ANNALES OF THE POLISH MATHEMATICAL SOCIETY
5TH SERIES: DIDACTICA MATHEMATICAE 30 (2007)

Anna Sierpińska, Georgeana Bobos
Concordia University, Montreal, Quebec, Canada
Christine Knipping
Acadia University, Wolfville, Nova Scotia, Canada

A study of university students' frustration in pre-university level, prerequisite mathematics courses: emotions, positions and perception of achievement[1]

1 Introduction

In some universities, secondary and college level mathematics courses are offered to people who, for various reasons, had not taken them when they were at the mandatory age for these institutions, but now need them as *prerequisites* for the academic programs of their choice. These are courses in basic algebra, linear algebra, functions and one variable calculus.

Few students take the courses to study mathematics at the university; most have other professions in mind. Some question the relevance of the math courses for their future studies or professions. They feel unjustly forced to take them. This is a well-known phenomenon:

> Mathematics is so pervasive that most university courses require, often implicitly, at least basic algebra and often more. Many students are literally *shocked* to find that the degree such as nursing or human resource management not only assumes pre-requisite mathematics but makes actual explicit demands in the course. Students who may have avoided

[1]The research presented in this paper was funded by Social Sciences and Humanities Research Council of Canada (SSHRC grant #410 2003 0799).

*Many thanks to students and instructors who agreed to respond to our questionnaire and granted us interviews.

> mathematics in choosing their area of study are now *forced* to confront it. (FitzSimmons & Godden, 2000, p. 28; our emphasis)

Others may accept the rationale of prerequisite math courses, but are frustrated with certain aspects of their functioning. Older individuals who come back to school to finish their education or change their profession may find this experience especially hard. In an interview, "JesTia" (an artificial label), a female student in her 30s, told us:

> I was working as a secretary for nine years and I was doing book-keeping where I worked and then I left it because my boss threw me out. 'Go find a corner where you belong' — he said — 'because you will never get paid for the work you do since you do not have a university education'. So I came back to school, *to the frustration of school*, which, right now, I'm *on the edge* of just, forget it!... I really wanted to do Accounting, but I got accepted into Math and Statistics, so I said let me try it. I haven't been in school for so long, so it was kind of *a rough time*. As I said, I like math and I really want to learn it, *but I just need good teachers* and so far at [this university], I haven't seen... I'm sorry, I heard there's good teachers, but I haven't experienced that yet. (JesTia, mature student, in the interview; our emphasis)

Some students are loud in expressing their frustration in classes and teachers have the impression that they represent the voice of the majority. In this study, we wanted to verify this impression, as well as find some systematic way of identifying the possible sources of this frustration. Our long-term goal (not achieved in this study) is to find ways, on the one hand, of helping students make the best of their frustrations rather than react by getting discouraged and giving up their studies, and, on the other, to help institutions and teachers to avoid causing unnecessary frustration.

We are saying this because we are far from considering frustration as an unconditionally unwelcome emotion in learning. In fact, we believe that overcoming frustration plays an important role in the development of new knowledge, both individual and collective. People act, plan, and work to achieve their goals based on some more or less conscious "theories" about what "normally" or "usually" happens. When they are *deceived* in their expectations or when their plans are *frustrated* (Mandler, 1975; Handa, 2003), they experience emotions that can range from mild surprise to frustration, anger or wild rage. Ideally, they try to overcome the frustration by trying to understand why the unexpected thing happened, revising their "theories" and finding new strategies for action, thereby developing new knowledge. Less ideally, they stop trying, and give up their actions, plans or goals, sometimes destroying what they

have achieved so far. Therefore, frustration may have negative effects when people stop trying, but some frustration is necessary to change one's ways of understanding or doing things.

