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Five-year-old children construct patterns, deconstruct them and talk about them[1]

Abstract: Patterning is related to important geometrical concepts and processes. Our experiment with kindergarten children was performed to examine their acquisition of patterns as revealed in their verbal and non-verbal acts and to identify the mathematical concepts that emerged.

1 Introduction

Many educators believe that mathematical abilities like generalization, abstraction, perceiving relations and understanding rules can be practiced within what we call "pattern environment". Therefore, pattern exploration has been identified as being central to mathematical inquiry and fundamental in children's mathematical growth (Fox, 2005; Swoboda, 2006). This fact has been also acknowledged by curricula designers. In NCTM's *Principles and Standards for School Mathematics* (2000) it is stated that children from Pre-kindergarten to Year Two are expected to be able to recognize, describe and correct patterns and also to perceive how patterns are created. However, Rudd, Lambert, Satterwhite and Zaier (2008) observed the lack of any planned activities around mathematical concepts in the kindergarten together with a small percentage of language use concerning patterns (0.3 %). At the same time, Fox (2006) who investigated the relationship of patterns with algebraic thinking concluded that "both the teachers and the children had limited understanding of the types, levels and complexity of patterns" (p. 94).

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In relation to relevant research in the field, Waters (2004) states that "exhaustive research revealed that mathematical patterning is generally not studied as a topic area per se, but has featured in the tasks of broader studies" (p. 566). Moreover, the relationship between the ability of making patterns by a child and her/his future mathematical development is not so clear (Clarke, Clarke, Cheeseman, 2006). This is evident in the following questions posed by Economopoulos (1998):

> What should be looked for when young children work with patterns? What kind of markers are there to identify development? What is really known about young children's engagement with patterns and the thinking that is being displayed? Are opportunities being provided for reasoning and justification that take the activity beyond "what comes next"?
> (Economopoulos, 1998, p. 232).

All the above have led us to perform research with a focus on patterns. We were particularly interested in studying how patterning can stimulate and shape young children's intuitions of geometrical transformations.

2 Patterns and geometrical knowledge

Patterning is a process of "discovering auditory, visual, and motor regularities" (Charlesworth, 2000, p. 190). These regularities can be related to a set of geometrical concepts and processes, namely recognition and manipulation of geometrical objects through various transformations. Such an approach could be seen as a repetition of human development during the process of establishing geometry as a scientific discipline and could be also identified in the words of many mathematicians and historians of mathematics.

According to some historians of this discipline, mathematical relations can already be identified in the geometrical decorations of items created by the late ice-age man.

> Except for the most innovative ideas, pre-historical art might be a proof of man's interest in the shape of figures, symmetry and the rhythm of linear and two-dimensional arrangements.
> (Polish Encyklopedia Szkolna, Matematyka, 1990, p. 140).

In (Kordos, 2005, p. 23) we read that:

> It is worth paying attention to the richness of geometrical forms used in decorations. In particular, it is worth seeing that the ribbon ornaments from the Neolithic period all had 7 one-dimensional crystallographic groups on the surface. (...) However, we cannot be certain that some kind of geometrical reflection was followed.

The history of mathematics shows the importance of the transition from static to dynamic interpretation of geometrical objects (Kvasz, 2000). This can be seen in Greek mathematics, in which the traces of general reasoning were based on dynamic object transformations. A significant part of this geometry was based on constructions, which – in an indirect way – required the use of translations, rotations and mirror reflections. This object-manipulation thinking can be recognized in Euclid's Elements – in Postulates, Common Notions and Propositions and their proofs (e.g. Book I, Common Notion 4: *Things which coincide with each other are equal to each other*). The overt description of symmetry as a transformation appeared rather late in mathematics – as it can be linked to the Erlangen Programme of F. Klein – but the dynamic approach itself is crucial for geometry. Geometrical reasoning requires mental transformation of objects.

Concerning teaching, the basic symmetries of the plane (including translation, rotation and axial symmetry) can be formulated twofold: as binary relations or as transformations. In the first approach the relation is understood as a specific mutual location of two congruent objects. Such a static interpretation can be identified in the perception of a symmetrical figure. This is when symmetry as a relation takes place between particular elements of the figure. In order to understand the isometric transformations it is essential to build connections between three components: (a) the initial position of a figure, (b) its movement and (c) its final position. In the visual, static representation of geometrical relations on the plane the components (a) and (c) are represented in a clear way. In order to understand a geometrical symmetry as a transformation it is necessary to conceive the specific movement that is transforming the initial figure into the final one. From a didactic point of view, it is important that such conception stems from dynamic aspects of transformations and mental reflection on the movement phenomenon.

