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Beata Matłosz
Thomas Kelly High School, Chicago, Illinois

A multiple intelligence approach to teaching algebra

Abstract: Since Howard Gardner first introduced the theory of multiple intelligences, asserting that individuals vary in their response to learning according to their dominant intelligence type, his theory has been applied to the didactics of a wide range of subjects. Yet despite the theory's wide application, its role in the instruction of algebra has received little attention in the literature. This article discusses the responses of two students with different multiple intelligence types (i.e., interpersonal and visual/spatial) to a series of algebraic exercises presented in worksheet form. Based on their responses as well as one-to-one conferences with the teacher and students, it was found that adapting learning support to the individual's multiple intelligence type improved that individual's understanding of the subject. Since the theory of multiple intelligences relates to various scholastic subjects, it is suggested that algebra instructors may wish to collaborate with their colleagues who teach other subjects at their educational institution in order to meet students' needs.

1 Introduction

Harvard professor Howard Gardner (1983) first introduced the concept of multiple intelligences to address the apparent cultural and scholastic bias of two forms of intelligence, namely linguistic and logical-mathematical intelligence. The current paper aims to discuss the application of multiple intelligences to the instruction of algebra, which has thus far received little attention in the literature. The extensive published material concerning multiple intelligence theory is briefly reviewed, and the author's experiences in using multiple intelligences within the context of teaching algebra are described. The paper will conclude with suggestions for additional ways to further algebraic instruction in the classroom using multiple intelligences.

2 The theory of multiple intelligence

2.1 Forms of intelligence

To fully understand the development of the theory of multiple intelligences, it is important to look back to the work of French psychologist Alfred Binet, who was asked by the municipal government of Paris to develop a way to predict the future academic success of young children (Gardner, 1995a). The result of Binet's work was a series of tests, which he claimed were able to measure a child's intellectual ability (Gardner, 1995a). However, Binet cannot be credited with the concept known as the intelligence quotient (IQ) (Montagu, 1999); this distinction must be given to the German psychologist William Stern, who introduced the concept in 1912 (Montagu, 1999; Locurto, 1991). Stern defined IQ as being equal to the ratio of a child's mental age to his or her chronological age (Locurto, 1991). Since then, IQ tests have become an extremely popular way for educational, sociological, and psychological professionals to measure intellectual abilities.

However, a major issue surrounding the concept of an IQ test is that no single objective definition exists for the concept of intelligence. In fact, several definitions of intelligence make use of the concept of learning while other definitions make use of a conflation of the concept of learning with the tasks of mental activity. One example of a "meta-educational/meta-psychological theory" that makes implicit use of a definition combining the concept of learning with the tasks of mental activity is Gardner's theory of multiple intelligences. This theory challenged the traditional notion of what constitutes intelligence by broadening its definition. According to Kornhaber (2004), "Gardner sought to illuminate the mental abilities underlying the range of actual human accomplishments that are found across cultures. How is it that there are not only capable mathematicians and writers, but also skillful historians, farmers, engineers, actors, teachers, lawyers, dancers, hunters, politicians, and, yes, even comedians? [. . .] Drawing on research in cognitive developmental psychology, Gardner considered whether or not abilities have separate developmental trajectories from infancy onward. He looked for evidence of distinct abilities from evolutionary biology, experimental psychology, and in findings from psychometric psychology. He also went beyond psychometric approaches by identifying computational systems that operate on information that enters via the several senses."

Gardner initially proposed seven types of multiple intelligences: linguistic, logical-mathematical, spatial, musical, bodily-kinesthetic, interpersonal, and intrapersonal intelligence (Gardner, 1993; Smith, 2008). Although the definitions of Gardner's seven types of intelligences are perhaps self-evident based on their names, it may be beneficial to briefly describe each of these seven types of intelligence. As implied by the name, linguistic intelligence relates to an individual's communication skills, whether spoken or written (Gardner, 1983; Gregory & Chapman, 2006). Similarly, logical-mathematical intelligence concerns an individual's ability to think critically, reason, and solve problems (Gardner, 1983; Gregory & Chapman, 2006). Spatial intelligence corresponds to the ability to visualize (Gardner, 1983; Gregory & Chapman, 2006). Musical intelligence concerns skills related to rhythm, harmony, and musical ability (Gardner, 1983; Gregory & Chapman, 2006). Bodily-kinesthetic intelligence refers to mental understanding connected with, and manifested through, the body (Gardner, 1983; Gregory & Chapman, 2006); as such, bodily-kinesthetic intelligence is associated with activities such as dancing, gymnastics, and other sports. Interpersonal intelligence is related to the set of skills involved within interpersonal relationships whereas intrapersonal intelligence concerns the understanding of one's own psychological state as well as the set of skills involved within self reflection (Gardner, 1983; Gregory & Chapman, 2006). More detailed definitions are provided in Table 1.

These seven original forms of multiple intelligences can be divided into three categories. The first category consists of linguistic and logical-mathematical forms of intelligences, which are the two types typically valued by traditional education. The next three forms of intelligence are commonly associated with the arts, with musical and bodily-kinesthetic intelligence being important within performing arts and spatial intelligence being more applicable to endeavors such as sculpture and painting. The remaining two types of intelligence concern an understanding of the human condition.

Gardner subsequently introduced naturalist intelligence, which refers to the ability to classify and recognize patterns within nature (Gregory & Chapman, 2006; Gregory & Kuzmich, 2004; Smith 2008). As such, naturalist intelligence may be thought of as being associated with those skills associated with the sciences (particularly the natural sciences, but some aspects of the social sciences are also pertinent to this ability to classify and recognize patterns). In this sense, naturalist intelligence parallels linguistic and logical-mathematical intelligence in that it is relevant to several subjects taught within the realm of academia. Gardner also proposed a ninth form of intelligence: existential intelligence (Smith, 2008). However, this form – defined as the ability to tackle deep philosophical questions (both ontological and epistemological in their essence) concerning human existence (Wilson, 2005) – has not yet been accepted.

Summary of the Eight Intelligences				
Intelligence Area	**Strengths**	**Preferences**	**Learns best through**	**Needs**
Verbal/ Linguistic	Writing, reading, memorizing dates, thinking in words, telling stories	Write, read, tell stories, talk, memorize, work at solving puzzles	Hearing and seeing words, speaking, reading, writing, discussing and debating	Books, tapes, paper diaries, writing tools, dialogue, discussion, debate, stories, etc.
Mathematical/ Logical	Math, logic, problem-solving, reasoning, patterns	Question, work with numbers, experiment, solve problems	Working with relationships and patterns, classifying, categorizing, working with the abstract	Things to think about and explore, science materials, manipulative, trips to the planetarium and science museum, etc.
Visual/ Spatial	Maps, reading charts, drawing, mazes, puzzles, imagining things, visualization	Draw, build, design, create, daydream, look at pictures	Working with pictures and colors, visualizing, using the mind's eye, drawing	LEGOs, video, movies, slides, art, imagination games, mazes, puzzles, illustrated book, trips to art museums, etc.
Bodily/ Kinesthetic	Athletics, dancing, crafts, using tools, acting	Move around, touch and talk, body language	Touching, moving, knowledge through bodily sensations, processing	Role-play, drama, things to build, movement, sports and physical games, tactile experiences, hands-on learning, etc.
Musical	Picking up sounds, remembering melodies, rhythms, singing	Sing, play an instrument, listen to music, hum	Rhythm, singing, melody, listening to music and melodies	Sing-along time, trips to concerts, music playing at home and school, musical instruments, etc.
Interpersonal	Leading, organizing, understanding people, communicating, resolving conflicts, selling	Talk to people, have friends, join groups	Comparing, relating, sharing, interviewing, cooperating	Friends, group games, social gatherings, community events, clubs, mentors/ apprenticeships, etc.
Intrapersonal	Recognizing strengths and weaknesses, setting goals, understanding self	Work alone, reflect pursue interests	Working alone, having space, reflecting, doing self-paced projects	Secret places, time alone, self-paced projects, choices, etc.
Naturalist	Understanding nature, making distinctions, identifying flora and fauna	Be involved with nature, make distinctions	Working in nature, exploring living things, learning about plants and natural events	Order, same/different, connections to real life and science issues, patterns

Table 1. Summary of the Eight Intelligences. *Source: Giles, Pitre, & Womack (2008)*

It is important to understand that no individual should be thought of as possessing only one type of intelligence. On the contrary, each of these types of intelligence is potentially and probably innate to every human. Gardner himself stated that these intelligences should be expected to be "seen in pure form only in individuals who are, in the technical sense, freaks" (Gardner, 1995a). Nonetheless, it may be reasonably conjectured that many individuals will express and use some subset of these intelligences more extensively than they will use the remainder. This conjecture provides an important opportunity for educators. In particular, it suggests that each individual may have their own learning style, implying that instructors should take into account the essence of each type of intelligence within their classroom strategies to ensure that these same strategies are as effective as possible in helping students acquire the knowledge and understanding relevant to the subject being taught.

The implication of Gardner's theory is that an individual can effectively learn any given subject if the subject material is presented within the appropriate manner. For example, an instructor who wishes to teach students a set of words using the method of multiple intelligences may simply provide a list of the words and their corresponding definitions for linguistic students[1]; however, this approach is not likely to work with students who use another type of intelligence to learn new material. Indeed, those students who possess spatial intelligence will need to use some sort of pictorial technique to memorize these same words, which can be accomplished by having them draw or examine images that incorporate the meanings of each of the new words. Similarly, those students who rely on musical intelligence to learn may be asked to write a song containing the new words. Thus, specific techniques are relevant to learners who use each of the intelligences.

Since the theory of multiple intelligences is particularly relevant to learning styles, one may erroneously think that all of the research concerning this theory focuses upon the way in which students acquire knowledge. In fact, several researchers have examined the nature of the intelligences possessed by teachers. One such study was performed by Chan (2003), whose research investigated the primary forms of intelligence used by secondary school teachers. He found that (1) teachers instructing courses within the arts, sciences, and music relied much more heavily upon music intelligence than those instructing courses on language and/or the social sciences and (2) guidance teachers (counselors) made extensive use of their interpersonal and intrapersonal intelligences

[1]This paper will occasionally refer to a student as a learner having the corresponding type of intelligence (e.g., students possessing linguistic intelligence may be referred to as linguistic learners).

(Chan, 2003). As the current article is concerned primarily with student learning, further discussion about teacher intelligence would be outside the scope of the present paper, although it offers possibilities for further research.

Simply put, MI refers to a preferential way of learning that makes use of certain sets of skills that learners appear to express and deploy by default. A plumber, by nature a special learner, trying to understand the best approach to a plumbing problem, would, if given a schematic diagram, understand the situation much more clearly than if given a detailed oral account of the problem. A student learning literature who has a preference for visual learning will perhaps learn the story more quickly and comprehensively if viewing a film than by reading the story alone. However, by combining these learning methods so that the student learns via several approaches is more likely to result in success than by one approach alone, as a several-approach method allows for differing methods to complement each other and reinforce what is learned by the other method. The concept of MI allows teachers and other to recognize that there are several helpful ways of reaching the positive learning approaches with which students come prepared.