This research deals with an "affective variable" in mathematics education. According to Reyes (1984), research on affect in mathematics education deals with questions such as,

> What is the nature of [an affective variable] A? What does it mean to be 'high' or 'low' in A? Is there a general construct in the psychological literature that is related to A? If so, how are they related? What are the important factors (e.g. student, peer, teacher, classroom, school, family, community) in the development of A? Are some points in A's development particularly important in terms of long-range implications? Is A stable over time? How does A vary in different mathematical contexts (e.g. computation vs. problem solving)? How does A vary in different instructional contexts (e.g. classroom, large group, small group, test)? (Reyes, 1984: 573)

In our case, the affective variable we studied was *frustration*. We certainly sought to understand how this construct was conceptualized in the psychology of emotion (second question in the citation above) and we will present the results of our efforts in this paper.

Our main research question in the study was the closest to the one about the "important factors in the development of A". We were interested in sources of frustration in students taking prerequisite mathematics courses ("PMC" in the sequel). But we also wanted to estimate the magnitude of the phenomenon of frustration among the PMC students and get an idea of how students experience it and explain it. This paper will focus on the latter questions; sources of frustration have been addressed in a separate paper (Sierpinska et al., in press).

Based on our initial hypotheses about these sources of frustration we constructed a questionnaire with 76 items which we sent to about 860 individuals registered in PMC in a large, urban North American university (hereafter called "The University") in the years 2003/4. Responses were obtained from 96 individuals. In this paper, we will present partial results of our analyses of the responses. We will focus on how the respondents expressed their frustration. More precisely, we will look at the relations between the character (positive, negative, ambivalent or neutral) of the emotions they expressed, and two other aspects of their responses: (a) the positions from which they spoke (Person, Learner, Student, Client); and (b) their perception of their achievement in the prerequisite courses.

2 Theoretical framework

Underlying our research were: a three-dimensional theory of learning (Illeris, 2004), a conceptualization of frustration (Handa, 2003), and a framework for the study of institutions (Crozier & Friedberg, 1977; Ostrom, 2005; Chevallard, 1992; 2002). In this paper we will speak about our conceptualization of frustration and refer only briefly to the other two elements of the framework. The institutional framework was already hinted at in (Sierpinska, 2006) and has been presented in a detailed manner in (Sierpinska et al., in press). Here we only define our distinction of four "positions" from which respondents could experience their situation as students of PMC.

2.1 A three-dimensional theory of learning

We were looking for an integrated theory of learning, taking into account not only the cognitive aspects of it but also the emotional and social aspects, and we found it in Illeris' work (2004). In this theory, the learner is considered as a subject of an educational institution, who may collaborate with or resist the imposed regime of study and forms of behavior, and thus learn what is expected, not learn it or learn something different instead. The attention devoted to the phenomenon of *resistance* in Illeris' work, which we saw as related to frustration, was one more reason why we considered it relevant to our work.

2.2 Conceptualization of frustration

In psychology, there is no special theory of frustration; there are theories of emotion. In a recently proposed theory (Scherer, 2000), emotions are complex processes, involving increased interactions amongst the five basic organismic systems: the appraisal by the *cognitive system* may be triggered by an arousal of the ANS (*autonomic nervous system*), expressed by the *motor system* and resulting in undertaking an action whose nature may depend on habitual or reasoned tendencies of the *motivation system*, as well as on feelings produced by the *monitor system*. Emotional processes change very quickly and one cannot really speak of frustration as a "state" of the organism in an interval of time. "Frustration", "anger", etc. are seen as "mere" mental constructs used by people to communicate their feelings. Still, we believe that when people speak of their frustration, they refer to something that is very real to them. It "feels" real in their nervous system and they appraise it as caused by some real situation. And it is this experience called "frustration" that we are interested in.

In general, in mathematics education, we are not interested in describing the physiology of emotions. Rather, we are interested in knowing what might be the reasons why students feel the way they do, about mathematics, or learning, or their teachers. In this research, we wanted to understand what makes students in PMC feel "frustrated". Therefore we had to conceptualize the meaning of this word in a way which would be as close to its use in everyday language as possible.