It is generally believed that the development of geometrical concepts is different from that of the arithmetical ones (Gray, Pinto, Pitta, Tall, 1999). The process of forming geometrical concepts has been the focus of a number of theories, of which van Hiele's (1986) is the most popular. He describes the first level of understanding as "visual", connected with non-verbal thinking. At this level the emphasis is placed on the ability of recognizing shapes, which are perceived as a 'whole' and connected with visual prototypes. Not much is mentioned about the role of action, although a didactical conception of the theory suggests activities with objects (de Lange, 1987).

Results of psychological research (Kaufman, 1979) confirm that in the process of grasping shapes pictorial designates are of great importance. But a next stage is needed. Acts of perception are important but they are not a sufficient source of geometrical cognition. Szemińska (1991, p. 131) states that "perception gives us only static images; through these, we can only catch some states, whereas by actions we can understand what causes them. It also guides us to possibilities of creating dynamic images". On the other hand, widely known Piaget's results (Piaget, Inhelder, 1973) show that children (on the preoperational level) have great difficulties in movement reproduction – they are not able to foresee a movement of an object in space. The process of acquiring such skills is lengthy and gradual. During manipulations, the child's attention should be focused on *action*, not on the very *result of action*. It requires a different type of reflection than the one that accompanied his or her perception.

The short juxtaposition above shows that the relation between visual recognition of geometrical objects and actions, which can lead to the creation of dynamic images of those objects needs further investigation. For this reason we designed an experiment in two stages, which were planned to resemble the passage from static to dynamic understanding of axial symmetry. We were particularly interested in connections within perception, actions and language used by children, which could help them obtain experiences needed for further development of their geometrical reasoning. Our basic aims were:

a) to examine a series of activities aimed at building the transition from the static presentation of geometrical relations to understanding them as transformations,
b) examine the role of language as a tool for reflection on actions performed.

In order to fulfil this aim, we sought answers to the following questions:

- How did children perceive a pattern that was built with geometrical figures?
- How did children continue a pattern according to a given initial part?
- How did children describe their work?

We needed a tool which would be relevant for our purposes, would account for phenomena observed and would be sufficiently reliable. The process of designing and implementing that tool is described in the next section.

3 Methodology

Our experiment took place in March and April 2008 in a typical public kindergarten in Rzeszów, Poland. This experiment was part of a broader study (Swoboda, Synoś, 2007; Swoboda, Synoś, Pluta, 2008), but for the purpose of this paper we focus only on our aims as expressed before. Our study group consisted of 19 five-year-old children, who were observed and interviewed. As a research tool we used two types of tiles (Figure 1).

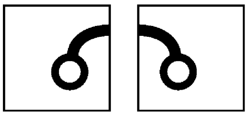

Figure 1. The research tool.

In *Part I* the tiles formed two separate piles on the table. The child had to continue the pattern shown by the teacher (Figure 2), and later to correct the mistake done by the teacher in the pattern.

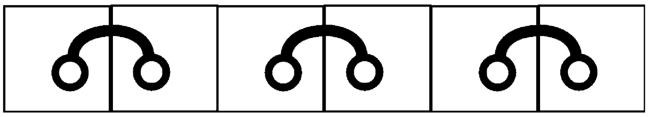

Figure 2. The pattern presented by the teacher.

The aim of the meeting was to provide the child with an opportunity to experience the tiles and making patterns. The child might work first at the visual level (at the beginning of the meeting) and later at the visual level combined with reflecting on the action performed (in the second part of the meeting).

The observations done in this part aimed at investigating the spontaneous ability of the child to create, investigate and correct regularities. We paid special attention to the children's ways of:

- constructing the initial motif from the provided elements,
- continuing the pattern.

During the correction of the pattern done by a child we tried to figure out whether the child was aware of relations between the manipulations and their results.

In *Part II* a few days later, the teacher asked the child to build the same pattern as in *Part I.* If the child did not remember it, the teacher started making the pattern, invited the child to participate and then asked her/him to show to a colleague how to make the pattern. They were invited to play each with the other in correcting it.

The aim of the second meeting (*Part II*) was to investigate how the child remembered and used the information and experiences gained in *Part I.* Especially, how much information expressed by the child in verbal and non-verbal ways (both during individual work and while playing with a friend) was related to mathematical terms relevant for pattern construction. Additionally, we wanted to distinguish these terms and characterize the scope of understanding among children. We also wanted to investigate the influence of verbal communication skills, in relation to the regularities presented graphically, which was linked with the skill of transition into a different representation scope (the transition from the visual to word/symbolic one). The analysis of verbal communication helped us in stating to what extent the child is aware of dynamic relations within the pattern during the building process.

