorithmisation used in the textbooks:

- trees-graphs showing the sequence of actions which replace a notation with brackets

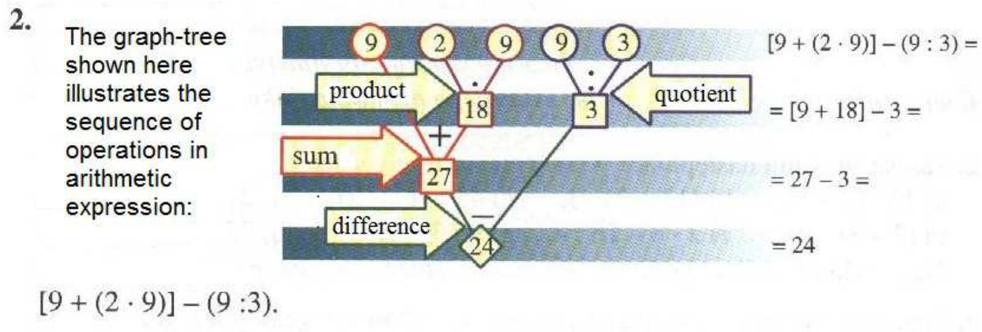


Figure 4.

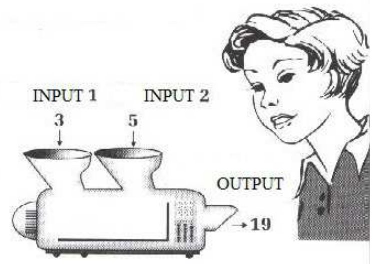
An arithmetic expression is shown graphically in a form of a tree. This scheme illustrates the sequence of actions by joining certain numbers by a line and writing between them what operations are to be performed. There are the results as well as the calculations presented in the form of an arithmetic expression at each level of the tree. In this way, a student can easily noitce an analogy between these different forms of presenting the same operation. A notation in the form of a tree is often more readable for students than a complicated notation with brackets. Moreover, the names of operations appear in the example. The commentary under the task explains to students how to read arithmetic expressions. This task helps them to learn how to use mathematical language. It can strenghten the belief in the usefulness of mathematical symbols but also develops awarness of the fact that using these symbols is associated with the ability to interpret them.

- number machines – a type of the graphical representation of very simple numerical algorithms

A2. This double-input number machine works according to the rule:

I multiply input numbers and add 4 to the result.

a) Choose the number for input 2 and complete the table. What could be the input numbers if the result was 24?



INPUT 1	3	4	0	2	0,5	12	21	30	8	
INPUT 2	5									
OUTPUT	19									24

Figure 5.

Students have to perform calculations according to the principle of operation of the machine. They are free to choose the second data item, which guarantees the diversity of examples as well as allows them to focus on the method and not on the result. While working on such a task, students get acquainted with the way numbers are processed according to the programmed principle of operation. In addition to developing the intuitive understanding of the concept of a function, numerical machines develop the ability to use patterns and search for a general procedure. Therefore, it is exactly the situation which is fundamental to the understanding of algorithms. The search for initial data which will give the result of 24 can be carried out in two ways. One way is a “trial and error” method of guessing the parameters and checking whether they meet the conditions of the task. The other method forces students to understand the machine’s principle of operation and through the inverse operations it leads to the discovery of the input parameters. Doing so, students get used to a careful analysis of the problem. Regardless of the choice of the method, the task shapes the relationship between the intuitive and formal thinking of students.

- the search for a general procedure – many tasks require students to understand the general method of creating the sequences and to write algebraic generalizations; the same skills are practised while creating algorithms by students

Excercise A. a) Look at the picture. Figures are arranged from matches according to some rule. What expressions should replace the question marks in the table?

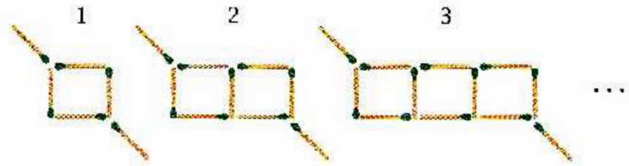


Figure number	1	2	3	4	5	n
Number of matches	6	$6+3$	$6+2\cdot 3$	?	?	?

Figure 6.