2.2 Criticisms of Gardner's theory

Gardner's theory is not without critics. The most fundamental criticism of his theory is that the intelligences are more of cognitive styles than stand-alone constructs (Morgan, 1996). Indeed, critics such as Morgan (1996) point to the way in which Gardner describes the intelligences (e.g., using such terms as abilities, sensitivities, and skills) to provide evidence. His "theory" is a semantic issue, not a new approach to multiple constructs of intelligence. Moreover, Morgan argues that Gardner's work is not new, but rather resembles earlier work by factor theorists of intelligence, such as L. L. Thurstone, who asserted that a single factor (g) cannot explain the complexity of human intellectual activity. Other criticisms leveled against Gardner's work include the idea that his theory is not empirical and that it is incompatible with g, heritability, and environmental influences. Some argue that Gardner's work expands the construct of intelligence too broadly – so much so as to render it meaningless (Gilman, 2001).

Yet Gardener's theory of multiple intelligences is not based on a semantics issue. He endeavored to define intelligence in a much broader way than psychometricians by establishing several criteria to be used when defining intelligence (Gardner, 1983; 1999). When defining an intelligence, he relied on data related to the potential for brain isolation by brain damage; the intelligence's place in evolutionary history; the presence of core operations; susceptibility to encoding; a distinct developmental progression; the existence of idiot-savants, prodigies, and other exceptional people; support from experimental psychology; and support from psychometric findings. Gardner (1995b) further responded to the criticisms of his work by addressed specific "myths" about his theory that emerged based on the major commentary on, and criticism of his work. For example, one myth argues that intelligences should be able to be measured using specific tests for each intelligence. Gardner stated that an intelligence is similar to a domain or discipline, although a domain is different in that it is "a culturally relevant, organized set of activities characterized by a symbol system and a set of operations" (Gardner, 1995b). Gardner used the example of a dance performance, explaining that the performance itself is "a domain that relies on the use of bodily-kinesthetic and musical intelligence" (1995b).

In response to the criticism that his theory is not empirical, Gardner (1995b) pointed out the numerous laboratory and field data that he used to help develop his theory as well as the continual re-conceptualization of the theory based on new scientific data. In addition, he explained that g has a scientific place in intelligence theory, but his work focuses more an understanding intellectual processes that g does not explain. Regarding criticisms that his multiple intelligence theory is incompatible with genetic or environmental accounts of the nature of intelligence, Gardner states that his theory attempts to understand intelligence primarily by examining the interaction between genetics and the environment. Finally, despite claims that his work expands the construct of intelligence too broadly, Gardner argues that narrowly defining intelligence to be equal to scholastic performance is too constrictive. He points out that his theory is not a theory of personality, morality, motivation, or any other psychological construct, but rather focuses on the intellectual and cognitive aspects of the human mind (Gardner, 1995b).

2.3 Measurement of multiple intelligence in students

Gardner (1993) believes that assessment should focus on obtaining information about individuals' skills and potentials in order to provide useful information for the individuals and the community at large. As such, Gardner distinguishes between testing and assessment: "Assessment elicits information about an individual's abilities in the context of actual performance rather than by proxy using formal instruments in a de-contextualized setting." According to Gardner, assessment should be incorporated into the learning environment in a natural manner, much like how constant assessment occurs in an apprenticeship or how experts internalize self-assessment based on teachers' previous guidance.

Gardner questions the usefulness of traditional intelligence tests beyond predicting academic performance, despite their predictive validity and psychometric soundness. He concludes that prediction can be enhanced by using assessments that more closely approximate real working conditions. Relying on a single IQ score without substantiating findings through other data sources results in producing insufficient information. Moreover, assessments must be sensitive to individual differences and developmental levels. As such, Gardner promotes using assessments to help students rather than classify or rank them.

A wide variety of instruments have been developed to assess the nature of the set of intelligences used by students. For example, Teele developed the Teele Inventory for Multiple Intelligences (TIMI) at the University of California (Sue Teele & Associates, 1992). Dr. Teele has focused on methods for integrating the theory of MI in the classroom, current brain research to classroom practices and instruction, multiple measures of assessment, approaches for teaching reading and writing and for teaching English language learners.

The TIMI is a forced-choice inventory of 56 items, which are numbered pictures of pandas. The pandas represent characteristics of the seven multiple intelligences, and the forced choices give students opportunities to unambiguously state a preference for one of the alternatives presented. After scoring, a profile of the responses for each student is given, and teachers can then use this information to identify the student's most dominant intelligences, which are indicated by the highest scores. The Teele inventory has been used in more than 10,000 locations and in 40 countries. It can be used with children as young as three years of age, and is used to gain an understanding of how these people learn.

This assessment is currently used in several schools across the United States. Teele claims that the test is statistically reliable; however, studies have challenged this claim. For example, McMahon, Rose, and Parks (2004) found that the instrument "does not provide consistent measurement and needs further development and refinement" (p. 48). Nonetheless, other studies have supported the validity and reliability of TIMI strongly enough to incorporate it into their experimental design (for example, Ozdemir, Guneysu, & Tekkaya, 2006).

A second type of assessment is the Multiple Intelligences Developmental Assessment Scales (MIDAS) inventory, administered by a private company (Multiple Intelligences Research and Consulting Inc.) and developed by C. Branton Shearer, a neuropsychologist with a doctorate in neuropsychological rehabilitation. He has studied multiple intelligences to discern how the brain functions after trauma. He created MIDAS as an alternative to other MI tests (MI Research and Consulting, 2010). The MIDAS inventory is three pages in length and can be used to assess the multiple intelligences characteristics of an individual of any age. It has four versions according to the age of the individual being assessed: adults and/or college students (age $\geqslant 20$), teenagers (ages 15 to 19), older children (ages 9-14), and younger children (4-8). The MIDAS inventory attempts to create a "multiple intelligences profile" by asking sets of questions strongly associated with the natures of each of the eight recognized types of intelligence. For example, the MIDAS inventory attempts to describe an individual's use of musical intelligence by asking a sequence of questions regarding that person's knowledge, experience, and passion of music. The MIDAS has been demonstrated to have good internal consistency; its alpha coefficients range from 0.78 to 0.89 (Harris & Sykes, n.d.). Furthermore, test-retest reliability tests demonstrated that the scale could reliably measure consistent traits (mean *gamma* value of 0.84).

Finally, several web pages provide free assessments/inventories to help identify a person's multiple intelligences profile. The Appendix lists the URLs of a few of these web pages. However, care must be taken in using these free online inventories because neither their validity nor their reliability may have been investigated by education professionals.

2.4 Application of multiple intelligence in mathematics teaching

Despite its extensive application in educational research (Section 2.1), the theory of multiple intelligences has been applied less often to the instruction of mathematics, although some authoritative resources on mathematical instruction implicitly and/or indirectly refer to the theory of multiple intelligences. For example, Furner, Yahya, and Duffy (2005) list what they believe are the 20 best practices for teaching mathematics. Although not all of their best practices relate directly to multiple intelligences, most can be implicitly, indirectly, and/or hypothetically related. For example, their first listed best practice suggests teaching mathematical vocabulary using realization and demonstration; they go further to state that instructors may implement this technique by making use of "concrete objects for hands-on activities" (p. 16). This is a particularly helpful suggestion for spatial and bodily-kinesthetic learners seeking to grasp mathematical vocabulary. Furthermore, the second and third listed best practices suggest relating mathematics to prior knowledge and background as well as to daily life situations. Students possessing a particular type of intelligence will likely use the inherent qualities associated with that intelligence within daily circumstances; therefore, a student's type of intelligence may also be interrelated to his or her knowledge and background — although this argument should be verified by future research.

An older article relevant this paper is a dissertation written by McGraw (1998), who investigated the use of the theory of multiple intelligences to develop methods of reinforcement for the topics covered by seventh grade mathematics students. For the purposes of McGraw's study, only the original seven types of multiple intelligences were used. His study had three goals: 1) to investigate the efficacy of didactic reinforcing strategies based upon Gardner's theory of multiple intelligences; 2) to investigate whether or not the degree to which the aforementioned reinforcing strategies were implemented (based upon students' ability profiles) had a measurable effect upon student performance; and 3) to examine whether or not any correlation existed between changes in student performance and the aforementioned student ability profiles when the reinforcement strategies were taken into account.

McGraw's results were mixed, depending on whether or not the use of reinforcement strategies based upon the theory of multiple intelligences was incorporated into the instruction of mathematics. Notably, a statistically significant improvement in student performance was observed when the reinforcement was connected to the student profiles, but no statistically significant improvement was evident when this connection was not made. This single study suggests no overall effect exists regarding the ability of students to learn mathematics when principles based upon the theory of multiple intelligences are incorporated into the classroom. One possible reason for this mixed result is the results of the study itself. In particular, the study found that the post-test scores did not correlate well with what the students perceived to be their "multiple intelligence strength" (i.e., which forms of intelligence they believed were their strongest). Therefore, the strength of each student within the context of each given type of intelligence was not accurately identified.

Other articles explicitly discuss the application of multiple intelligences to mathematics. Adams (2000) explored how multiple intelligences can be applied to teaching mathematics to young children. Similarly, Martin (1996) addressed a number of topics relevant to the mathematical instruction of young children as well as middle school and high school students (e.g., geometry and probability). However, none of these resources addresses the application of the theory of multiple intelligences to the instruction of algebraic concepts and principles, which is the focus of the present paper.

3 Aims and objectives

The literature discussed thus far has shown that students learn in a variety of ways. In a typical school setting, most of this learning is accomplished didactically; historically, this method has been of the one-size-fits-all variety.

However, with knowledge of learning styles, didactic learning may still be possible and even preferable in some cases. Therefore, the current study aims to examine how the use of intelligence types can maximize students' learning. Specifically, the objective of the study is to determine whether a significant difference exists between solving algebra problems using (a) standard didactic methods regardless of preferred learning style and (b) a preferred style based on Multiple Intelligences theory. To this end, a case study approach was adopted to address the following hypothesis:

H1: Pairing the didactic methods students use with their dominant intelligence types will improve their learning.

This hypothesis proposes that didactic methods and the actual activities be conceived in terms of learning style rather than simply based on the method. Improvement then, in this case, refers to demonstrated gains made with respect to specific learning concepts as measured by the results of activities engaged in that are relevant to those concepts. Improvements in learning were assessed by examining students' response to arithmetic tasks. For example, one task presented to students is:

> Suppose that a quarterback had the following plays in a football game: a pass for a gain of 7 yards, a sack for a loss of 2 yards, a sneak for a gain of 6 yards, and a sack for a loss of 13 yards. What was the total yardage gain or loss on these four plays?