All sessions were videotaped and then transcribed. The transcripts were analyzed by using grounded theory techniques (Strauss, Corbin 1990).

The process of identifying, characterizing and classifying dominant phenomena led us to distinguish three sets of categories, related to constructing the patterns, describing the construction to another child and deconstructing the patterns. These categories were devised for the whole experiment and were useful in comparing the behaviour of children belonging to different age groups and in different phases of our experiment.

The first set of categories refers to the behaviour observed while the children were asked to construct the pattern.

Symbols such as AAAA and ABAB will denote general pattern types. AAAA denotes continuation by using simple repetition (one object is repeated), whereas ABAB denotes continuation by repeating two objects. Symbols such as a_1, a_2 will denote single tiles.

The second set of categories refers to behaviours observed while the children were asked by the teacher to describe the way of making the pattern to a colleague. They were devised to examine the children's ability of using verbal means of communication in a situation which had strong visual aspects. Moreover, we were interested in examining how non-verbal aspects of language supported the communication (Table 2).

The third set of categories refers to behaviour observed while the children were asked by the teacher to deconstruct the pattern they had made (to make any "mistake" in the pattern). These categories were devised to describe the children's ability to change the pattern (deconstruction phase) and their awareness of features of the tiles (correction phase). They were also used to explain the results obtained in the construction phase of our experiment (Table 3).

A1.	being unable to start
A2.	continuing after the teacher's prompt
A3.	continuing without any prompt
A4.	starting the pattern from the edge of the table
A5.	starting the pattern from the middle of the table
A6.	making motifs without breaks
A7.	making separate motifs
A8.	making motifs in one direction
A9.	making motifs to the right and to the left
A10	simultaneously taking tiles from the two piles (making AAAA pattern)
A11.	taking one tile at a time, each time from a different pile interchangeably (making ABAB pattern)
A12.	taking tiles only from the same pile
A13	visually checking the tiles before putting them in the pattern
A14.	randomly putting tiles in the pattern and then checking
A15.	finishing the task after making only one motif
A16.	making patterns in one line
A17	making patterns in more than one lines
A18.	creating different types of motifs than the one shown

Table 1. Categories related to the construction of patterns

B1.	presenting the pattern non-verbally by showing how to construct it
B2.	presenting the pattern by construction accompanied with talking
B3.	presenting the pattern initially by talking and then by constructing it
B4.	presenting the pattern by talk supported by gestures
B5.	focusing on the difference of the two piles
B6.	describing the motif as consisting of two halves
B7.	describing the motif with everyday words
B8.	focusing on the alternating nature of the process of picking up tiles from the two piles
B9.	focusing on continuity by the use of words
B10	focusing on continuity by the use of gestures
B11.	describing verbally the one-dimensional character of the pattern by calling objects of everyday use
B12.	describing verbally the two-dimensional character of the pattern by calling objects of everyday use
B13	unspecific prompt
B14.	focusing on actions which are not important for the construction of the pattern
B15	not talking and not performing any construction

Table 2. Categories related to communication

C1.	single action related to deconstruction
C2.	multi-activity related to deconstruction
C3.	taking care about proper relation tile to pile
C4.	action with using tiles from pattern
C5.	actions with using tiles from piles
C6.	putting two identical tiles next to each other
C7.	putting two identical tiles next to each other in two different places of the pattern
C8.	removing tiles from the pattern without filling the empty places
C9.	rotating one tile
C10	translating one tile
C11.	putting two identical tiles next to each other and rotating the new tile
C12.	exchanging two tiles from different parts of the pattern without any effect
C13	putting a tile of the same type with the original taken from the pile

Table 3. Categories related to the deconstruction of patterns

To make the categories valid, the authors performed two independent analyses of the same phenomena. In the next section we present the results with examples of implementation of our categories.

4 Results

4.1 Examples of analysis

The categories mentioned in the previous section were helpful in studying the behaviour of each child and its evolution in time. Work done by three children is shown and analysed in terms of the three sets of categories. The examples illustrate the variety of children's behaviour. The discussions have been translated from Polish; as literally as possible, including the children's grammatical mistakes.

A. Constructing patterns

Example 1A – Bartek (5-year-old boy)

After some thinking (A3) he takes in his hand one tile from one pile, later on a second tile from the second pile (A11), joins them into a motif. He puts them directly in front of him (A5). After that, he takes simultaneously two tiles from two different piles (A10), compares them with those lying in front of him (by moving his hands over them) and then connects them into another motif. He locates them separately from the previous one (A7), on the right. He then constructs the third motif quickly, by taking tiles from the two piles simultaneously (A10) and putting them again on the right side of those lying at the table (A16) and keeping the same distance as before (A7).