The excercise above necessitates a thorough analysis of the given sequence. In order to create another figure, students should first notice the rules for creating it. In the same way as when they create algorithms, students should identify which parts are fixed and which of them change. Thus, they will focus not on the result but on the method. In the table, there is a number of matches required to create an initial figure. The student, in order to complete the missing values, probably will look for the relationship between the number of the matches and the number of the element of the sequence. The unusual way of presenting the number of matches highlights this relationship. It allows the student to see the cyclical changes, which leads to devising the formula for the nth item of the sequence. Therefore, the student makes algebraic generalizations, and thus fulfils some objectives of teaching mathematics. This example shows that creating and analysing algorithms can be treated as a tool for introducing students to generalizations, which form the basis for understanding the algebraic notations.

- ready-made algorithms in the form of block diagrams – a student has to create an algorithm, often to understand its principle of operation or to find out what mathematical operation is performed in the diagram. I will show it on the example from a workbook for second-grade middle school students.

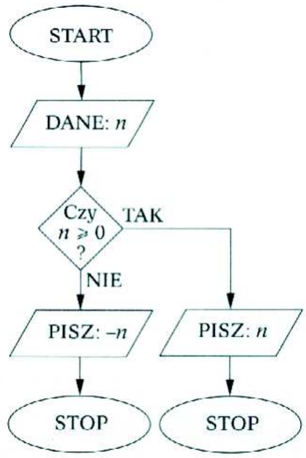


Figure 7.

Before doing the excercise that contains the algorithm, there is the explanation of what a block diagram is and how to understand the notation and the functions performed by each "box" in the diagram. Undoubtedly, it is the IT-oriented teaching because in this way students learn about the basis of algorithmisation. The students' task involves analysing the algorithm, understaning its principle of operation and determining which mathematical operation it performs. While using the algorithm with arbitrarily chosen inital data, the students observe its principle of operation. In addition to discovering the relationship between the input number and the result that the diagram gives, the student is required to adapt this procedure to the already known operation of determining the absolute value of a number. This gives the students a chance to operationalize the definition, while on the other hand they have a chance to observe that mathematical operations can be presented in a very formal, schematic way. The notation itself makes the algorithm more readable by a clear and unequivocal presentation of the order of the operations. This excercise also develops the students' ability to use patterns and symbols, the ability to draw conclusions or make and verify hypotheses. In this way, the algorithm has become here a method of achieving the objectives of teaching mathematics.

- creating analogous patterns – on the basis of a known model students have to create an algorithm for solving a similar problem (the example comes from "Matematyka 2001" textbook for fifth grade).

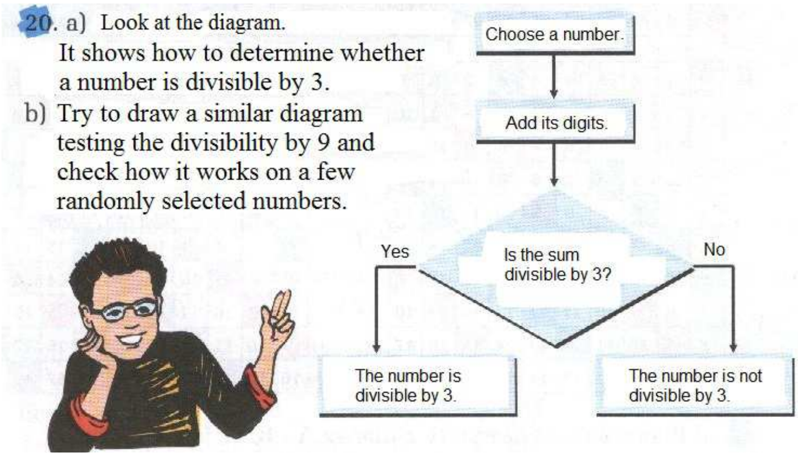


Figure 8.

The algorithm itself summarizes the earlier discussion about divisibility, and thus it is one of the ways of presenting the theorem of divisibility by 3. The theorem has been now broken down into a sequence of operations, and therefore the diagram shows how the theorem should be used. Students should draw such a conclusion by themselves (this is what they are asked for in point a). Writing a diagram showing the divisibility by 9 can be an ivitation to formulate a similar theorem themselves. In order to do that, students have to once again follow the argumentation which is the basis for the theorem concerning number 3 and find correspondence with the arguments for the divisibility of 9. Another analogy that a student should use is the same sequence of steps in determining the divisibility of 9. Therefore, by solving such a task in the classroom a student fulfils some of the objectives that I have presented in the discussion.

4 Summary

The algorithm is a concept that is often neglected in teaching mathematics. It is worth changing it because the development in mathematics occurs inter alia when general problem-solving patterns are discovered. Exploring mathematics by students can be supported by constructing or analysing algorithms as weel as by the intentional use of algorithms. Because of their unambiguity, algorithms make students get used to a precise and well-organised deductive thinking, which is associated with the development of formal reasoning – the main way of thinking in mathematics. By applying the elements of algorithmisation many objectives of mathematical education can be achieved at every level of education. A teacher who uses the existing textbooks and who is aware of these benefits can, for example, use algorithms to develop students' logical and mathematical thinking, to revise the material or to develop students' habit of validating the results.