In this example, H1 proposes that a significant improvement will exist between students' first attempt at solving the problem (i.e., without accounting for their learning styles) and the subsequent attempt to solve the problem after conferencing with the students and engaging their preferred learning styles. Such improvement will be measured based on four areas: mathematical concepts, mathematical errors, explanations, and mathematical terminology and notation. The didactic concept is the operation of arithmetic in solving real-world problems; the proof of improvement is in successfully solving a range of problems after intervention or conferencing with a teacher and applying the preferred learning style to the problem solution. The important thing here is not the specific problem, but the problem type. Given the Multiple Intelligences theory, if a student can solve a range of similar problems in a second activity using a preferred learning style when such accomplishment was not evident after a first activity, there is clear evidence that learning has occurred.

4 Methodology

The case study method is a qualitative approach that one uses to conduct a "holistic inquiry into a contemporary phenomenon within its natural setting" (Harling, n.d). It is generally based on theory (in this case, the Multiple Intelligences Theory). A case study is defined by the phenomenon being studied (here, the abilities of two students to learn algebra), the natural setting (here, the classroom), and the holistic inquiry itself, which "involves collection of in-depth and detailed data that are rich in content and involve multiple sources of information including direct observation, participant observations, interviews, audio-visual material, documents, reports and physical artifacts. The multiple sources of information provide the wide array of information needed to provide an in-depth picture" (ibid).

Several different kinds of case study can be used. *Intrinsic* case studies are designed so that we learn about something unique that the study focuses on. The uniqueness (e.g., ways in which specific students learn) is based on a collection of features or a sequence of events (e.g., algebra problems presented). Case studies can also be *instrumental,* in which a general understanding can be determined using a particular case – usually a typical case. A *collective* case study uses a number of case studies that become analytical generalizations, presenting themes that are common to all (ibid). In the current study, an instrumental case study is used to develop understanding about the role of multiple intelligences in learning based on two typical cases from an algebra class.

As previously mentioned, theory also plays a role in the case study. Theory is the starting point for the study in that it gives structure and direction to the research question(s). The theory is then used to filter and organize the received data. This has a tendency to confirm existing theory. However, a researcher must be careful to ensure that the existing theory does not predetermine the results. The researcher must always be aware of paradoxes between case situations and theory (ibid).

The present study adopted a case study approach to analyze the work of two students (introduced in Sec. 4.3). In each case, the students were given two multiple intelligence tests spaced over a three-day period in order to verify their preferred learning styles. After the administration of Activity 1, in which the students were asked to complete a variety of mathematical problems (applying the basic operations of arithmetic to real numbers), both students were asked to conference with the teacher, who proceeded to explain the solution parameters to such problems using methods that were more properly aligned with each student's preferred learning style. The purpose of the conference was to help the student understand the meanings of the terms used and the way in which the problem could best be solved using inputs based upon students' preferred learning styles. Upon completion of this intervention, students were given the problems in Activity 2, which were of the same type (applying the basic operations of arithmetic to algebraic expressions) as Activity 1, but not the same specific problems. Although some whole classroom discussion took place on the nature of the problems to be solved, this was seen as reinforcing students' needs and not as a part of the specific intervention designed to improve individual students' performance on Activity 2.

4.1 Class and classroom characteristics

The class from which the case studies were drawn was a two-period Algebra I class (92 minutes of instruction per day) involving 22 ninth grade students aged 14 to 15. The featured unit was performing operations of arithmetic with real numbers and algebraic expressions. Questionnaires distributed to parents and students in September 2008 revealed that 19 (86%) were from non-English-speaking homes (18 Spanish and 1 Cantonese). The math vocabularies of these 19 students were far less developed than those of their native-English speaking peers. Approximately 60% of the students admitted that they feel they were programmed into this double period class because they are slow in math and, consequently, they could be considered as discouraged math learners. According to elementary school records, all students in the study scored below the 45^{th} percentile in their seventh grade state-wide standardized math test. Observations made during class work showed eight of the students used their fingers when adding and subtracting, while twice as many consistently obtained wrong answers to multiplication and division problems without calculators. In addition, before starting this unit, the author observed that all but four students lacked a solid grasp of the abstract notion of a variable.

4.2 Learning goals and approach to teaching

The substantive mathematical idea of the instructional sequence involved applying the basic operations of arithmetic to real numbers and algebraic expressions, thereby moving from the concrete to the abstract. The learning goals (LGs) for this sequence were that students gained the ability to (1) apply the basic operations of arithmetic to real numbers to solve word problems, (2) write algebraic expressions to solve word problems, and (3) simplify algebraic expressions to solve word problems using properties of real numbers. To test the students' achievements in terms of the learning goals, the students were asked to complete a series of activities, as outlined in the following paragraphs. Activity levels were set to create a manageable entry point for students in understanding the relationships between real numbers and variables. The instructional resources were a worksheet and associated rubric, which were distributed to the students.

The unit started with a whole class discussion of opposites in everyday life. For example, some students mentioned a door opening and closing, hot and cold temperature, etc. This analogy was used to explain the number line as a representation of the set of real numbers. Students then worked in pairs with algebra tiles on building mats to compare integers. Student pairs modeled addition with algebra tiles, with special emphasis on neutral pairs.

The next activity was the featured Activity 1. This employed the worksheet, in which students were prompted to apply the skills of adding and subtracting integers by solving word problems related to statistics, banking, and chemistry. Students completed word problems that required them to identify and add specific integers (e.g., yards gained/lost in a series of football plays). They also answered the question, "What is meant by adding the opposite?" The author then graded the students' responses using a rubric previously supplied to the students. The author looked at students' abilities related to mathematical concepts, mathematical errors, explanations, and mathematical terminology and notation.

After completing Activity 1, the two students chosen for the case study were asked to participate in a student-teacher conference to discuss their responses and identify new approaches to learning based on their preferred learning style. Given that properly identifying the dominant type of intelligence was of key importance for the study, both students were assessed using two multiple intelligences tests spaced across three days. This procedure allowed for identification of their intelligence types with greater validity. The tests were also supplemented by a questionnaire distributed to the students' parents. This instrument asked the parents about their children's predispositions for particular learning styles.

Four weeks after the conferences, Activity 2 was completed. Using similar types of tasks as in Activity 1, students were asked to combine algebraic expressions by solving word problems related to geometry, manufacturing, and inventory. They defined a variable, wrote an algebraic expression, and evaluated the expression by substituting integers for the unknown quantities. For example, they were asked to identify specific algebraic expressions with integral coefficients to determine the perimeter of several figures and to simplify expressions that contained variables. In addition, students defined a variable and then wrote an algebraic expression. They also wrote an explanation in their own words of what is meant by adding the opposite expression. To complete Activity 2 successfully, students needed to recognize and apply the basic operations of arithmetic to simplify algebraic expressions, which are abstract representations of the same concept (adding signed numbers) presented in Activity 1, and apply the procedures used in Activity 1 to solve the word problems in Activity 2.

4.3 Student case studies

The responses of two students, Student A (Araceli) and Student B (Fatima), during Activities 1 and 2 were studied in further detail.

Student A (Araceli) is a female Spanish speaker with a similar background to many students in the class; the student shows good attendance, regularly completes homework, and is in the mid-range for math content knowledge and skills. The analysis of this student's responses is therefore transferable to other students, thereby providing insights into the ways by which many other students deal with math concepts. The multiple intelligence test results identified the student as an interpersonal learner.

Student B (Fatima) is also a female Spanish speaker. The student is a low range learner in terms of math content skills due to reading difficulties – a common problem among students in the class. As this lesson involved word problems, this student's responses could demonstrate the level of success of a student with reading problems in achieving the identified LGs. This student requires additional time to process new ideas and needs extra support in explaining math concepts and vocabulary. She also needs further instruction using visuals and books to ensure that she fully understand ideas and the step-by-step processes involved. Student B (Fatima) is a visual/spatial learner, utilizing graphical representations to recognize groups in algebraic expressions.

5 Scoring

The activities were scored in order to quantitatively assess students' progress between activities. Scores were awarded based on four categories: mathematical concepts, mathematical errors, explanation, and mathematical terminology and notation. Scores ranged between 1 and 4 based on predefined attainment levels (see Table 2 for details). A maximum of 16 points was available for each activity.

	Score criteria				Student scores			
Assessment Category	**4**	**3**	**2**	**1**	**A A1**	**A A2**	**B A1**	**B A2**
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written.	**2**	**4**	**2**	**4**
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	More than 75% of the steps and solutions have mathematical errors.	**3**	**4**	**2**	**4**
Explanation	Explanation is detailed and clear.	Explanation is clear.	Explanation is a little difficult to understand, but includes critical components.	Explanation is difficult to understand and is missing several components OR is not included.	**2**	**3**	**2**	**3**
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are used, but it is sometimes not easy to understand what was done.	There is little use of terminology and notation OR there is significant inappropriate use of terminology and notation.	**1**	**3**	**2**	**3**
Total points: (Max possible: 16)					**8**	**14**	**8**	**14**

A – Student A (Araceli), B – Student B (Fatima), A1 – Activity 1, A2 – Activity 2

Table 2: Student assessment and scores: Math-Problem Solving

6 Results and discussion

Sections 5.1 and 5.2 summarize in more detail the responses of the two selected students (i.e., A-Araceli and B-Fatima) participating in Activities 1 and 2. The corresponding discussion is based on the scanned copies of the two students' responses to Activities 1 and 2 (see Figures 1, 2, 3 and 4) and on the scores achieved in each of the four assessment categories (see Table 2).

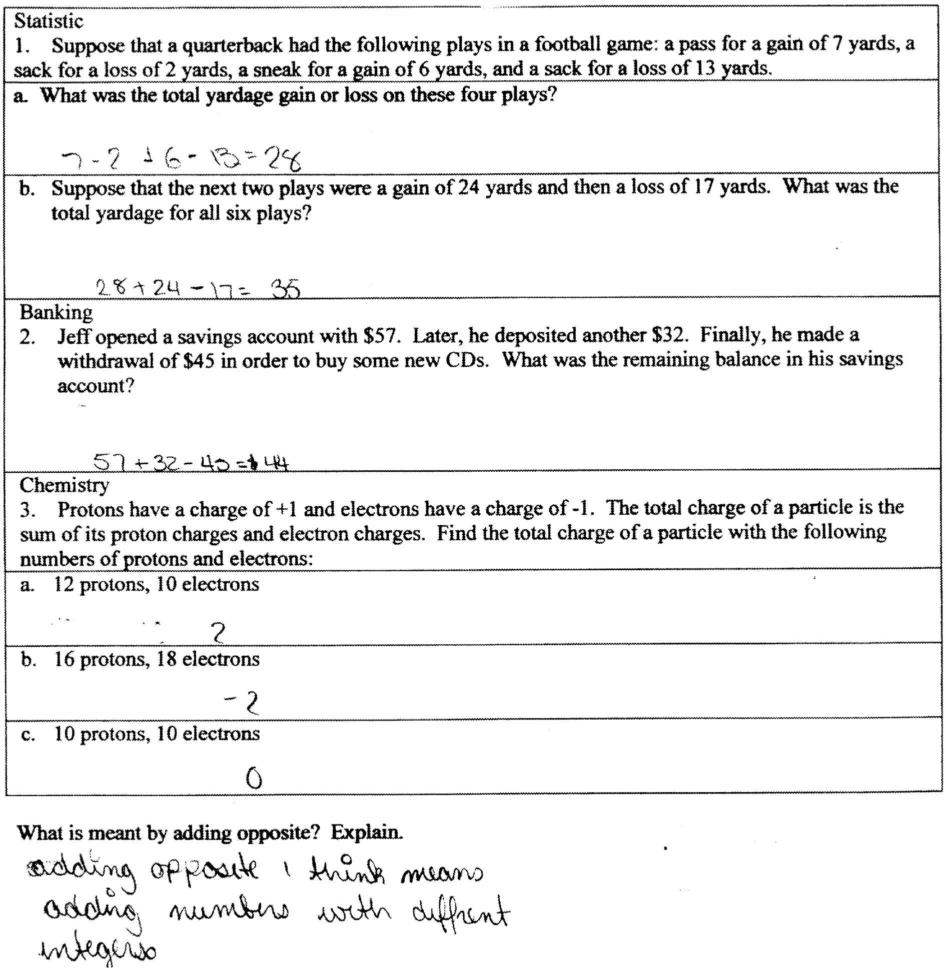
Since Student A (Araceli) is an interpersonal learner, and as students with this MI classification do not generally possess a typical academic skill set, she is likely — and probably has been likely — to be viewed as a socially adept person who is not particularly bright but does her work adequately. She is clearly not the best student in the class and would not be regarded as such. After years of being seen this way, it is highly likely that she sees herself in the same way, and her effort is probably commensurate with that image. Although activities were included that addressed each type of MI, there is very little academically that she could do to compensate for her preferential learning style.