Comments: From the beginning the boy clearly emphasized the difference between the tiles from two piles by performing distinct activities (e.g. by using his right and left hand). He used two tiles for making one basic motif A = a_1a_2. The construction of the motif is treated by us as the main part of the task realized by the boy. He is aware that he extended the original task to many motifs, but does not seem to grasp the relations between them.

Example 2A – Olivia (5-year-old girl)

Olivia starts at the middle of the table (A5), takes one tile at a time, each time from a different pile (A11) and stops (A15). After the teacher's prompt (A2) she takes tiles from the two piles (A10), makes continuous motifs (A6) to the right and left directions (A9) in a single line (A16). (Fig. 3)

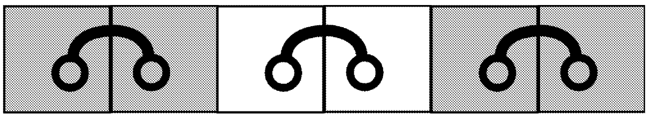

Figure 3. Olivia's construction of the pattern.

Comments: At the beginning Olivia focuses mostly on creating the motif. For her this is the first step, definitely disconnected from the other steps (A2). This stage looks like a separate unit; after completing one motif the girl is able to finish the work. She does not plan space for further work – she only does the work in front of her (A5). This does not mean that her understanding of the pattern is limited to the construction of one motif. It seems rather to be several motifs connected (A6) and placed in one line (A16). Olivia adds one motif on each side (A9), which seems to be a 'global seeing' of many motifs rather than an iterative continuation.

The motif itself has got a certain structure. Olivia clearly distinguishes between the two piles of tiles – this distinction is not only visual but also time-related (A11). She knows that the time-related distinction is not crucial, as she can easily change her behaviour for simultaneous actions with two different tiles (A10).

Example 3A – Monica (5-year-old girl)

M: She starts her work by taking simultaneously two tiles from the two piles (A10) and then she joins them on the table into an a_1a_2 motif. She puts this motif close to the edge of the table, at the left-hand side (A4).

T: Moves the motif to the middle of the table. – *Start here, your work will be more visible.*

M: She takes the motif into her hands, for a moment keeps it without moving it, and after that she pushes it a little to the left, moves it again in the same direction and finally places it close to the edge of the table (A4). *It will be from this place to that one …* (B9) She shows, moving her head, the opposite edge of the table (B10). She takes two other tiles (A3). She manipulates them in the air by joining them into a motif (A10), but after that she takes only one tile from the motif and very carefully joins it with the part lying on the table (A6); after that she joins the second one. In the same way she works with the third motif – at the beginning she connects the couple of tiles and then disconnects them in order to achieve the continuation of the pattern. She continues her work by putting motifs in one line (A8). She makes 4 motifs (A8) and then the teacher stops her.

Comments: Watching the evolution of Monica's work we concluded that the arrangement of the two tiles a_1 and a_2 was functioning for her as a whole ($a_1a_2 = A$). She felt a strong need to create a long pattern by taking up the whole accessible area. The motifs were added one after another – in this way they formed an ordered sequence. Each motif was joined to each other in a very precise way. For Monica the pattern continuation without any break was so important that she was able to overlook for the while the idea of seeing the motif as a whole.

B. Describing patterns

Example 1B – Bartek and Damian (5-year-old boys)

T: *And now I ask Bartek to teach Damian.*

B: He shows by his hands lenghtways the table (B10) *It needs to do so afterwards* (B9). [*3 seconds silence*]

T: *So?*

B: *And then you close your eyes and it needs to be connected in this way* (B6).

T: *Not everything at once, first we will ask Damian to build the pattern – is that true?*

B: *Yes.*

T: *So, tell Damian what he has to do.*

B: *Take that in such a way, to come out...* (B13).
T: *You can show him how to do it.*
B: He takes immediately two tiles from the two piles (B1), creates a motif and joins it to the pattern.
T: *And now what do you say to Damian?*
B: *Let's continue* (B13).
D: He takes one tile from the one pile and he wants to take the second tile from the same pile. Bartek interrupts him and shows him the second pile.
B: *Now from that one* (B5).

Comments: Bartek's description is focused on continuation – first he does it by gestures, then by using words. Clearly, this is an important message because it appears several times. He probably considers this information sufficient for his colleague to undertake some action. Simultaneously, he does not clarify under what kind of condition the pattern will 'come out' (B13). The construction is described by showing it. Bartek consciously differentiates the tiles; he is aware that there is no other way to obtain the motif than by using tiles from both piles.