In this study I have only shown the examples of algorithms that are explicitly or almost explicitly present in textbooks. However, widely understood algorithmisation can be present in the classroom in various forms, while at the same time, thanks to the extensive use of the elements of algorithmisation in mathematics classes, various educational objectives can be achieved. The analysis of the scheme for solving a particular problem can be a pretext for developing students' algorithmic thinking. For example, a teacher instructs the class to create a similar task in group work. Each group presents the results in front of the class and then the discussion starts. Through analogies, the students can see that the construction of the solving method does not change. Thus, they make a simple, yet independent generalization, the outcome of which will be an algorithm. An algorithm should be optimal, i.e. it should lead to the solution in the simplest possible way. It may be beneficial for students to be able to compare their own schemes for the solution of the same task and choose the most economical methods together. Such actions affect the mental discipline of students solving a mathematical problem and help them to understand the structure of the tasks better. Optimalization of algorithms is the same type of mental activity that always accompanies mathematicians looking for the easiest way to solve the problem.

In teaching mathematics, creating algorithms independently and representing them graphically is also the IT-oriented teaching through the introduction of new concepts and through teaching the IT-oriented thinking, understood as algorithmic thinking. According to the IT method of creating new programmes on the basis of existing ones, mathematicians can also use previously generated algorithms as "subprogrammes" (elementary operations). This allows them to create complex patterns in a relatively simple and clear way and helps to constantly revise the knowledge. A collection of such algorithmised constructions can be used for revising the material.

It is worth mentioning that also geometric constructions are all operation plans which can be represented in a form of algorithms. It is enough to treat some skills as elementary so as to be able to create a lot of different schemes. Such an approach can help students to understand the sense of geometric constructions. The introduction of algorithms in the realm of geometry classes may be one of the ways of making students familiar with this field of mathematics, which, as we know, is difficult to implement and which is often marginalized in school.

References

B a z y l u k, B., D u b i e c k a, A., D u b i e c k a - K r u k, B., G ó r a l e w i c z, Z., M a l i c k i, T., P i s k o r s k i, P., S i e n k i e w i c z, H., Z i e m i e ń c z u k, A.: 2006, *Matematyka 2001. Gimnazjum klasa 2. Zeszyt ćwiczeń 2*, WSiP, Warszawa.

C i e ń s k a, M., Jędrychowski, W. N o w a k o w s k a, A.: 1997, *Matematyka 2001 Podręcznik dla klasy 5 szkoły podstawowej*, WSiP, Warszawa.

C i e ń s k a, M., Jędrychowski, W. N o w a k o w s k a, A.: 1996, *Matematyka 2001. Zeszyt ćwiczeń dla klasy 5 szkoły podstawowej, Część I*, WSiP, Warszawa.

D o b r o w o l s k a, M., Karpiński, M., Lech, J.: 2004 *Matematyka II. Podręcznik dla liceum i technikum zakres podstawowy z rozszerzonym*, Gdańskie Wydawnictwo Oświatowe, Gdańsk.

D o b r o w o l s k a, M., Z a r z y c k i, P.: 2007, *Matematyka 4. Podręcznik dla klasy czwartej szkoły podstawowej*, Gdańskie Wydawnictwo Oświatowe, Gdańsk.

J u c e w i c z, M., K a r p i ń s k i, M., L e c h, J.: 2009, *Matematyka z plusem Program nauczania matematyki dla drugiego etapu edukacyjnego (klasy IV-VI szkoły podstawowej)*, Gdańskie Wydawnictwo Oświatowe, Gdańsk.

J u c e w i c z, M., K a r p i ń s k i, M., L e c h, J.: 2009, *Matematyka z plusem Program nauczania matematyki dla trzeciego etapu edukacyjnego (klasy I-III gimnazjum)*, Gdańskie Wydawnictwo Oświatowe, Gdańsk.

K a r p i ń s k i, M. D o b r o w o l s k a, M. B r a u n, M. L e c h, J.: 2003, *Matematyka I. Podręcznik dla liceum i technikum, zakres podstawowy z rozszerzonym,* Gdańskie Wydawnictwo Oświatowe, Gdańsk.

Kompetencje kluczowe w uczeniu się przez całe życie – Europejskie Ramy Odniesienia, Wspólnoty Europejski, 2007, Luksemburg.