Student B (Fatima) is described as a low-range achiever in math and exhibits a visual/spatial learning style. Because she is not a good reader, and given that reading skill is important in mathematics learning, this student has likely had years of frustrating encounters with mathematics concepts, and her effort is likely also commensurate with this image of herself. However, in mathematics instruction, graphical representations are fairly common, providing an important way in which she can compensate for her low levels of reading ability. In her learning mathematics, she has a fair number of opportunities to use her intelligence. However, unfortunately, her inability to read well intrudes on her other mathematical learning; when visual/spatial opportunities are not present (when determining the appropriate equations to use or deciding which operation will solve a problem, and then executing the equation or operation, for example), she has no strong intelligence upon which to rely.

6.1 Response of Student A (Araceli)

The responses of Student A (Araceli) to Activity 1 are shown in Fig. 1.

Activity 1 tested LG1 (apply the basic operations of arithmetic to real numbers to solve word problems). In Q1a, the student failed to achieve this goal because she added the absolute values of the numbers rather than their real values. She wrote the equation correctly, suggesting the correct understanding of the question, but when evaluating the successive totals ($7-2=5$; $5+6=11$; $11-13=-2$), she was likely confused by having to subtract a bigger number



Statistic

1. Suppose that a quarterback had the following plays in a football game: a pass for a gain of 7 yards, a sack for a loss of 2 yards, a sneak for a gain of 6 yards, and a sack for a loss of 13 yards.

a. What was the total yardage gain or loss on these four plays?

7-2+6-13=28

b. Suppose that the next two plays were a gain of 24 yards and then a loss of 17 yards. What was the total yardage for all six plays?

28+24-17= 35

Banking

2. Jeff opened a savings account with $57. Later, he deposited another $32. Finally, he made a withdrawal of $45 in order to buy some new CDs. What was the remaining balance in his savings account?

57+32-45=$44

Chemistry

3. Protons have a charge of +1 and electrons have a charge of -1. The total charge of a particle is the sum of its proton charges and electron charges. Find the total charge of a particle with the following numbers of protons and electrons:

a. 12 protons, 10 electrons

2

b. 16 protons, 18 electrons

-2

c. 10 protons, 10 electrons

0

What is meant by adding opposite? Explain.

adding opposite I think means adding numbers with diffrent integers

Figure 1. Responses of Student A (Araceli) to Activity 1.

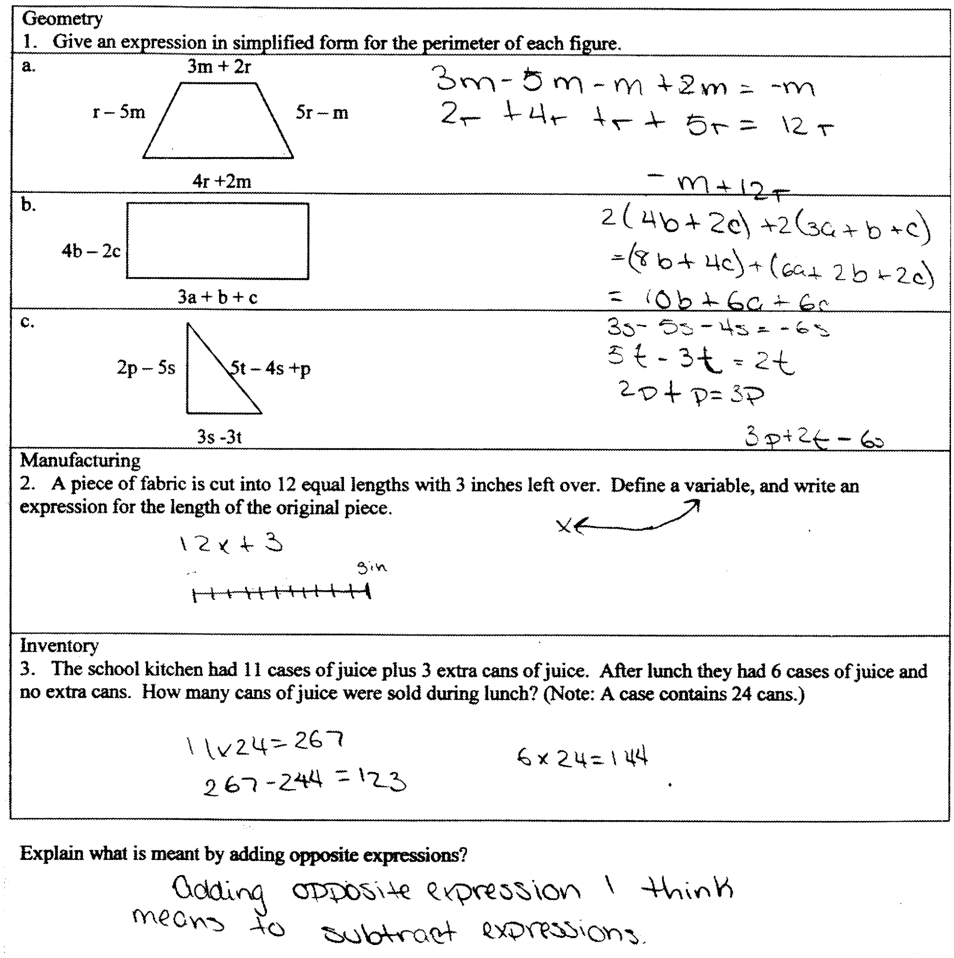
(13) from a smaller number (11). In part (b), she again wrote the correct solution, but this time evaluated the sum correctly, although the final answer was nominally incorrect as she carried forward her incorrect answer from part (a). Q1 therefore showed she could add signed numbers but was confused by a negative result, as supported by her correct answers to the banking problem in Q2. However, she successfully achieved LG1 in the chemistry problems in Q3, where she correctly obtained a negative answer in Q3b; unfortunately, in this part, she did not show how she reached her answers.

In her response to the question "What is meant by adding opposite? Explain," Araceli wrote "Adding opposite I think means adding numbers with different integers." This answer highlights her limited math language and her lack of understanding of the meaning of integer. It also suggests that she did not understand the meaning of "opposite" in the mathematical context and did not see subtraction of a number as the addition of its opposite. Therefore, she might not perceive negative and positive numbers as two different groups of numbers, even if she recognizes the opposite actions.

During the conference with the student, the author reviewed with her the rubric from Activity 1, pointing out what she was doing correctly. The student was asked to explain her answers verbally to determine how she derived her answers. As an interpersonal learner, Araceli tends to learn best by talking to and cooperating with others. The student was offered choices in her study plan by discussing several tools available for developing a better understanding of the concepts (e.g., keeping a personal dictionary of key terms, developing flashcards to help memorize the concepts, using self-monitoring checklists, using supportive materials such as math tables). To promote her understanding of how to write verbal explanations, the student was given correctly solved problems and asked to analyze and talk about them. This enabled the student to practice examining problems and identifying the processes (e.g., subtraction) used without having to solve the problem. The author encouraged her to seek assistance from more knowledgeable students, as appropriate (not simply "answer giving"); such collaboration with others would benefit her as an interpersonal learner.

During the four weeks between Activity 1 and Activity 2, the author worked with Araceli and her parents using interactive assignments. As part of these assignments, the student read her work out loud to her parents. She was assigned a partner to work day-to-day activities in class. These activities aim to maximize her mathematics learning as an interpersonal learner so that she was better prepared to complete Activity 2 more successfully.

In Q1 of Activity 2 (see Fig. 2), Araceli simplified two different groups of like terms separately. As such, she demonstrated understanding of subtraction in each group as adding of the opposite and showed she had achieved LG1, LG2, and LG3. However, her responses did contain some errors. First, in parts (a) and (c), she failed to use proper mathematical notation to represent the final sum because she did not include the plus sign between two different groups of variables or an equals sign to represent her simplification of algebraic expression for the perimeter of the given figure. In part (b), she wrote $(4b+2c)$ instead of $(4-2c)$; thus, although she grouped terms correctly, her final answer was incorrect, perhaps suggesting she does not always see symbols correctly.


Geometry
1. Give an expression in simplified form for the perimeter of each figure.

a. 3m + 2r, r – 5m, 5r – m, 4r +2m

3m - 5m - m + 2m = -m
2r + 4r + r + 5r = 12r
- m + 12r

b. 4b – 2c, 3a + b + c

2(4b + 2c) + 2(3a + b + c)
= (8b + 4c) + (6a + 2b + 2c)
= 10b + 6a + 6c

c. 2p – 5s, 5t – 4s +p, 3s -3t

3s - 5s - 4s = -6s
5t - 3t = 2t
2p + p = 3p
3p + 2t - 6s

Manufacturing
2. A piece of fabric is cut into 12 equal lengths with 3 inches left over. Define a variable, and write an expression for the length of the original piece.

12x + 3
3in
x

Inventory
3. The school kitchen had 11 cases of juice plus 3 extra cans of juice. After lunch they had 6 cases of juice and no extra cans. How many cans of juice were sold during lunch? (Note: A case contains 24 cans.)

11x24 = 267
267 - 244 = 123
6 x 24 = 144

Explain what is meant by adding opposite expressions?

adding opposite expression I think means to subtract expressions.

Figure 2. Responses of Student A (Araceli) to Activity 2.