Example 2B – Olivia and Martyna (5-year-old girls)

T: *Come on Olivia, show Martyna how to make such a pattern.*
O: Without a word (B4) she takes one tile from the first pile and adds it to the pattern (B1). Then, with the same hand, she reaches for the other piles (B5) and continues the pattern.
T: *Tell her what she is supposed to do.*
O: Looks at her friend and smiles. She reaches the tiles again and places two more in the same way she did before.
T: *Tell Martyna so that she know.*
O: [6 s silence] (B3) *You have to arrange* (B9) *this* (shows one pile) *and this* (shows the other pile) (B8).

Comments: At this stage Olivia shows a slightly different understanding of the task than at the previous one. Now the action is directed to creating the alternate set from the tiles from both piles, which is an example of the ABABAB structure.

She finds it hard to describe the construction – she definitely prefers demonstration (B1). It is only after the clear, twofold suggestion from the investigator that she uses verbal expressions (B9) backed by adequate gestures (B3, B5). In her verbal descriptions she restricts herself to distinguishing between the tiles.

Example 3B – Monica and Ola (5-year-old girls)

T: *So Monica, show Ola how to make this pattern.*
M: She immediately takes one tile from each pile (B1). *Put them like this* (she firstly puts the tiles together over the table and then puts them one by one on the table and connects them with the pattern she had already built (B2), (B6)) *you put (them) like this* (she points with her hand (B4)) *and do the same until the end* (puts another motif at the end of her pattern (B2) and moves her hand over the table (B9), (B10)).
O: She begins by repeating Monica's movements and makes one correct motif. Then she picks two tiles from the same pile.
M: *No, Ola. Look. There are two different piles* (B5). *You take from them and go on like this* (B9).

Comments: Teaching is based here on presentation of actions, aiming at showing the results of them (B1, B10). However, verbal descriptions also appeared quite often (B2). Sometimes the verbal description anticipates presentation, but this happens rarely (B4) and only in special cases, when Monika intends to underline the meaning of the presentation (B5).

Monika does not disconnect the process of creating a single motif from the creation of the whole pattern. Her pattern consists of many motifs, created in a process-related activity 'till the end' (B9). She created the motif by taking two tiles at once (B6). At the same time, she is much aware of the differences between the tiles.

C. Deconstructing patterns

Example 1C – Bartek (5-year-old boy)

He takes a tile a_2 from the pile with his left hand (C5). Then he takes with his right hand a tile a_1 (C4) from the left end of the pattern lying on the table, puts it away at the right pile (C11) and moves the last tile from the pattern – an a_2 – (C10) to the free place. Then he fills the gap with the tile kept in his right hand (C6) (C2).

Comments: Bartek does not act 'blindly'. He knows precisely what can affect the pattern's deconstruction. At the beginning he exchanges a tile (C6). (Firstly he took a tile form the pile (C5), and only after that he started deconstructing the pattern). Going back to the visual representation (to the existing pattern) he performed several additional actions (C2). He shattered the a_1a_2 motif by taking away one a_1 tile (probably planning to replace it by an a_2 tile). However, for this purpose he did not use the tile in his hand, but the one he was looking at.

Example 2C – Olivia (5-year-old girl)

O: *Close your eyes.*
Decisively she removes a tile from the middle of the pattern, puts it at the correct pile (C3), takes a tile from the second pile and inserts in into the free place – in this way she produces an arrangement of two identical tiles next to each other (C6). She continues (C2) by taking with her left hand an a_1 tile from the pile; then she takes the last a_1 tile from the pile; then she takes the last a_2 tile from the pattern with her right hand and wants to put it on a pile. She cannot decide which pile is correct (probably because the top tile was turned by 90^0) – she comes back with the tile to the pattern, puts it back at the same place, takes a neighbouring a_1 tile off and puts at its place the one from her left hand (C13). Although regularity is not damaged by this action, she says: *That's it.*

Comments: Here the base for the action is the differentiation of the tiles according to their location in a particular pile. She knows that there are only two piles and therefore if a tile does not fit one pile, it must fit another. She visually verifies the relation between a tile and a pile – but she is able to do it properly only when a tile is lying in the privileged position. Partly mechanized actions lead her to incorrect results. The girl did not feel any need to check the result of her work in any additional way.

Example 3C – Monica (5-year-old girl)

Monica, using both her hands, pulls two tiles a_1a_2 from the middle of the arrangement (C2). She puts them back to the right pile (C3) and then she takes two identical tiles from one pile, and puts them in the empty spaces (C5). She sits back showing that she has finished her work.