Dictionaries define frustration as a feeling of tension or insecurity caused by encountering an obstacle to satisfying a need or achieving a goal. This sounds like a description of Mandler's "visceral arousal" (Mandler, 1975: 8) accompanied by an appraisal of something as an "obstacle" which "interrupts" an action or "frustrates" a plan. The definition identifies three elements in the situation of a person experiencing frustration: assessing something as desirable or necessary (e.g. "*I like math and I really want to learn it*"), perceiving an obstacle to obtaining it (" *but I just need good teachers and so far at [this university], I haven't experienced that yet*") and therefore feeling a tension or insecurity ("*so, right now, I'm on the edge of just, forget it!*").

In Mandler's theory of emotion, all emotion seems to be a result of an interruption and therefore of some kind of frustration, understood as in "frustration of a plan, of an expectation", so this theory didn't seem very useful. Looking for theories that would distinguish frustration from other kinds of emotions, we came across Roseman's structural theory of emotion (described in Cornelius, 1996: 140-144).

Roseman's theory focuses almost exclusively on appraisal, and treats emotions as states. The theory proposes a framework to analyze the structure of emotional states. Emotions are distinguished based on how different aspects of the situation are appraised. In particular, in frustration, one appraises the events in the situation as creating an *obstacle* to reaching one's goals, blames the circumstances beyond one's control for this state of affairs, and feels *strong* with regard to the development of events. Disliking differs from frustration in two aspects: other people are blamed for the state of affairs, and the individual perceives him/herself as weak. Anger differs from frustration in only one aspect — the events are appraised as other-caused. Regret is, like frustration, on the negative side — perceiving the situation as inconsistent with one's goals — and feeling "strong", but the state of affairs is blamed on oneself.

We were not satisfied with this theory because we could think of situations where one could feel frustrated with self (e.g., "*I'm not stupid, so why don't I get it?!*") or with others (e.g., "*How am I supposed to learn this stuff, if the teacher is unable to explain it?*"). But if we admit that frustrations can be circumstance, other and self-caused, then, within Roseman's theory, there is

no difference between frustration, anger and regret.

It is Handa's (2003) explanation of frustration that we found the most appealing, because it did not exclude being frustrated with self, was close to the common meanings of this word, and also focused on the causes and contexts of the feeling rather than on inner psychological processes. Moreover, the explanation was elaborated from the perspective of a study of mathematical activity and not general psychology of emotion. Handa derived the meanings of "frustration" from the Latin verb *frustrare* which can be used in the sense of "to deceive" or "to disappoint". In deception, the emotion can be caused by the discovery of being wrong in one's expectations. For example, I work long and hard on a problem, I finally solve it, and am very proud of it. But then I discover that the problem was very simple and I feel deceived. Disappointment refers to situations of not being able to complete a plan: "A certain aspect of frustration involves disappointment in a plan that has been foiled, or frustrated." (Handa, 2003: 24) For example, I wanted to solve a problem; I have thought out a strategy or approach, and it turned out not to lead to a solution.

2.3 Positions

In our research, we assumed that PMC students may experience and construct their frustration in different ways, depending on the position they take relative to their situation, as:

— *Persons* in the world, frustrated with aspects of their out-of-school life, or commenting on cultural and general scientific value of mathematics in the world,

— *Learners* of mathematics (cognitive subjects),

— *Students*, i.e. as subjects of a particular educational institution, with certain duties towards it and some resistance to its demands, or

— *Clients*, that is, persons who ask for services, pay for them and have the right to evaluate their quality.

For us, "position" was an answer to a question such as,

Where is the speaker positioned with regard to the situation of taking a PMC course: In the city (metaphorically)? In the library? In class? In the registrar's office?

This idea of position relative to a situation has a certain similarity with Evans' notion of "positioning" in a situation (Evans, 2000). Evans used this term to classify adults' approaches to mathematical problems according to

the mathematical practice whose discourse they were using. For example, "she was addressing the problem from within business/financial discourses, rather than college mathematics or statistics. Therefore her positioning was classed as work practices" (Evans, 2000: 153). We would say that she was speaking from the position of a Person; but for the purposes of a research on adults' numeracy (as in Evans' research) we would have to refine the classification and perhaps distinguish between the positions of Lay-Persons (Evans' common sense discourses) and Professional-Persons (work discourses). We didn't see the need of making such distinctions in our research. We were looking not at adults in concrete mathematical situations/problems but in a more general situation of being an adult student returning to university, which involves not only solving problems but many other activities, as well. Evans claims that positioning is an important aspect to take into account when trying to understand the context of people's thinking in specific situations: proficiency and functional conceptions are not adequate to understand the context of subject's thinking in specific situations. Indeed, it is necessary to determine which practice(s) a subject has called up, and hence his/her positioning, in the situation. (Evans, 2000: 181). We would agree with a similar claim regarding the need to take into account the subject's position relative to an academic situation.