K r y g o w s k a, Z.: 1979, *Zarys dydaktyki matematyki. Cz. 1*, WSiP, Warszawa.

N o w e c k i, B., Klakla, M., Malicki, T.: 1996, *Błękitna Matematyka 4. Podręcznik*, KLEKS, Bielsko-Biała.

R a m s, T.: 1982, Problemy algorytmizacji, w: I. Gucewicz-Sawicka, (ed.), *Podstawowe zagadnienia z dydaktyki matematyki*, PAN, Warszawa, 17-22, 119-141.

S e m a d e n i, Z.: 2009, Edukacja matematyczna i techniczna w szkole podstawowej, gimnazjalnej i liceum. Tom 6, w: *Podstawa programowa z komentarzami*, MEN, Warszawa, 29-50.

Links:

http://www.mini.pw.edu.pl/MiNIwyklady/algebra/t_galois.html
http://www.wmie.uz.zgora.pl/∼aszeleck/W61_CELE_NAUCZANIA_MATEMATYKI.pdf
http://www.rea-sj.pl/pub/File/download/Program_matematyka_zsz.pdf

Algorytmizacja w nauczaniu matematyki

Streszczenie

Matematyka szkolna ma dwa oblicza: pojęciowe i algorytmiczne. Nie można jednak dokonać podziału na matematykę pojęciową i matematykę algorytmiczną, gdyż oba te aspekty stale się mieszają i oba są jednakowo ważne. Badając elementy pojęciowe potrzebujemy metod obliczeniowych. Algorytmy dostarczają środków technicznych do pogłębienia pojęciowego poznania matematyki. Z drugiej strony nauczanie samych algorytmów spowodowałoby, iż w świadomości ucznia matematyka byłaby zbiorem automatycznych schematów umożliwiających rachunki. Biorąc pod uwagę obie cechy matematyki nauczyciele powinni podkreślać rolę elementów algorytmicznych w matematyce, przy równoległym docenianiu znaczenia jej elementów pojęciowych.

Wykorzystanie algorytmów w nauczaniu matematyki może przynieść bardzo dużo korzyści dydaktycznych. Poznawanie matematyki przez uczniów może być wspomagane przez konstruowanie czy analizowanie algorytmów, jak i świadome stosowanie algorytmów. Algorytmy dzięki swej jednoznaczności przyzwyczajają ucznia do bardzo precyzyjnego i uporządkowanego myślenia dedukcyjnego, co związane jest z rozwojem rozumowania formalnego, głównego sposobu myślenia w matematyce. Analiza celów nauczania matematyki zawartych w podstawie programowej oraz dydaktycznych korzyści płynących z zastosowania algorytmizacji w nauczaniu matematyki pokazała, iż dzięki zastosowaniu elementów algorytmizacji, na każdym poziomie nauczania można realizować wiele celów kształcenia matematycznego. Wykorzystując istniejące podręczniki, nauczyciel, który ma świadomość tych korzyści, może używać algorytmów np. do rozwijania u uczniów logicznego, matematycznego myślenia, powtarzania materiału czy wytworzenia nawyku sprawdzania poprawności uzyskanych wyników. Oczywiście algorytmizacja nie może być traktowana jako jedyne narzędzie realizacji konkretnych postulatów, lecz raczej jako jedną z wielu metod służących do pogłębiania wiedzy, zdobywania nowych umiejętności oraz nabywania dobrych nawyków.

Algorytmy i elementy algorytmizacji pojawiające się we współczesnych podręcznikach pełnią różnorodne funkcje. Niektóre z nich są schematami działań prowadzącymi do automatycznego uzyskania rozwiązania zadania np. algorytm dzielenia wielomianów. Uczniowie zobowiązani są do poznania podstawowych algorytmów arytmetyki i algebry - jest to warunek konieczny bardziej twórczej pracy w dalszym etapie nauczania. Oprócz uczenia algorytmów są one wykorzystywane często jako metoda do zdobywania innych umiejętności między innymi takich jak: logiczne myślenie, odczytywanie zapisów graficznych, operacjonizacja definicji, tworzenie uogólnień, stosowanie języka matematycznego. Algorytmy mogą być pomocne przy wyrabianiu nawyku sprawdzania uzyskanych wyników i ewentualnej korekty błędów, czy też kształceniu umiejętności prowadzenia merytorycznej dyskusji mającej na celu wspólne dojście do optymalnego rozwiązania. Dzięki wielorakim korzyściom dydaktycznym płynącym z zastosowania algorytmizacji w nauczaniu matematyki, warto dokładniej przyjrzeć się temu zagadnieniu.