Comments: In this example the pattern deconstruction is equivalent to the deconstruction of the motif. The girl knows that the basic motif a_1a_2 consists of two different tiles. These are in such a strong relation that she cannot decide for changing only one of the tiles. Such behaviour suggests that the tiles a_1,a_2 create a supplementary whole. From this we can conclude that her pattern has an AAAA structure. Destroying this structure means destroying the relation within the motif; she achieves this by connecting two tiles of the same kind.

Example 4C – Emilka (5-year-old girl)

Emilka starts by exchanging two tiles from two different parts of the pattern; she places one tile without changing its position (C12) and then she rotates the second one (C9). Then she removes one tile from the left border of the top row and places it at the left border of the bottom row (C6) (Figure 4).

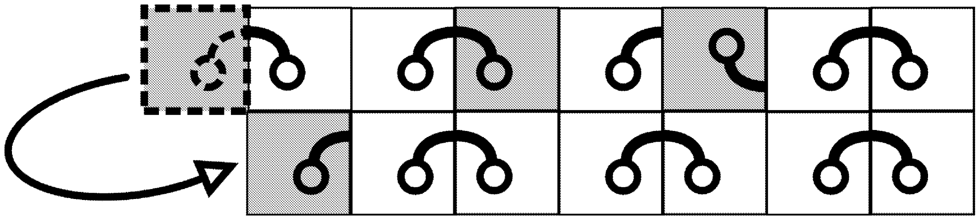

Figure 4. Emilka's deconstruction

Comments: The girl understood the pattern deconstruction as 'some' manipulations (C12), including replacement of tiles, which should destroy the regularities. The result of her first action did not cause it (she could see it).

She was possibly influenced by the visual information when she turned one of the newly placed tiles by 90 degrees (C12). Probably the result was not satisfactory for her either and she attempted further action. She took one of the most remote tiles from the upper row and added it to the second row.

It seems that Emilka's actions outpaced her reflection. Her actions were rather chaotic and she could only visually confirm the soundness of her actions. Emilka knew what she wanted to achieve but did not know how to do it.

4.2 Results of analysis

The dominant characteristics of the children's behaviour concerning constructing patterns are presented in a set of tables. These tables consist of data that came from the analysis of each child's behaviour during the sessions and they consist of the frequency of each category observed in the whole group of children. Some categories overlap, since there were cases when a child 'switched' between two or more behaviours. The examples of children's work presented here show that at different stages of their participation in the experiment, children's behaviours were occasionaly different. This is a sign of gaining experience and deeper understanding of the relations occuring in this task.

The data from the table were used as a basis for creating some generalisations, which we present separately under each table.

1. Constructing patterns (category A)

Meaning of the category	Under-standing task			Continuity						Way of cre-ating regulari-ty					Understanding the pattern as a concept			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Number of children	0	5	14	7	12	18	2	13	6	9	13	0	6	6	3	16	3	0

Table 8. Frequency of categories related to the construction of patterns

Making pattern was easy enough, so each child attempted to do it. Many children (one fourth of them) perceived the whole task as divided into two stages: creating a motif (the first stage) and creating the whole pattern. Generally, when children started working they usually did not conceived the pattern as a long one. They did not plan space for a long sequence of motifs and rather put the tiles in front of them.

Creating the motifs the children visually recognized the differences between the tiles. Sometimes, before they decided to finally connect two tiles into one motif, they tried to compare them.

Most children created the motifs by taking two tiles simultaneously with both hands, connecting them into a motif and adding such a motif to the existing pattern. At this stage the children understood the patterns as a repetitive sequence of motifs (i.e., a pattern built in the form of AAAA, where the motif A consisted of two elements a_1a_2). These motifs were placed closely to each other, along a single line, and the extension of the pattern went on in both directions.

2. Describing patterns (category B)

Meaning of the category	Connection between ges-ture and talk				Describing motif				Describing continuity				Difficulties		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of children	6	6	2	6	10	7	1	3	6	8	0	0	9	1	0

Table 9. Frequency of categories related to communication

Table 5 shows a number of children who gave unclear prompts to their partner (B13), e.g., "Could you do something like this?". These children appeared unable to discover and communicate regularities of the pattern. The visual information seemed clear enough so that a different type of information was not needed. This was usually observed at the beginning of the session; later on, most children proceeded with a more detailed verbal description. One third of the whole group of children did not use any verbal communication (B1) to describe features of the pattern, while the majority of children often used verbal means to support non-verbal means of communication such as showing how to construct and using gestures (B2, B3, B4).

As the most important thing, children pointed out the difference between the tiles in the two piles. They talked to their peers about different "piles" and about two different "halves" of the motif. The feature of "continuity", described both verbally and by gestures, was also important for them.