The distinction of the various positions from which respondents expressed their frustrations is already a part of the institutional theoretical framework in the study, which are in more detail in (Sierpinska et al., in press).

3 Sources of data

3.1 A general description of the institution from which we recruited the participants

For The University, the main tasks of the prerequisite mathematics courses are:

— to prepare students for understanding and using mathematical models in a variety of academic programs, and

— to select candidates that can be considered prepared and can therefore be admitted into these programs.

Most students registered in the courses take them as prerequisites. A few students take the courses as electives to complete their non-mathematical programs. Instructors are mostly members of the mathematics department,

professors as well as master's and doctoral students. Occasionally, invited professors from other, sometimes foreign, universities are assigned to teach the courses. The University also hires, on a part-time basis, experienced college instructors.

The University offers ten PMC (we call them Course 0, Course 1, ..., Course 9). Each course has its particular clientele, e.g. candidates to Psychology in Course 0 (basic algebra), Industrial Engineering or Computer Science in Course 4 (elementary linear algebra), Business School in courses 6, 8 and 9 (functions, one-variable calculus). Most courses are multi-section courses. All sections of a course follow the same course outline, the same homework assignments and the same final examinations, all set up by the so-called "course coordinator", usually a full-time member of the mathematics department, appointed for this function by the chair of the department.

In the span of 13 weeks and about 33 hours of class time, in groups of 60-70 students, the courses "cover" material which, in a secondary school or college, might be spread over two terms and much more class time in smaller groups. The focus is on basic algebraic techniques (including elements of linear algebra), functions and one-variable calculus, with applications to solving word problems. Quantitative relations in the word problems are expected to be represented using functions and equations. The word problems are usually about everyday life or work situations in basic algebra courses, networks and predator-prey systems in linear algebra courses, financial situations or physical phenomena in pre-calculus and calculus courses. These are mostly "closed" problems with a unique solution. Open problems, projects, exploratory activities are not part of the 'normal' practice. Neither is systematic presentation of theory or deductive proof. Generality is achieved through formulas and procedures for solving types of problems. Concepts and procedures are given meaning not so much by definitions and theorems, as by means of examples, informal explanations, metaphors, graphical and other visual representations.

The selection of candidates to academic programs is based on their grades in the prerequisite courses. In PMC, the grades are based on homework assignments[2] (0%-10% of the grade), one or two class tests (worth 30-40% of the grade) and a final examination (worth 60-70% of the grade). Students can choose to have their final to count for 100% of the grade. Attendance and class tests are not compulsory. The questions on the final examination are of the types practiced in class. Questions very different from those discussed in class are rare. If they occur and the failure rate is high on a final examination,

[2]In some courses, homework assignments are not marked. Students are free to do them or not. They are provided with model solutions of these assignments.

students complain more or less formally to the chair of the department or even to the dean of the faculty.

3.2 The student-participants

We recruited the student-participants in our study from Courses 0, 1, 6 and 9 (in years 2003 and 2004). Course 0 focuses on basic algebraic techniques, normally covered in high school. This course is a prerequisite, among others, for admission to Psychology, and it is a prerequisite for Course 1. Course 1 studies algebraic properties of elementary functions, including trigonometric, exponential, and logarithmic functions. This is the so-called "pre-calculus course"; differentiation and integration techniques are not used. Course 6 is also a pre-calculus course, similar to Course 1, the difference being the absence of trigonometry in Course 6. Course 1 is a prerequisite to engineering programs; in Course 6, the focus is on exponential and logarithmic functions and applications to financial problems. Course 9 requires Courses 1 or 6 as a prerequisite; it is a one-variable calculus course, covering topics such as limits of functions (treated in an intuitive fashion, without the epsilon-delta definition), differentiation of rational, exponential and logarithmic functions, theory of maxima and minima; integration. Most students in Course 9 are candidates to studies in commerce.

