ANNALES OF THE POLISH MATHEMATICAL SOCIETY
5TH SERIES: DIDACTICA MATHEMATICAE 32 (2009)

DOCTORAL THESES

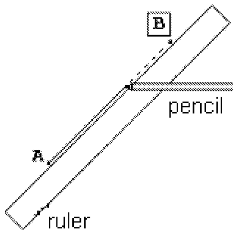
Witold Pająk
Mathematics teacher, Oświęcim (Poland)

Role of the CABRI computer program for the assimilation of mathematical contents by polish middle school (gymnasium) students[1]

Introduction

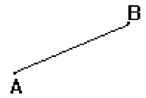
CABRI computer program is commonly rated among teaching aids. One of the main purposes of using teaching aids during lessons of mathematics is to make it easier for the students to "transform" mentally the objects of the physical world into the objects of the world of mathematical abstraction. *Even if, in the teaching of geometry, in its initial phase, the teacher organizes students' work done on physical objects, the target of this work is beyond physical reality* (Krygowska 1977a, page 67, my translation). On the road from the concrete to the abstract *CABRI* program uses, to a great extent, indirect representations in the form of a static or dynamic drawing (Konior: 2002, page 5). Drawing a figure, for example, a segment or circle on a paper sheet or blackboard (drawings 1a, 2a) by hand, the student takes advantage of motor abilities of his or her hand (letter B in the box stands for the point "which is not yet in the drawing, but will be").

[1]The paper is a modified version of the author's lecture presented 21st January 2009 at the Pedagogical University of Krakow during the doctoral defense. Prof. Henryk Kląkol (Pedagogical University of Krakow, Poland) was a supervisor of the PhD thesis; reviewed by: Prof. Maria Korcz (University of Poznań, Poland) and Prof. Ryszard J. Pawlak (University of Łódź, Poland).

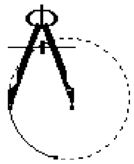


Drawing 1a.

Drawing 1b$_1$.

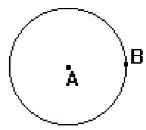


Drawing 1b$_2$.



Drawing 2a.

Drawing 2b$_1$.



Drawing 2b$_2$.

Whereas, while drawing an analogy figure in *CABRI* (compare the drawings 1b$_1$, 1b$_2$ and 2b$_1$, 2b$_2$ respectively with a traditional method) only the initial state and final effects are recorded. As a picture, they are created "complete", always in the same, very precise way, they appear quickly and firmly. Even this episodic comparison shows the differences between a drawing made in a traditional way and a drawing made in *CABRI*, on condition we consider making a drawing to be an important method of shaping concepts.

The comparison of both ways of drawing illustrated in this example is a part of a more general thesis, which can be formulated as follows: *CABRI* **and a traditional way of drawing are not competitive (or alternative) systems, but in an educational process they complement each other**.

The subject, targets and kinds of research; The construction of research tools

In the year 1999, a curriculum which combines teaching mathematics with the elements of computer science was created[2] The curriculum is intended for the middle school (gymnasium) students. One of the computer programs, which are recommended by the authors of the curriculum as a geometry teaching tool, is *CABRI*. Therefore, the forms that work with this program

[2] *Program nauczania matematyki z elementami informatyki w gimnazjum* (PROGRAMME OF TEACHING MATHEMATICS WITH THE ELEMENTS OF COMPUTER SCIENCE). Worked out by a team under the supervision of H. Kąkol and approved by the Ministry of National Education; decision number: DKW-4014-289/99 (Kąkol 1999).

could be used as a natural testing ground for experiments and research on how *CABRI* program itself functions.

The main research was conducted in the school year 2001/2002. For organizational reasons, in some classes it was completed in October, 2002. The organizational pattern of the research included the following main steps.

1. Students of four first-year classes of the middle school (gymnasium) who worked according to the above mentioned curriculum even before they started working with *CABRI* program in class, which was in September and October 2001 – underwent tests called *reconnaissance tests*. Those tests made it possible to identify intellectual predisposition of the students and the level of their abilities as well as specific geometric skills.

2. *Identification tests* were conducted, at the same time (which was in September and October, 2001), among students of seven first-year classes that did not work according to the mentioned curriculum and did not use *CABRI* program during mathematics lessons.

3. As a result of the identification tests, pairs of students were chosen (one student from the *CABRI* class, the other one from the non-*CABRI* class. They were selected in such a way that both students had identical characteristic traits.

4. Individual tests and observations of students' work were conducted and lessons were observed.

Type of research, the number of the students tested for the sake of individual observations and the number of lessons observed are shown in the table below.

Type of tests	Number of students
Reconnaissance tests carried out in the *CABRI* classes	109
Reconnaissance tests in the non-*CABRI* classes	149 students
Individual observation 1 – **The height of a triangle**	65 students
Individual observation 2 – **Understanding the general validity of a theorem and awareness of the need of proving it**	45 students
Developing geometrical imagination (final test)	56 pairs
***CABRI* program in the organizational structure of a lesson and in its contents**	20 lessons observed

Table 1.