3. Deconstructing patterns (category C)

Meaning of the category	Work organisation					Breaking symmetry		Breaking continuity	'Some' transformation			Action without visible results	
	1	2	3	4	5	6	7	8	9	10	11	12	13
Number of children	7	12	12	8	12	11	4	3	2	2	1	8	1

Table 10. Frequency of categories related to pattern deconstruction

Individual deconstruction of the pattern was not an easy task for the children. This can be seen in categories C12 and C13. The child moved tiles with no result, replacing a tile by an identical piece.

Still, in this age group there was already a number of children who could follow the form of deconstruction suggested by the experiment supervisor. Sometimes children performed several different transformations. These children appeared to grasp the idea of pattern regularity. Surprisingly, some children regarded the break of the continuity of the arrangement as a major feature of 'destroying' the pattern; moreover, such behaviour was not uncommon.

5 Conclusions

The data presented in the tables combined with an analysis of each child's behaviour led us to the following observations.

- During the series of activities the children gained significant experience concerning differences between changes resulting from axis symmetry (as a transformation) and those resulting from rotations and translations. Children seemed to be convinced that for creating a symmetrical object (in which the two halves could be distinguished) it is not enough to move one of the halves. The two parts which make a symmetrical object are essentially different, although closely related. Such experience was confirmed by the way children created and destructed the patterns. However, their work was oriented not to replacing a_1 by a_2 but to creating a certain whole, constituted by both a_1 and a_2. Therefore, these activities were oriented to axial symmetry visible in axially symmetrical figures. Thus, the change (overlapping) of one part of the figure over the other is devoid of sense – both fragments must function together. Yet we cannot claim that children have an intuition about movements needed to put one object over the other (a_1 over a_2) preserving their symmetry axis. Up to this point children only gained knowledge of those actions, which were not successful for obtaining a symmetrical object; translating or rotating could not transform a_1 in $a_{2.}$ This experience is very important for children's geometrical development.

- Our experiment contained different forms of engagement with patterns. This enabled children to experience different aspects of mathematical activity, to perform different actions (constructing-deconstructing) and to communicate about them. At each stage children acted differently. Experiences gained during activities of one type did not influence actions at the next stage. This is coherent with the action-related concept of mathematical instruction (Krygowska, 1977; Siwek, 2005), in which different types of tasks could be distinguished (mutually inverse tasks, different sequences of activities aiming at the same result, etc.). Such tasks are to help the child to pass to a more general level. Hejný (2008) writes about *isolated models*, which concern some concepts or procedures functioning separately before they are integrated into one 'generic model'. In our case the set of activities (pattern continuation, reperation of a destroyed pattern, verbal description, deconstruction) could be a trigger of supplementary sequences, helping the child to integrate the experiences into one 'generic model'.
- Some children started the pattern from the edge of the table and appeared to leave space for a long repetitive sequence of motifs. This – together with children's descriptions – indicates their prospective awareness of potential infinity.
- The child's conversation was prompted in a direct way ("tell your friend how he/she should create the pattern"). This influenced the content of the talk. The discussions were all directed to the actions and their anticipated aim. This agreeded with the aim of the research, as it directed the child's attention to the dynamic approach, related to the transformation. However, this limited the possibility for spontaneous comments.

Children searched for ways of describing the pattern features verbally, but this was difficult for them. Very often the verbal language was accompanied by gestures. This is coherent with other observations (Steinbring, 2005). It is now widely accepted that gestures help to formulate mathematical thoughts, especially to convey the meaning (Arzarello et al., 2009). The results of other researchers confirm the hypothesis that the passage from visualization to language requires an in-between stage, which could perhaps be gesture-based (Swoboda, Synoś, Pluta, 2008). In our case gestures do not only indicate some difficult-to-express relations, but could also be the result of encapsulation of dynamic actions.

The most common concept that emerged during the sessions was continuity of the pattern. Continuity was observed not only in the patterns (A1, A4, A6), but it was also present in children's expressions (B5, B9, B10). Mirror symmetry of pairs of tiles was a basic feature of our pattern. Children were able to recognise the difference of the tiles (A10, A11, A13, B5, B6, B8, C6, C7). Almost all children were aware that the one of these tiles cannot be obtained by translating and/or rotating the other (C9, C11).

The results of our study show the significance of patterns as a way to engage children in a meaningful communicative activity that can be rich in mathematical ideas. Children, to some extent, can use language to accompany or support their actions. This is an indication of their potential in acquiring some mathematical intuitions at an early age.