We chose these courses for two reasons.

Firstly, these courses were often mentioned in stories of teachers' and students' frustration. Why other courses were not mentioned? Here is a possible explanation. Courses 2, 3, 5 and 7 are rarely taught. Course 4, a basic linear algebra course, is filled by candidates to engineering programs who do not contest the necessity of mathematics for their future studies. The material is also new for the students, and therefore students do not have the feeling of having to unnecessarily repeat something they already know. Course 8 contains very little mathematical theory and focuses on financial applications; candidates to commerce programs, for whom this is a prerequisite, appear to find it interesting and useful.

Secondly, we were initially particularly interested in "mature students'" experience in PMC and Courses 0, 1, 6 and 9 normally contain large numbers of mature students. In The University, candidates 21 years old or over and having spent some time outside of formal education are considered "mature entry students". Other students in the courses come straight from secondary or collegial institutions, but are required to take the courses because they had not taken mathematics courses at the required level or obtained poor grades in such courses.

We sent our questionnaire to about 60 students by ordinary mail, and about 800 requests to fill the questionnaire by e-mail. Only 96 students (63 qualifying as "mature") responded to our questionnaire in writing, and one (JesTia, whom we quoted above) called and said she preferred to talk about her experience rather than write. We interviewed 6 students altogether, 5 of whom also responded to the questionnaire. Four of the interviewed students were "mature".

In the interviews, we asked students to tell us a bit more about themselves, their histories, their goals and sources of frustration, if any, beyond what they had written in the questionnaires. We were also able to observe their mathematical behaviors, while asking them to explain their answers to the mathematical questions in the questionnaire, especially if they did not write any comments or explanations.

We will use the abbreviation "ms" for mature students and "nms" for non-mature students.

Of the 63 ms, 56% (35) were female; 79% (26 out of 33) nms were female. The distribution of respondents among the courses is given in Table 1.

This must be considered an "opportunistic sample" of the population of students in PMC 0, 1, 6 and 9. We had to rely on volunteers, because the subject of the questionnaire, related to emotions in studying mathematics, is a sensitive issue and not everybody wants to talk about it, maybe exposing one's feelings of shame or failure. Still, the number is not small (about 100 students), and the sample represents rather well the diversity of the population of the students in the courses, with both mature and non-mature students, male and female, local and immigrant, and a large variety of ethnic and educational backgrounds.

Course	ms	nms	Total students
0	17	0	17
1	17	9	26
6	11	6	17
9	12	15	27
unknown	6	3	9
Total	**63**	**33**	**96**

Table 1. The distribution of respondents among the courses.

3.3 Teacher-participants

We started our research by interviewing 4 instructors. All except one were male except for TDF. This represents rather well the distribution of male

and female instructors in the courses. One was a part-time instructor with an experience of at least 20 years teaching in a college and as many years teaching the PMC. Two were doctoral mathematics students with two years experience of teaching the 0-9 courses at The University. The female instructor was a beginning doctoral mathematics student and, at the time of the interview, she had twice taught the Course 0.

The stories the instructors told us about cases of students' and their own frustration, and their conjectures about the reasons of frustration were an inspiration for us in designing the questionnaire.

4 Methodology

In this section we first present our research instrument — a questionnaire — and then the procedures we used to analyze the data[3].

4.1 The questionnaire

Items of the Questionnaire were mainly statements in the first person with which the respondent could agree, disagree or remain neutral. E.g.

Item 56. *I need the teacher to tell me if I am right or wrong.*

Agree Disagree Neutral

The statements were speaking about the different elements of the situation of being a PMC student, such as, for example,

— the rule which makes certain math courses a prerequisite for some programs (we call it "the PMC rule"),

— mathematics (as knowledge, as object of learning, of study),

— achievement,

— teachers,

etc.