In Q2, Araceli wrote the correct expression representing the prompt, thereby demonstrating achievement of LG2. However, she failed to name or define the variable correctly – she only showed (with an arrow) that she had chosen the letter x to represent the length of a piece of fabric, without providing an explicit definition. Also, she did not follow the proper mathematical notation. Her answer shows she did not define variable x as a piece of fabric and did not chose a letter to define the length of the original piece, because she drew a segment representing only 11 units and then wrote "3in" at the end of her sketch. To obtain a result of 12 pieces of fabric, she counted "marks" that she made on the segment she drew instead of units (there were only 11 units) and then above the segment she wrote 3in – which probably represented the additional 3 inches left – which she didn't show on her picture.

In Q3, Araceli wrote an incorrect expression ($11\times 24 = 267$) for the number of cans before lunch and the number of cans used ($267 - 244 = 123$), yet wrote the correct answer to the question. The correct expressions should have been $(11 \times 24) + 3 = 267$ and $267 - 144 = 123$. Her correct answers combined with sparse and incorrectly written work suggest that she may have used a calculator for this part, despite the fact that the use of calculators was disallowed in this activity. Her use of a calculator indicates she may still have problems with mental arithmetic, as do the majority of students in her class.

Her answer to "Explain what is meant by adding opposite expression" shows her correct understanding of adding the opposite as being equivalent to subtraction.

Although the student's work demonstrated achievement of all three goals of this sequence, she did not correctly use the subtraction symbol consistently in Activity 1. To illustrate, in Q1a related to football yards, the student added the absolute value of the given yards (lost or gained) instead of correctly computing the numerical sentence she had already written. She then used the sum from 1a in prompt 1b, resulting in a wrong answer. Thus, although she had practiced identifying and organizing data during the conference, she did not apply basic operations to real numbers to solve word problems in prompt 1a. She subsequently achieved this goal in the remaining parts of Activity 1 and in all the tasks in Activity 2.

In Activity 2, the student demonstrated in her definition of adding opposite expressions that subtracting expressions meant adding the opposite expressions. She applied the skills of adding/subtracting integers by solving word problems (part of LG1) and defined her understanding of adding opposite numerical expressions. However, she demonstrated an inability to utilize math notation correctly, indicating a gap in her prior knowledge. For example, in the first set of problems (1a, b, and c), she simplified two groups of like terms separately, thereby achieving LG3 of this lesson and indicating her understanding of subtraction in each group as adding the opposite. However, she failed to use proper math notation to represent the final sum (not including the plus sign between groups or the equals sign in 1a or 1c) to represent the simplified algebraic expression. The student gave the correct notation in finding the perimeter of the rectangular figure, achieving LG2, but incorrectly copied the expressions representing the width of the sides for this figure (Question 1b) – she wrote $4b + 2c$ instead of $4b - 2c$. Student A (Araceli) further demonstrated her math reasoning by using the letter x to write an expression representing parameters identified in prompt 2 in Activity 2, where she wrote $12x + 3$ (LG2). In addition, she drew a segment representing only 11 units to report a result of 12 pieces of fabric, thereby indicating a lack of recognition of a variable as a piece of fabric; drawing 11 units implies she counted “marks” on the segment instead of the units. However, it is also possible she simply miscounted the units.

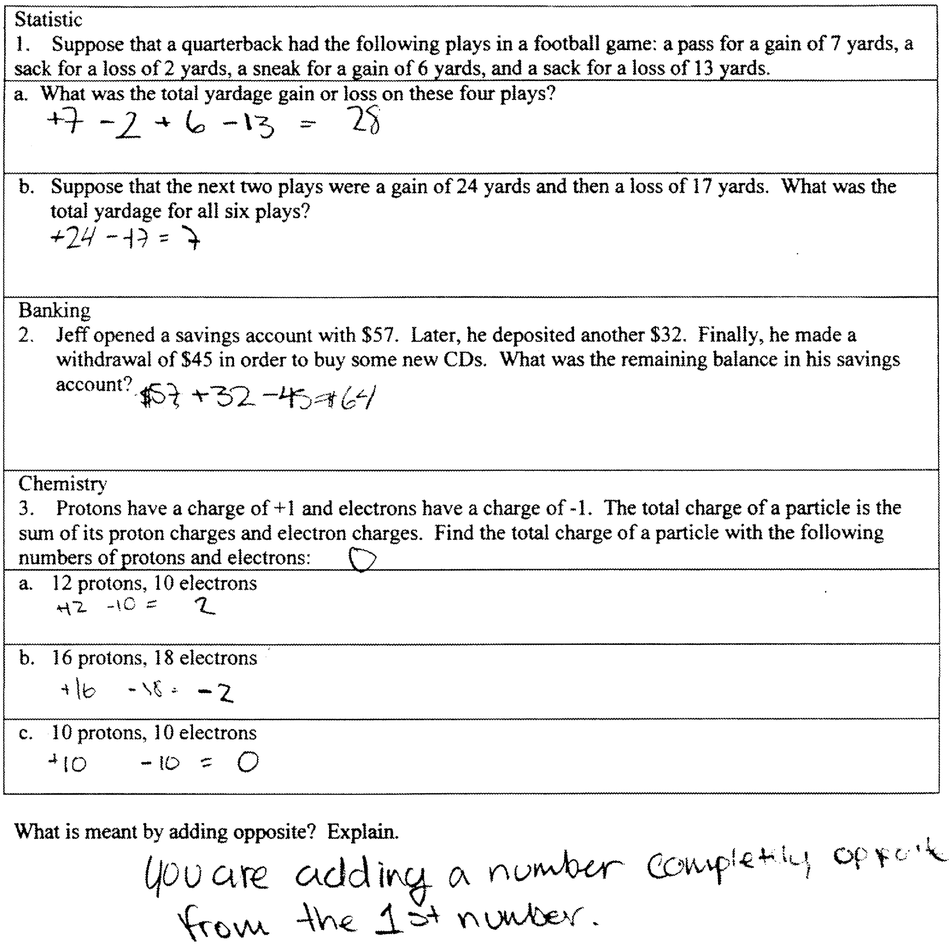
Student A (Araceli) was able to correctly identify data in both activities and showed improvement in her use of proper math notation from Activity 1 to Activity 2. The student failed to write equations in Activity 1 (providing answers for only 3a, b, and c), but attempted to write out the equations for all problems in Activity 2. However, as the student wrote the equations in both activities – particularly Activity 2 – she did not consistently use or apply the $+$, $-$ and $=$ signs appropriately. For example, in Activity 2, Questions 1a and 1c, the student did not use the $+$ and $=$ signs appropriately, despite correctly grouping the like terms. This inconsistency suggests that she may have started to understand some of the concepts involved, but needed additional clarification and practice in using terminology and notation. Her explanation of answers improved slightly between activities, since she showed full working for all questions in Activity 2 but only showed working for Q1 and Q2 in Activity 1.

The marks achieved by Araceli in the two activities showed an overall improvement (from 8 out of 16 points on Activity 1 to 14 out of 16 on Activity 2), as reflected by her increased scores in each of the four assessed categories (see Table 2). Such an achievement further supports the inference that the student-teacher conference targeting the student’s MI characteristics directly improved her performance.

6.2 Response of Student B (Fatima)

The responses of Student B (Fatima) to Activities 1 and 2 are shown in Figs. 3 and 4, respectively.

The student’s work in Activity 1 reveals that she did not achieve LG1 in either of Questions 1 or 2. In Activity 1, Q1a, Fatima made the same mistake as Araceli (adding the absolute values rather than their signed values). The numerical sentence was written correctly in 1a, but she added the absolute value of given yards (lost or gained), resulting in an incorrect answer. The student also demonstrated a lack of understanding of the subtraction symbol in writing the numerical sentence: $+7 - 2 + 6 - 13 = 28$. This indicates the inability to visualize two different groups of numbers, suggesting that she did not understand the meaning of opposite. She then failed to read the question properly in part (b) and computed the wrongly formulated expression $+24-17 = 7$ rather than carrying forward her previous answer and computing $28 + 24 - 17 = 35$. This answer also reveals a failure to understand positive and negative numbers as opposites or as a neutral-pair concept and a failure to correctly use written symbols. Indeed, in Activity 1 (Question 1b), students



Statistic

1. Suppose that a quarterback had the following plays in a football game: a pass for a gain of 7 yards, a sack for a loss of 2 yards, a sneak for a gain of 6 yards, and a sack for a loss of 13 yards.

a. What was the total yardage gain or loss on these four plays?

+7 -2 + 6 -13 = 28

b. Suppose that the next two plays were a gain of 24 yards and then a loss of 17 yards. What was the total yardage for all six plays?

+24 -17 = 7

Banking

2. Jeff opened a savings account with $57. Later, he deposited another $32. Finally, he made a withdrawal of $45 in order to buy some new CDs. What was the remaining balance in his savings account? $57 +32 -45=$64

Chemistry

3. Protons have a charge of +1 and electrons have a charge of -1. The total charge of a particle is the sum of its proton charges and electron charges. Find the total charge of a particle with the following numbers of protons and electrons: 0

a. 12 protons, 10 electrons

+12 -10 = 2

b. 16 protons, 18 electrons

+16 -18 = -2

c. 10 protons, 10 electrons

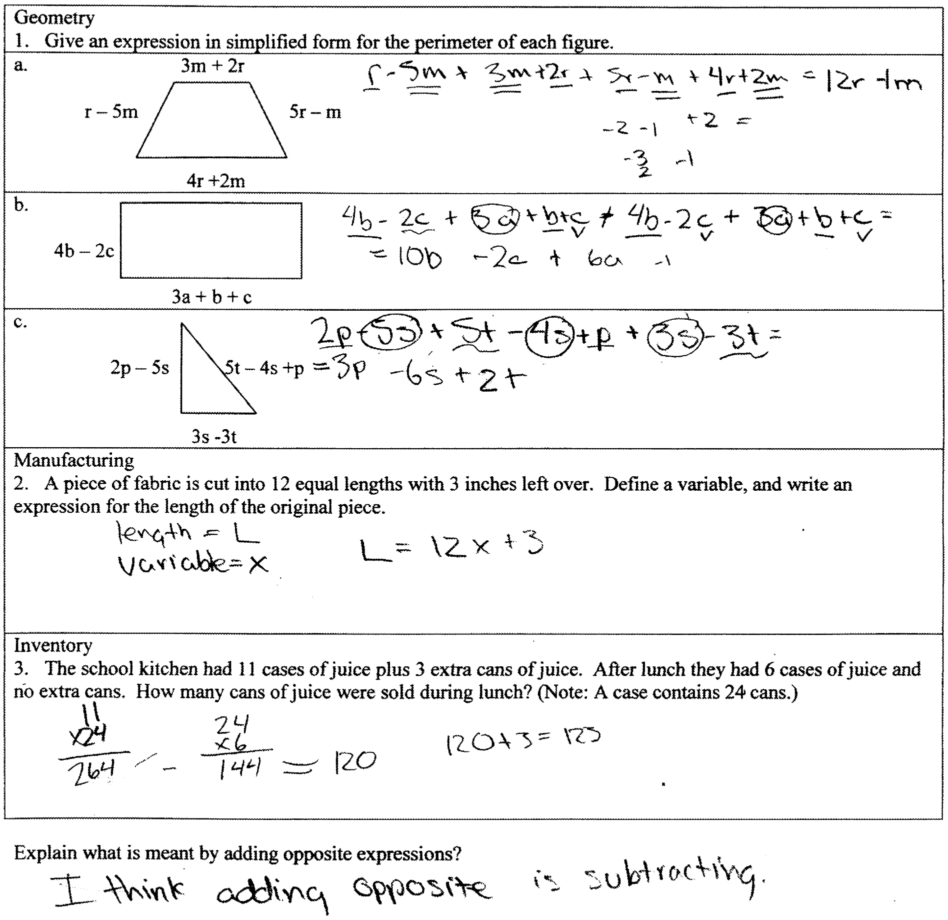
+10 -10 = 0

What is meant by adding opposite? Explain.