General aim and structure of the research

There were two types of empirical research in the course of work, whose role was to decide about the whole structure of the research procedure:

1) Laboratory research conducted separately with individual students. This type of research was done out of classroom.
2) Research conducted in the conditions of *a natural teaching/learning process* – as it has been defined before – with the use of *CABRI* program and concerning the process which takes place in a classroom.

Laboratory research is mainly the research on the *student's behavior patterns*, while the research called here group research is related, first of all, to *all direct effects of the teacher's activities.*

Two types of research imply two groups of general goals, which are to be achieved by means of the research. First, those goals will be formulated as a scope and then their detailed contents will be specified.

Goal

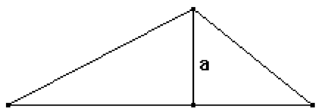
1) Determining what attitudes, activities and ways of thinking in the conditions of individual work on a mathematical problem are inspired (generated) by *CABRI* program
   a) used for the first time by a student as a completely new tool for him/her,
   b) used while solving a problem as a means already known (from mathematics lessons),
   c) functioning not by a direct use of the computer in work on this problem, but as an effect of former training, which was organized with the planned use of this means in class.
2) Working out elements of the educational value of *CABRI* program used as a tool in teaching geometry in the first class of middle school (gymnasium); so, it is about selective characteristics of the usefulness of this program within the range of shaping mathematical concepts, using imagination and justifying mathematical facts by students.

Examples of research

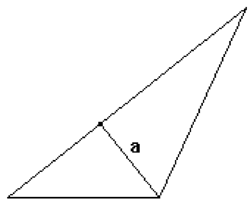
1. Laboratory research – the height of a triangle

To examine whether *CABRI* program may help to weaken the impact of „natural” directions in the process of reception of a drawing by a student, an individual observation was undertaken, *The height of a triangle*. The course of the session was the following:

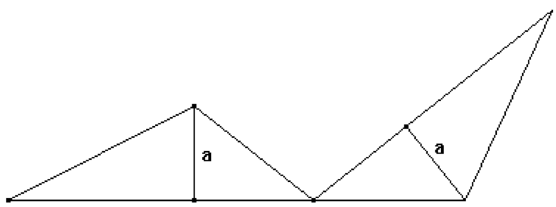
1) In preliminary talks, the student was reminded the definition of the height of a triangle (it was either a descriptive form or a functional presentation in the form of enumeration of successive stages of making a drawing, sketched by hand at the same time); it was not about reciting a “bookish” formula.
2) The subject was observing on the computer screen ready-made, static drawings 3a) and 3b) which illustrated triangles with equal sides. In each of those cases he/she answered the question: *Is segment* a *a height of the triangle?*



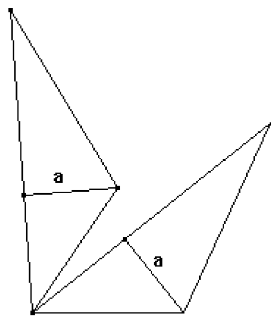
Drawing 3a.



Drawing 3b.



Drawing 4a.



Drawing 4b.

3) Having recorded the individual answers, the student was encouraged to observe transformation of the triangle from the initial position 3a in-

to the final position 4b on the monitor screen. First, a parallel shift to the position visible in drawing 4a was done, next, a rotation around one of the vertices allowing to achieve the final position. In this kind of examination, it is best when it is the student that executes the transformation; this operation might be reversed and repeated many times by the student.

4) In subsequent phases of the transformation – especially, when the triangles superimposed each other – the question formulated in point 2 was asked again. In this part, in which movement in the *CABRI* environment was used, only those students who had given wrong answers concerning drawing 4b) earlier (which is, in the static part of the research) took part.

5) In case of any change of the student's attitude in the course of the experiment being done on the screen, the attempt was made again and, after about three months, the question formulated in point 2 was asked one more time. Let us call this stage *repeated test.*

This examination was intended to answer the following questions.

1) **Can continuous motion of a geometrical object on the monitor screen created by means of *CABRI* help to weaken the impact of „natural" directions (vertical and horizontal) on the student's perception of a geometrical drawing and, as a consequence, to interpret the concept represented by this drawing in the right way?**

2) **If a student, under the influence of motion, has changed his/her initial answer, how permanent are the changes that took place just once under the influence of *CABRI* in his/her system of knowledge and skills?**

2. Developing geometrical imagination

In solving tasks in plane geometry and general geometry as well as in mathematical activity of a student much depends on what store of images of the objects on the plane he/she possesses and how well and flexibly he/she can use them. A success at the stage of coming up with ideas how to solve a given problem may depend on this store and the ability of using it. The ability to see a richer structure and a variety of figures on the plane is expressed in the readiness to draw these figures. The ability to bring the elements of this structure up to date seems to be very significant. Such an attitude may be considered a conducive circumstance in the assimilation of a new contents. The elements of this structure are the building blocks of new concepts. Therefore, a student should not only know individual geometrical concepts, but he/she should be able to imagine them in various situations and geometrical contexts, seeing, at the same time, the whole family of figures, not just a clearly defined, single case.