The statements were meant to cover the different "positions" participants might be taking relative to the above aspects of their situation in the PMC: from "outside", as *persons* in the world, or, from the "inside", as *learners* of mathematics, as *students* of PMC or as *clients* of The University.

The statements represented our initial hypotheses about the possible sources of students' frustration, that is, causes of their *disappointment* in relation to plans they had in mind when embarking on the course or *deception* relative to expectations, based, perhaps, on their past experience as learners and students of mathematics in high school.

[3]The questionnaire can be viewed from the first author's webpage at http://www.ajdomain.ca

The details about the questionnaire are given in sections 4.2, 4.3 and 4.4 below. Section 4.2 presents the content of the items structured along the four positions from which participants could experience their frustration. In section 4.3 we speak about the purposes of some open items in the questionnaire, and in section 4.4 we dwell on the mathematical items.

4.2 Hypotheses about sources of frustration underlying the questionnaire items

Hypotheses about the frustrations of the Person

From the Person's point of view, the participant sees his or her situation globally, taking into account both the out-of-school life and the school, speaking of the issues of transition, of the differences of roles played and power possessed in the two settings, and seeing mathematics not only as a school subject but knowledge developed and applied in sciences and professions. We expected this perspective to be revealed to some extent in responses to open items such as 66 (reasons for liking mathematics or not) and 76 (completing the sentence, "Math is") and closed items such as 7 (*I am wiser and more motivated than my younger colleagues*) or item 65a (*I took this course because I'll need the math in my profession*).

We expected mature students to be enthusiastic about coming back to school (item 9), and all students to be enthusiastic to study at the university, where they go from their own free will and not because somebody tells them to, as is the case of mandatory education for children and adolescents. We imagined that this initial enthusiasm could be frustrated later on by the difficulties of a mature person to adjust to the role of a student (item 11), sometimes perhaps feeling angry at the teacher (item 14). Frustrations could easily arise in a situation where the Person has to juggle the family and job commitments and school duties (30), to the point, perhaps, of giving up hope of ever getting a university degree (item 25).

The first items of the questionnaire identified the respondent as a Person being male or female (item 1), and mature or non-mature according to The University's definition (items 2, 3).

Hypotheses about the frustrations of the Learner

The Learner's task is to learn mathematics. We assumed that the accomplishment of this task can be seen as obstructed by, for example,

- teaching approaches, e.g.,

  - a teaching approach that could be incompatible with one's learning style; for example, the learner may like it when the teacher helps him or her to see "the big picture' (item 19); this can be difficult if the course focuses on memorizing rules (item 35) and the student doesn't like it (item 5);
  - a teaching approach incompatible with the learner's habitual ways of thinking (item 43);

- the learner's own relations with the task of learning mathematics, e.g.

  - not liking math (item 66) or algebra (17, 18, 70, 71); finding it too abstract (items 15, 16);
  - not liking to solve problems (item 4);
  - having difficulties with math: not understanding (item 6, 52); having a history of difficulties in math (53); finding it hard (item 62);
  - not being used to the kind of math taught in PMC (item 63);
  - not having a good judgment with regard to mathematical truth (items 56, 72, 73, 74, 75);

- the learner's knowledge and ways of knowing, e.g.,

  - poor background knowledge (item 61);
  - memorizing rules (item 5) but noticing that one also needs to understand them (51);
  - wanting to understand every single detail (item 20);
  - not spending enough time thinking about a problem (item 21);
  - ways of thinking that are very different from other students' (59) or the teacher's (60).

Hypotheses about the frustrations of the Student

We were looking for the frustrations of the Student mainly in the obstacles to his or her exercising the "job of the student", i.e. taking and passing the prerequisite math courses. Items 12, 67, and 68 were aimed at revealing respondents' perceptions of their actual achievement in the courses and reasons for them. Item 32 asked if respondents thought they used to be good students. Agreement with that, in combination with the respondents' present dissatisfaction with their results in PMC (items 12 and 68) could point to a feeling of deception with one's performance as a student. Various aspects could be blamed for this deception and spontaneous explanations were expected in items 67 and 68.