You are adding a number completely opposite from the 1st number.

Figure 3. Responses of Student B (Fatima) to Activity 1.

were asked to add data from Question 1a into the equation; Student B (Fatima) failed to do this, focusing instead on data in 1b. She showed the correct notation of two different groups of numbers (+24 and −17), but then ignored the information that 17 is a negative number, treating it as +17 and performing the take-away action, thereby reducing her 'mathematical error' score. In Question 2, the student again incorrectly computed the numerical sentence she wrote, further indicating a lack of understanding of the subtraction of negative numbers. Thus, although the student previously practiced arithmetic with negative numbers using the deposit/withdrawal metaphor, she failed to achieve LG1 in either Q1 or Q2. However, in Q3, she correctly wrote down and correctly evaluated expressions for the particle charges, thus achieving LG1. Yet Fatima's answer to "What is meant by adding the opposite" shows limited understanding of math language and of the concept of subtraction as adding the opposite, suggesting poor recognition of positive and negative numbers as being two individual groups, even if she can recognize their opposite actions.


Geometry
1. Give an expression in simplified form for the perimeter of each figure.
a. 3m + 2r; r – 5m; 5r – m; 4r +2m
r-5m + 3m+2r + 5r-m + 4r+2m = 12r 4m
b. 4b – 2c; 3a + b + c
4b - 2c + 3a + b+c + 4b-2c + 3a + b +c =
= 10b -2c + 6a
c. 2p – 5s; 5t – 4s +p; 3s -3t
2p-5s + 5t - 4s + p + 3s - 3t =
= 3p -6s + 2t
Manufacturing
2. A piece of fabric is cut into 12 equal lengths with 3 inches left over. Define a variable, and write an expression for the length of the original piece.
length = L
variable = x
L = 12x + 3
Inventory
3. The school kitchen had 11 cases of juice plus 3 extra cans of juice. After lunch they had 6 cases of juice and no extra cans. How many cans of juice were sold during lunch? (Note: A case contains 24 cans.)
11 × 24 = 264 – 24 × 6 = 144 = 120
120+3 = 123

Explain what is meant by adding opposite expressions?
I think adding opposite is subtracting.

Figure 4. Responses of Student B (Fatima) to Activity 2.

As a visual/spatial learner, Student B (Fatima) tends to learn from visual presentations. She can read diagrams and maps and enjoys solving mazes and jigsaw puzzles. Therefore, during the student-teacher conference, the student was encouraged to draw pictures of the problems. For example, while reviewing the tasks from Activity 1, she was asked to read a problem, draw a picture of the problem, and then restate the problem in her own words. This helped the student realize that she was being asked to subtract or add amounts. In addition, in light of the student's failure to obey the rules for subtraction of negative numbers in Activity 1, Question 1b, and recognize the subtraction of a negative as the addition of its opposite, the student was encouraged to use graphical representations (e.g., underlining) when grouping like terms. She was also encouraged to mark steps for adding negative numbers on a number line, helping her understand (in a graphic representation) that adding negative numbers is not simply the "take-away" function of subtraction. She was further recommended to use color-coding to help build awareness of how and when rules are used (for example, using different colors to identify plus or minus signs). This new approach aimed to help her organize data and focus on the appropriate notation. In addition, the student was asked to break tasks into smaller steps to help focus on the most salient features, for example, by color-coding the symbols (e.g., $+$, $-$) for a particular math calculation before calculating the answer or highlighting the most important information in a story problem.

It should be noted that, during the conference, it was determined that Student B (Fatima) did not always understand questions, leading her to focus only on the numerical data and ignore or overlook the written information. The student's response to Question 1 in Activity 1 implied possible problems with reading. In Question 1b, the student failed to incorporate data from 1a despite the fact that the prompt directly instructed students to include all data. In response to her work, the student was taught to read for meaning rather than search for key words when trying to identify the operation to use for a math word problem. Although this is not specifically related to her multiple intelligence, this information is included here to provide a more comprehensive understanding of the factors affecting this student's learning.

Following the student-teacher conference, Fatima completed Activity 2. In her first answer, for Q1a, she wrote the expression for perimeter as $\underline{r} - 5\underline{\underline{m}} + 3\underline{\underline{m}} + 2\underline{r} + 5\underline{r} - \underline{\underline{m}} + 4\underline{r} + 2\underline{\underline{m}}$; having clearly separated the r and m terms with respective underlining, she incorrectly simplified the expression as $12r + m$ (rather than $12r - m$). She therefore successfully grouped terms, but either made a mistake adding the m terms or incorrectly copied the answer from her work. However, overall, she demonstrated the successful achievement of LG1, LG2, and LG3 in this answer. Her answer to part (b) showed a similar ability to group terms, as demonstrated by her underlining and circling of different groups. Furthermore, and in contrast to Araceli's answer, Fatima wrote the full expression rather than applying the commutative rule of addition. However, she again made an arithmetic error, this time in the coefficient of the c term. In part (c) she again used underlining and circling of terms to provide a fully correct answer, demonstrating the achievement of LG1, LG2, and LG3.

In Q2, Fatima wrote a correct expression representing the prompt. She did not name both variables correctly, as she chose the letter x to represent a piece of fabric without defining it as such. She correctly defined the length as L (although she did not specify it fully as the length of the original piece of fabric). This answer demonstrated her achievement of LG2. In Q3, Fatima wrote down the correct operations for determining the number of cans and reached the right answer, but – like Araceli – showed no work for the difficult multiplication (for example, 11×24), again suggesting the use of a calculator despite the fact that calculators were not permitted. Finally, in the last question, Fatima showed her full understanding of subtraction as adding the opposite in her response to being asked to "Explain what is meant by adding opposite expressions."

The student's work in Activity 2, where the perimeters were correctly determined, reveals that all three goals were achieved. Each group of like terms was underlined in Question 1a (r terms once; m terms twice) and calculations were written on her paper, demonstrating the application of basic operations of arithmetic to real numbers to solve word problems (LG1) and the writing of algebraic expressions (LG2). This clear evidence of written working also earned the student greater marks in the explanation category on the rubric. Based on these written calculations, the student found the correct answer and simplified algebraic expressions to solve word problems using the properties of real numbers (LG3). The student applied the skills of adding/subtracting integers by solving word problems and defined her understanding of adding opposite numerical expressions. In the second question in Activity 2, the student further demonstrated her mathematical reasoning by using the letters L to represent the length of the original piece of fabric and x to write an expression representing parameters identified in the problem (LG2).

The higher standard of responses in Activity 2 relative to Activity 1, as seen in Table 2 and suggested by the discussion above, was confirmed by this student's scores, which improved from 8 out of 16 points on Activity 1 to 14 out of 16 on Activity 2. As with Araceli, the higher score achieved by Fatima in Activity 2 was due to improvements in each of the four assessed categories. The graphical representations that she incorporated into her work, as discussed in her conference with the teacher appeared to maximize her visual/spatial learning and enable her to answer the questions correctly.

6.3 Future Activities

Between the two activities, an individual teacher conference took place with each of the two students. Each student's preferential learning style had been previously ascertained; during the conference, that knowledge was used to teach the basic concepts of mathematical operations, geometric concepts, and other relevant information. Some time was allotted for practice before the completion of Activity 2, which was used to determine if the students utilized what they had learned to solve the problems. The results indicated that the students had indeed understood the concepts after they had been presented via learning preference rather than via the usual method of just giving them the problems to solve.

The general consensus amongst teachers, as well as most researchers, indicates that complete learning is a process rather than a point-in-time accomplishment. According to Dewey (2007), when a task is completed successfully, learning occurs, and a person becomes more effective at solving the problem or completing the task successfully. With repetition (i.e., solving a range of problems of the same type), that learning becomes stabilized over time. Dewey believed that learning follows an S-shaped curve, with skill measured on the Y-axis and the number of attempts to complete the problem on the X-axis. The curve is illustrated in Fig. 5.

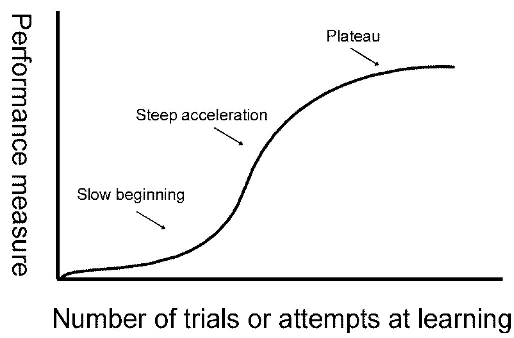


Figure 5. The S-shaped "learning curve" typical of complex learning[2].

The beginning of any learning process proceeds slowly until confidence in one's own ability improves competence at the task. At this point, a very rapid acceleration of learning occurs until a stable plateau is reached.

In the case of students A and B (Araceli and Fatima), their learning was at the beginning stage. Prior experience of incompetence and the interference

[2]Russ, Dewey.: 2007, *Psychology: An Introduction*, Chapter 7, "Cognition", http://www.intropsych.com/ch07_cognition/tofc_for_ch07_cognition.html

of negative learning that had to be unlearned likely played a factor in their ability to complete all problems successfully; this study did not allow for sufficient follow-up to determine whether the learning was still continuing. Future similar activities, of course, would help researchers make a more positive determination of progress and improved learning. Another similar activity, perhaps a week later, would show the strength of learning and its direction. Further follow-up would reveal more about the learning situation. Class time instruction, even if not geared specifically to learning style, would also play a role in student learning, but this might be a negative influence, depending on the strength of the negative prior experience.

In any event, it is the conferencing and the additional instruction between tasks or activities that are of significance here. The improvement between Activity 1 and Activity 2 is solely attributable to the intervention that occurred between activities and not a result of a joint discussion in the classroom or the mere effect of an intervention taking place.

6.4 Activities 1 and 2: Learning strategies

The students built on what they had learned in Activity 1 to answer the questions in Activity 2, thereby advancing from the concrete to the abstract. Both activities provided real-life examples in which students identified and applied the basic properties of real numbers to solve word problems. Activity 2 required students to collect like terms in algebraic expressions using the rules of addition of signed numbers as practiced in Activity 1.