In order to check whether *CABRI* program, used on regular basis in geometry lessons, creates the conditions conducive to the development of so called "simulative imagination" (Młodkowski 1998, p. 254) in students of the first class of middle school (gymnasium), a special test was designed. It was administered after one year of learning and, both among the *CABRI* and non-*CABRI* students. The test comprised tasks that had to be solved without the use of this program. The pairs of students selected before (on the basis of *reconnaissance tests*) were the base for comparison. The purpose of the test was to obtain the answer to the following question:

Is a regular use of *CABRI* program as a teaching aid visibly conducive to the development of geometrical imagination in a student?

Selected general conclusions

Evidence collected justifies formulation of the following thesis:

The use of *CABRI* program should be connected with the exchange of views on the observations made and their mathematical interpretation between students or students and the teacher. The dialogue seems to be an indispensable factor in a correct analysis of a mathematical situation and in correct comprehension of the role of a teaching aid by students.

Let us remark that the teaching value of *CABRI*, although, undoubtedly, objective, is conditioned by the maturity of the teacher who uses this program in lesson. We differentiate here between the evaluation of *CABRI* program and its use during lessons. If a teacher takes care of the correctness in didactics and methodology in the lesson conducted with the use of computers, then, the assessment of the use of *CABRI* in lesson will be very high. However, if this program is treated superficially, the assessment may be much lower. *CABRI* shall be considered a modern technological means possessing high teaching potential, although, its true values might be visible only if it is used properly.

After analyzing the results of individual examination as well as of the final test two groups of students were distinguished. They were called *the top* and the *far behind*. In the former group there were students who had achieved very high results in reconnaissance tests. The analysis of their results allows to formulate the following observation:

13-year-old students who achieve very good results and possess high intellectual possibilities are able to use *CABRI* effectively as far as the perception of the contents shown.

The students from the other group – the *far behind* – working with *CABRI* program, can make a progress in the perception and, later, in the reception of certain mathematical contents. It requires, however, a big intellectual effort, thoroughness, and patience from them, which is not always easy.

The role of *CABRI* program in solving mathematical problems

What can a student do when his/her attempts to solve a problem do not bring about the desired effects? **Can the use of *CABRI* program in such cases help this student solve a mathematical problem?**

As a result of computer observations as well as empirical verifications made (by means of *CABRI* functions such as: measurement, trace etc.) hypotheses can be formed and verified. We are aware of the fact that relations obtained in this way require a correct mathematical proof based on reasoning without using a computer. Let us remark, however, that for middle school (gymnasium) students searching for a theoretical proof seems to be an activity that goes beyond their capabilities. At the same time, we realize that *the basic task of a proof is to modify the comprehension of mathematical facts and conclusions* (Prus-Wiśniowska, 1995, page 174). We can also understand a proof as a **research process**, which comprises, *formulating a problem, solving a problem, arranging various parts of the solution into a logical whole, generalizing and proving* (Prus-Wiśniowska 1995, page 175). Looking at the issue from such a point of view, we can risk saying that forming hypotheses and their empirical verification (the one done with the use of a computer and the one using traditional methods) are a part of such a research process. What is more, *both these elements introduce the student into the process of mathematical creativity, without which one cannot understand the beauty and sense of mathematics* (Krygowska 1977b, page 13). So, working with middle school (gymnasium) students we will be satisfied just with their forming hypotheses, attempts to verify them empirically as well as their awareness of the need of mathematical justification. The teacher's role, in turn, is to make students aware of what hypotheses, even the ones verified empirically, are, and what a mathematical proof is.

References

K ą k o l, H. (red.): 1999, *Program nauczania matematyki z elementami informatyki w gimnazjum*, Wydawnictwo Dla Szkoły, Wilkowice.

K o n i o r, J.: 2002b, Repetytorium z CABRI, część II, *Matematyka i Komputery* **11**, 5-8.

K r y g o w s k a, Z.: 1977a, *Zarys dydaktyki matematyki, część 1*; WSiP, Warszawa.

K r y g o w s k a, Z.: 1977b, *Zarys dydaktyki matematyki, część 2*; WSiP, Warszawa.

M ł o d k o w s k i, J.: 1998, *Aktywność wizualna człowieka*, Wydawnictwo Naukowe PWN, Warszawa, Łódź.

P r u s - W i ś n i o w s k a, E.: 1995, Dowód matematyczny i jego rola w dydaktyce matematyki: przegląd literatury współczesnej, *Roczniki Polskiego Towarzystwa Matematycznego, Seria V, Dydaktyka Matematyki* **17**, 167-181.