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Pięciolatki układają szlaczki, burzą je i rozmawiają o regularnościach

S t r e s z c z e n i e

Wśród dydaktyków matematyki istnieje przekonanie, że takie matematyczne umiejętności jak uogólnianie, dostrzeganie związków, rozumienie reguł może być rozwijane w tzw. „środowisku regularności”. Badania prowadzone w tym zakresie wciąż jednak nie odpowiadają na wiele zasadniczych pytań, przede wszystkim – jaki jest związek między umiejętnością kontynuowania szlaczka przez dziecko a jego dalszym rozwojem matematycznym. Badania opisane w tym artykule były próbą określenia, w jakim stopniu szlaczki geometryczne mogą pomóc w kształtowaniu dziecięcych intuicji związanych z przekształceniami geometrycznymi. Interesowało nas przejście od statycznej, wizualnej

reprezentacji relacji geometrycznych (symetrii osiowej, obrotu, przesunięcia) do rozumienia związków między figurami, określanych przez ruch nakładający jedną figurę na drugą. Badaliśmy również rolę języka jako narzędzia umożliwiającego zwrócenie uwagi na dokonywane manipulacje.

Nasze pytania badawcze były następujące:

- Jak dziecko postrzega szlaczek zbudowany z figur geometrycznych pozostających w określonych geometrycznych relacjach?
- Jak dziecko kontynuuje szlaczek?
- Jak dziecko opisuje swoją pracę?

Badania zostały przeprowadzone w grupie 19 dzieci w jednym z miejskich przedszkoli w Rzeszowie. Składało się ono z dwóch sesji, w których dzieci uczestniczyły indywidualnie. Narzędziem były następujące kafelki (rys. 1, strona 157).

W pierwszej sesji na początku nauczyciel przedstawiał dziecku szlaczek (rys. 2, strona 157) i prosił, by dziecko układało go dalej. W dalszej części wymieniał jeden kafelek w szlaczku na kafelek innego typu (dziecko nie widziało procesu wymieniania), a potem prosił o naprawienie zepsutego szlaczka. W trakcie drugiej sesji (kilka dni później) to samo dziecko pracowało jako ekspert z drugim dzieckiem – najpierw samo układało początek szlaczka, potem tłumaczyło drugiemu dziecku, jak szlaczek należy zbudować, następnie „psuło" szlaczek, aby kolega mógł naprawić.

Wyróżniliśmy szereg kategorii opisujących zachowania dziecka podczas badań. W tym artykule przedstawiamy tylko te, które dotyczą zachowań podczas drugiego spotkania. Są to więc kategorie dotyczące sposobu budowania szlaczka (18 kategorii), opisywania szlaczka (15 kategorii) oraz psucia szlaczka (13 kategorii). Pokazujemy również, w jaki sposób te kategorie były stosowane w konkretnych przykładach prac dzieci, prezentując opisy zachowania trójki dzieci. W naszym badaniu praca każdego dziecka była opisana w ten sposób, później te wyniki zostały zebrane w zbiorczych tabelach. Na ich podstawie mogliśmy wyciągnąć następujące wnioski:

1. Podczas zajęć dzieci zdobyły istotne doświadczenia dotyczące symetrii osiowej, odróżniające to przekształcenie od przesunięcia i obrotu. Dzieci wydawały się przekonane, że dla stworzenia symetrycznego obiektu (składającego się z dwóch symetrycznych połówek) nie wystarczą manipulacje jedną połówką – dwie połówki w istotny sposób różnią się między sobą. Są to doświadczenia istotne, chociaż ograniczone do rozumienia figury osiowosymetrycznej.

Jedną z najczęściej występujących własności, którą dzieci wiązały ze szlaczkami, była jego kontynuacja. Podkreślana była ona przez gesty, słowa, jak również sposób budowania szlaczka – część dzieci rozpoczynała pracę od samego brzegu stolika, przygotowując miejsce na długą sekwencję motywów.

2. W eksperymencie dzieci realizowały serię zajęć związanych ze szlaczkami. Dało im to możliwość zbierania różnych doświadczeń: konstruowania, dekonstruowania i rozmawiania o szlaczkach. Dzieci w różny sposób reagowały na przedstawione im zadania, nie przenosząc w prosty sposób doświadczeń z etapów wcześniejszych, raczej je wzbogacając. Takie obserwacje są zgodne z koncepcją czynnościowego nauczania matematyki (Krygowska, 1977, Siwek 2005), według której zadania różnych typów pomagają dzieciom przejść na bardziej ogólny poziom rozumienia pojęcia czy procedury. Propozycja pracy w różnych *izolowanych modelach* jest również podstawą teorii wypracowanej wśród praskich dydaktyków matematyki (Hejný, 2008).