The author's previous experience teaching this unit has shown that the concept of negative numbers is confusing for most students because they think that negative numbers lack connections to the real world and that negative numbers act differently from the positive numbers. Some students have difficulty adding two negative numbers or adding numbers with different signs, failing to understand why subtracting a negative number would result in a number becoming greater. To address and correct these misconceptions, students were directed to work with algebra tiles, collaborating with their fellow students to figure out the rules while reinforcing their understanding of negative numbers. This exercise activated their visual/spatial intelligence.

For most people, there is a learning curve when new information is presented or when one must demonstrate a skill or perform some task that is a new experience for them. In most cases, such as learning to be a restaurant server or a researcher in a crime lab, there are many tasks that must be learned – some concrete, some abstract. However, interventions are also available, usually in the form of help from someone who knows how to perform the tasks required;

this is similar to moving from Activity 1, with conferencing, to Activity 2, where positive results from the lessons learned in conferencing were applied. The key part, in all cases, is the conferencing or mentoring aspect of learning, in which various intelligences can be applied to a situation in order to learn it better. Thus, not only did the students learn the required material, they also learned that with the application of other ideas and other remedies, they are able to succeed in situations that at first seemed difficult or impossible.

It is important that any assessment of student success in learning be associated with problem types rather than single specific algorithms. The concept of mathematical operation is what is being taught, along with the ways in which that operation is perceived by the student. Merely answering a question correctly or incorrectly is, for most purposes, irrelevant. Knowing how to solve similar problems when they are encountered is the evidence of understanding. Of course, the knowledge of how to solve them will likely result in correct responses, and that is what is measured, with understanding inferred. If only one such problem is given out of a type, there remains uncertainty. When a range of problems of a type are solved, even if they are not all done so correctly, the student's understanding of that problem type can then be inferred.

6.5 Matching tasks to intelligence types

Traditional lectures on algebra benefit students who possess linguistic and logical-mathematical intelligence, while the strategies implemented in the activities described herein helped spatial and interpersonal learners use their inherent skills to learn algebra. Thus, it remains necessary to develop strategies to help students possessing musical, naturalist, intrapersonal, and bodily-kinesthetic intelligences use their inherent skills, as associated with their type of intelligence, to more effectively learn the concepts and principles of algebra. Table 3 outlines possible ideas for the specific intelligences not discussed herein.

Understanding is a commonly used term in education, but few really understand its importance; nor do they assess understanding in many meaningful ways. Neither paper-and-pencil multiple-choice or fill-in-the-blank assessments nor national standardized tests necessarily assess true understanding of a concept. Rene Diaz-Lefebvre (2004), a professor of psychology at Glendale Community College, stated that, "To understand simply means that a person can take something learned – concepts, theories, knowledge – and apply it appropriately in new situations ... the challenge becomes the student's ability to demonstrate an understanding of terms, concepts and knowledge as they are applied to the real world outside of the classroom setting. Many students

Type of Intelligence	Suggested Activities	Algebraic Focus
Musical	Developing chants and/or song lyrics (Ediger 2005)	Fundamental algebraic concepts/ principles
	Physically demonstrating how the volume of a set of musical instruments varies according to distance	Squares (i.e., variables raised to the second power) and square roots
	Striking tuning forks with nearly the same frequency to create a beat representing how like terms can be combined when adding them together or subtracting them from one another (conversely, two tuning forks with very different frequencies that are struck together will not create a beat, but simply overlaying sounds, suggesting unlike terms)	Algebraic principle that allows like terms to be either added together or subtracted from one another
Bodily Kinesthetic	Asking a physically healthy bodily kinesthetic learner to run first at a slow speed for some given distance and then the same distance at twice the speed, focusing on the effort required to perform each of the tasks	Squares (i.e., variables raised to the second power) and square roots
	Constructing a relevant object to demonstrate concepts (Ediger, 2005)	Fundamental algebraic concepts and principles
Naturalist	Engaging students in performing an experiment regarding the relationship between time and distance with regard to a falling object (namely, in the absence of dissipative forces, the distance which an object falls is proportional to the square of the time with it takes the object to fall the distance)	Squares (i.e., variables raised to the second power) and square roots
	Applying algebraic principles to an observation or experiment relevant to the understanding of nature (Ediger, 2005)	Fundamental algebraic concepts and principles
Intrapersonal	Spending quiet time in self-reflection during a class (Ediger, 2005)	Fundamental algebraic concepts and principles

Sources: Author's ideas, except as noted

Table 3. Suggested Activities according to Type of Intelligence.

thought to be lazy are actually bored and frustrated. Because, even though they are 'smart,' they crave multiple methods of stimulation. Providing various methods of intellectual stimulation may be more effective in helping them master new material." A student needs options to access when needed; in lower grades, teachers can provide such options. Learning options, even in the mathematics and sciences classroom, include acting/role-playing, creative dance, collage, mime, book report, poetry, drawing/sketching/painting, computer simulation, sculpture, interview, creative journal writing, musical/rhythmic interpretation, and traditional tests. Students can demonstrate their true understanding of academic material by performing their understanding for the teacher and classmates. Creative grading rubrics complete the evaluation process (ibid).

The key, of course, is in the application of understanding. Both students A and B were able to do this to a limited degree when asked to do so after they had been given learning options. Although not quite to the degree suggested, both students took the options provided during conferencing and applied these alternative ways of seeing mathematical problems to demonstrate some understanding where none existed before. The options naturally relate to the various multiple intelligences and how students use them to acquire new material. As one of Diaz-Lefebvre's students said, "I think the type of student that would benefit would be the one that usually doesn't want to participate, the one that is just sick of regular school, you know, it's never ending, it's always paperwork and book work. And they don't try and so they do badly on tests and stuff like that. But this gives them an opportunity to do something completely on their own. And like I saw with everybody in my algebra class that did the learning options, they all worked hard. None of them slacked off. So I think the kids in the back of the classroom that would usually never participate are given an opportunity to okay, if you don't enjoy learning this way, you have your own choice. Then they are trying harder in learning the material and then they feel better about themselves as far as you know, they can excel in class. It's just those they get into a slump with the old style of teaching."

Such varied activities may interfere with a given class meeting that already has limited time and resources. Therefore, instructors may wish to place students with similar types of intelligence into teams that can meet outside of class as opposed to the in-class groups described herein. To ensure that each team carries out its assigned task, the tasks could be assigned as homework. Each team member could be asked to fill out a form once the team submits its task to the instructor, assessing the contributions of the other team members regarding the completion of the assigned task. Such forms would be anonymous in order to ensure that the assessment provided by students is not adversely influenced by the possibility of a student's assessment becoming known to classmates and peers.

7 Conclusions

The introduction of instructional strategies that incorporate the theory of multiple intelligences can greatly increase educators' effectiveness as teachers of algebra. One of the teachers in Diaz-Lefebvre's (2004) study said that, for him, a major way of assessing understanding is in the application of theory and concepts. When a student relates how an idea or concept applies in his or her own personal life, the student understands the material. If the student can further explain the connections to others, such understanding is even more evident. When students use something abstract to show their understanding, they are close to mastery. It is reasonable to assume that a proper selection of didactic methods based on dominant intelligence types can accelerate the process of acquiring knowledge on a wider range of topics, thereby promoting the inclusion of students requiring additional help in regular academic courses.

Ultimately, the theory of multiple intelligences relates to various scholastic subjects, such as techniques to address the needs of spatial learners within the context of algebra that ultimately connect algebra with art. Similarly, musical learners connect mathematics and music within the algebra classroom. Therefore, algebra instructors may wish to collaborate with their colleagues who teach other subjects at their educational institution in order to meet students' needs.

7.1 Limitations and areas for further research

Hodkinson and Hodkinson (2001) identified eight limitations inherent in case studies. Although this method was appropriate for the current study, there are specific limits to its usefulness both generally and specifically to this study. The eight identified limits to case study research generally are that case studies 1) generate too much data for easy analysis, 2) are very expensive if attempted on a large scale, 3) attempt to represent complexity in a simple manner, 4) do not lend themselves to numerical representation, 5) are not generalizable in the conventional sense, 6) are strongest when researcher expertise and intuition are maximized, although this raises doubts about their "objectivity", 7) are easy to dismiss by those who do not like the results, and 8) cannot answer a large number of appropriate and relevant research questions.

For this particular research study, expense was a negligible factor since the study took place within a normally operating venue and the research was basically a part of what was occurring within the classroom already. However, if this study were to be conducted on a wider scale, with perhaps some modifications mentioned below in areas for further research, it might indeed become quite costly when one considers the time involved in analyzing the data. Should the researchers apply their ordinary hourly rates to the time involved, the study could very quickly become financially untenable.

The data generated from even this small study were considerable, including data from each student and her preferred way of learning, administrative data that had to be considered, data from the two activities, data from the conferencing, and the analysis of all of the data for a study that – with the students – took much less time. Some of the data was numerical, which had to be graded or evaluated and subsequently analyzed in terms of the stated learning goals and the hypothesis that guided this study. Other information (e.g., from the conferencing sessions) had to be incorporated into what was known about performance so that inferences could be drawn. Again, if this study were much larger, the analytical component could present serious challenges to the researcher.

Furthermore, this study is not generalizable. Both of the participating students were Spanish-speaking females who were 14 years of age. Neither was particularly good at mathematics, and one of them had an intelligence preferential learning style of interpersonal skills, which does not correlate well with academic achievement. Of more generalizable benefit would have been a case study of a set of students' representative of all students in the school, with identified academic intelligences (e.g., linguistic intelligence or mathematical/logical intelligence), who were having difficulty with math. A series of studies with several students with differing abilities would make a good comparison study for the current study as well.

Since there was only one hypothesis, the information obtained from the study is only useful for that specific hypothesis and no other. This is also a limitation in that only one thing was learned about multiple intelligences from the study. Absent were hypotheses about conferencing and its effects, teacher influence in the selection of activities, and the effects of being in a classroom in which the relative performance of all students is below average. If these same students had been in a classroom with high-performing students, would the results have differed? That is an unanswered question that may have extreme relevance in the discussion of multiple intelligences and achievement.

Finally, concerns about the theory of multiple intelligences itself pose potential limitations to the current study. Several criticisms have been leveled against Gardner's work, as identified in section 2.2. of this study. Until these issues can be unequivocally addressed, questions will remain about the validity of this theory. Of particular note is the difficulty of quantifying performance based on an individual's singular intelligence as defined by Gardner's theory.

Future studies should focus on addressing these limitations. Studies that compare students with the same intelligence could prove useful for understanding the effects of that kind of intelligence. Students with other types of intelligences, rather than the two examined, might also be of some importance. The point is to get at what multiple intelligences look like in a classroom and the effects that they have on student's achievement and learning. Another approach would be to give accomplished students new tasks in algebra that they had not previously experienced (e.g., logic problems or community issues that are difficult to solve) and observe how they manage to solve them. Using this approach, higher order thinking skills come into play, and the combination of fluent thinking and intelligence would reveal a whole new set of data that differ remarkably from the data obtained in this study.

Given the premise that very little research has been done in this area with high school algebra instruction, this study is a good start toward filling that gap. As No Child Left Behind Act results show and are apparent in studies linking US mathematics achievement with other countries, the traditional ways in which mathematics has been taught are not particularly effective with the current generation of learners. Perhaps this is because we underestimate the importance of multiple intelligences and the ways in which students really learn best. Creating classrooms that give students the best opportunities to learn the content in whatever ways work best for them is a goal that is worthy of study. In the meantime, linking effective studies to create a realistic picture of how students learn is an appropriate way to address 21^{st}-century learning in our schools.

Although further analysis of the results herein is warranted, it is still worth stressing that this subject area can play a significant role in the didactics of mathematics at almost every level. Further research is needed to test new hypotheses suggested by the results presented here.

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http://www.standards.dfes.gov.uk/sie/documents/mod6_handsB.pdf
(only takes into account the original seven types of intelligence described by Howard Gardner)

http://www.businessballs.com/howardgardnermultipleintelligences.htm#multiple intelligences tests (contains two tests in two different file formats)

http://www.teachervision.fen.com/intelligence/teaching-methods/3678.html (a questionnaire to help parents identify the learning style of their children)

http://www.scribd.com/doc/267246/free-multiple-intelligences-test-manual-version (manual)

Nauczanie i uczenie się elementów algebry w świetle teorii wielorakich inteligencji H. Gardnera

Streszczenie

Opisane w pracy badania opierają się na pracy Howarda Gardnera i jego teorii inteligencji wielorakich. Gardner zakłada, że inteligencja nie ogranicza się do jednego atrybutu, ale może przejawiać się w wielu odrębnych kategoriach. Odkąd przedstawił on teorię inteligencji wielorakich, twierdząc, że jednostki różnią się pod względem predyspozycji do uczenia się zgodnie z ich dominującym typem inteligencji, jego teoria została zastosowana do dydaktyki nauczania w szerokim zakresie tematów/przedmiotów. Gardner wyróżnił osiem różnych typów inteligencji: językową, logiczno-matematyczną, przestrzenną, muzyczną, kinestetyczną, interpersonalną, intrapersonalną i przyrodniczą. Spośród nich, zwykle tylko pierwsze dwa rodzaje są doceniane w tradycyjnym nauczaniu. Pomimo szerokiego zastosowania tej teorii w nauczaniu wielu przedmiotów, jej roli w nauczaniu algebry poświęcono dotąd niewiele uwagi w literaturze.

Podjęte badania, o charakterze studium przypadków, miały na celu zweryfikowanie następującej tezy: „Powiązanie stosowanych zabiegów dydaktycznych oraz metod i form pracy z dominującymi rodzajami inteligencji wielorakiej uczniów poprawi ich wyniki w nauce.” Zgodnie z tą hipotezą, podejmowane przez nauczyciela zabiegi dydaktyczne oraz stosowane metody i formy pracy powinny być dostosowane do dominującego typu inteligencji ucznia i związanego z tym stylu uczenia się. W moim artykule przedstawiam pracę dwóch uczennic szkoły średniej w klasie algebry wyrównawczej (klasa dziewiąta, 14 lat), z różnymi typami wielorakich inteligencji (interpersonalne i wizualno-przestrzenne preferencje uczenia się), nad serią ćwiczeń-problemów z algebry, przedstawionych w formie sprawdzianu.

Badanie polegało na analizie odpowiedzi uczennic w dwóch sprawdzianach (tzw. Arkusze 1 i 2). Oba arkusze zawierały podobne zadania matematyczne sprawdzające umiejętności zastosowania podstawowych operacji matematycznych do rozwiązywania różnorodnych zadań z elementarnej algebry, ukierun-

kowanych na zaplanowane przez nauczyciela cele: (1) zastosowanie podstawowych operacji matematycznych do zbioru liczb rzeczywistych w rozwiązywaniu zadań tekstowych, (2) zapisywanie wyrażeń matematycznych do rozwiązywania zadań tekstowych, (3) upraszczanie wyrażeń algebraicznych w rozwiązywaniu zadań tekstowych przy użyciu własności zbioru liczb rzeczywistych. Uczennice rozwiązywały zadania tekstowe powiązane ze statystyką, bankowością i chemią, geometrią, produkcją i kontrolą zapasów. W okresie między sprawdzianami odbyły się rozmowy nauczyciela prowadzącego badania z uczennicami.

Najpierw u badanych uczennic zdiagnozowano dominujący typ inteligencji, ujawniający ich preferowany styl uczenia się. Zastosowano do tego celu kwestionariusz wielorakiej inteligencji wypełniony przez uczennice. Następnie zostały poproszone o rozwiązanie sprawdzianu matematycznego z Arkusza 1. Zadania z Arkusza 1 należało rozwiązać przy użyciu podstawowych operacji matematycznych w zbiorze liczb rzeczywistych. Po rozwiązaniu zadań z Arkusza 1 uczennice zostały zaproszone na indywidualne rozmowy z nauczycielem, który omówił popełnione przez nie błędy, naprowadzając je na poprawne rozwiązania przez stosowanie zabiegów dydaktycznych odpowiednich do dominującego typu inteligencji każdej z nich, sugerując stosowanie podobnych środków w procesie uczenia się. Po pewnym czasie uczennice otrzymały do rozwiązania zadania z Arkusza 2, przy czym mogły w rozwiązaniach wykorzystać metody, na temat których dyskutowały z nauczycielem podczas rozmów. Poprawa osiągnięć uczennic była oceniana w odniesieniu do konkretnych nauczanych pojęć, poprzez pomiar uzyskanych wyników w zadaniach istotnych dla tych pojęć. Osiągnięcia te były oceniane zarówno pod względem jakościowym (w analizie rozwiązań poszczególnych zadań), jak i ilościowym (poprzez porównanie poprawnych i błędnych wyników), i były podzielone na cztery obszary realizacji celów kształcenia: pojęcia i błędy matematyczne, objaśnienia, terminologia oraz zapis matematyczny.

U uczennicy A dominuje inteligencja interpersonalna, co oznacza, że najłatwiej jej przyswajać informacje poprzez jej relacje interpersonalne i konwersacje. Po wypełnieniu Arkusza 1, nauczyciel przedyskutował z A prawidłowe odpowiedzi i poprosił, aby znalazła partnera, z którym może o nich porozmawiać. Nauczyciel również zasugerował dodatkowe metody pomagajajace jej prztwarzać informacje i wzmacniające pamięć, takie jak wykonanie flashcards z informacjami na temat nowo poznanych metod lub definicji pojęć matematycznych. Uczennica została również zaproszona do współpracy z innymi uczniami podczas pracy w klasie w grupach, a jej zadania domowe opracowano jako interaktywne z rodzicami i/lub z rówieśnikami.

Porównanie wyników sprawdzianów 1 i 2 pokazuje wyraźną poprawę. W Arkuszu 1 tylko 8 z 16 zadań zostało rozwiązanych poprawnie, a w Arkuszu 2, poprawnie rozwiązano 14 z 16.

Dla uczennicy B określono, jako dominującą, inteligencję wizualno-przestrzenną. Osoby posiadające ten typ inteligencji jako dominujący często rysują na marginesach esy-floresy, wyraźnie ożywiają się, gdy w książkach obok tekstów pojawiają się obrazki, diagramy, mapki, labirynty. Lubią, kiedy na zajęciach prezentowane są filmy lub pokazy multimedialne. Myślenie obrazami, dobry odbiór świata wizualnego to ich domena. Podczas rozmów z nauczycielem uczennica została zachęcona do korzystania z tych możliwości. Tak jak A, również uczennica B zdobyła 8 z 16 punktów w sprawdzianie 1, a po rozmowie z nauczycielem 14 z 16 punktów w sprawdzianie 2.

Uzyskane w sprawdzianach wyniki obu uczennic wskazują na znaczną poprawę. Z analizy odpowiedzi wynika, że znacznie podniosły się ich umiejętności we wszystkich czterech obszarach realizacji celów matematycznego kształcenia, jak również, że dostosowanie wsparcia nauczania do dominujących typów inteligencji danej osoby wyraźnie poprawia zrozumienie tematu i zwiększa szanse uczniów na osiągnięcie lepszych wyników. Wybór zabiegów dydaktycznych, które są dostosowane do dominujących typów inteligencji, może przyczynić się do zdobycia przez uczniów umiejętności samodzielnego uczenia się przez poznanie swoich mocnych stron. Ponieważ teoria inteligencji wielorakich dotyczy różnych podmiotów szkolnych, wydaje się, że współpraca nauczycieli matematyki z kolegami, którzy uczą innych przedmiotów w danej placówce oświatowej, poprzez poznanie przez uczniów swoich dominujących typów inteligencji, może wzmocnić ich wiarę w swoje możliwości i jest kluczem do osobistego sukcesu. Nauczyciele, wiedząc, w przypadku każdego ucznia, jakie są preferowane przez niego, dostosowane do dominującego typu inteligencji, metody uczenia się, mogą stosować na lekcjach takie zabiegi dydaktyczne, dzięki którym praca uczniów będzie efektywna i skuteczna.

Anna Zofia Krygowska Competition for the best diploma dissertation in mathematics education

The competition has been organized since 1992. In 2010 the Jury composed of
dr Stanisław Domoradzki,
dr hab. Ewa Swoboda,
dr Antoni Pardała – chairman
decided to aword the following graduates:

1^{st} place – Marta Korczyk (Pedagogical University of Krakow, Prof. H. Siwek supervisor) Theme: *The didactic conceptions of developing the idea of on the algebraic expression in textbooks for different types of schools*

2^{nd} place –Anna Jurewicz (Pedagogical University of Krakow, Dr L. Zaręba supervisor) Theme: *The impact of visualization on noticing and formulating regularities by students at different levels of mental development*

Also, Ewelina Holender (Pedagogical University of Krakow, dr A. Urbańska supervisor) was honored for an interdisciplinary, novel, and consistent approach to the theme *On the integration of mathematics, computer science, and art – a lesson sequence with M. C. Escher's mosaics.*

Konkurs jest organizowany od roku 1992. W edycji 2010 Jury w składzie
dr Stanisław Domoradzki,
dr hab. Ewa Swoboda,
dr Antoni Pardała – przewodniczący
postanowiło przyznać

1. miejsce – mgr Marcie Korczyk (Uniwersytet Pedagogiczny, promotor – prof. Helena Siwek) za pracę na temat *Dydaktyczne koncepcje kształtowania pojęcia wyrażenia algebraicznego w podręcznikach matematyki dla różnych typów szkół*,

2. miejsce – mgr Annie Jurewicz (Uniwersytet Pedagogiczny, promotor – dr Lidia Zaręba) za pracę na temat *Badanie wpływu wizualizacji na dostrzeganie i zapisywanie regularności przez uczniów na różnym poziomie rozwoju intelektualnego.*

Jury postanowiło także wyróżnić mgr Ewelinę Holender (Uniwersytet Pedagogiczny, promotor – dr Aleksandra Urbańska) za „interdyscyplinarne, nowatorskie i spójne ujęcie" tematu *O integracji matematyki ze sztuką i informatyką na przykładzie cyklu lekcji w klasie VI z mozaikami M. C. Eschera.*