Reasons were also suggested in the closed items. For example, participants could find the fault with the tightness of the didactic contract, not allowing them to use whatever methods they liked (item 44).

They could also blame their own relations with the task of taking math courses; for example, not liking to study math (item 8), not doing well on tests (item 31), finding it more difficult than in the past to learn for an exam (item 57), or having hard time concentrating in class (item 58).

They could blame the "tools" they thought were necessary to achieve the task and which they did not use or did not have at their disposal, e.g. knowing a lot of math already (item 34), hard work (items 22, 26), mathematical giftedness (item 23), good organization (item 24), attending classes (items 27, 28, 29), memorizing rules (item 35).

Hypotheses about the frustrations of the Client

We assumed that, as clients, participants would blame, mainly, the "objective", institutional aspects of their situation and not themselves; that they would be concerned with

- the PMC rule, i.e. being forced to take math as a prerequisite (item 65c), rather not taking the courses if they had a choice (13), being terrified with having to study math again (item 10), doubting the relevance of the math in PMC for their future studies and professions (item 64);
- the way the PMC courses are run, in particular,
  - assuming university students to be autonomous learners (item 38), even though PMC are pre-university level courses and students expect the courses to be run like high school courses, thinking they can study just by following teacher's instructions (item 48);
  - the scarce feedback on students' performance (item 45) since assessment is mainly evaluative (and not formative) and based on one or two class tests and one final examination, usually worth 60 or 70% of the total marks in the course; students may feel they need feedback in order to learn (item 55);
  - the amount of work left for students to do on their own because of small number of classroom hours (item 36);
  - the fast pace of the courses (item 37), because of a lot of material to be covered in one semester, while a student may feel he or she learns better at their own pace (item 54);

- the quality of the teaching services regarding, in particular, the clarity and fairness of the didactic contract[4] in class (items 39, 40, 41, 42, 49, 69);
- the quality of the moral support expected from teachers (items 47, 50).

A confrontation of our hypotheses about sources of students' frustration and the participants' responses is presented in (Sierpinska et al., in press).

4.3 The open items

Some of the questionnaire items were open, asking respondents to explain their reasons for liking mathematics or not (item 66), or for having done well or not so well in a course (items 67, 68). They were also asked to finish the sentence "Math is" (item 76), where we expected the participants to express their opinions and attitudes and perhaps vent all their frustrations with PMC. We expected responses to open items to reveal students' prevailing positions (Client, Student, etc.) and relations between positions and the coloring of emotions (positive, negative, ambivalent, neutral). This paper focuses on these relations and therefore looks mainly at responses to the open items.

4.4 The mathematical items

Six items dealt with mathematical situations. Of these, two (70, 71) were meant to get a glimpse of the respondents' attitudes to algebra, while two other (72, 73) probed their judgment regarding the correctness of an answer to a mathematical problem. In item 72 respondents were asked if writing "2/4" instead of "1/2" should be considered a minor, major or no mistake. In item 73, a similar question was asked about "$y = 2x - 3$" and "$y = 2 - 3x$". PMC instructors perceive algebra as difficult for students, especially mature ones, and a major cause of frustration. The correctness of an answer is often a contentious issue in discussions between instructors and students and that's why we chose to include these items in the questionnaire.

Items 74 and 75 dealt with absolute value inequalities. We state these items below.

> Item 74. Given a problem: Solve $|2x - 1| < 5$. Which solution do you like better?
>
> Solution a.
>
> $|2x - 1| < 5$

[4]The concept of didactic contract is owed to Brousseau (1997).

$2x-1=5$ neg. $2x-1=-5$

$x=3$ $x=-2$

Answer: $-2<x<3$

Solution b.

We use the theorem: $|a|<b \Leftrightarrow -b<a<b$

$|2x-1|<5 \Leftrightarrow -5<2x-1<5 \Leftrightarrow 2x-1>-5$ and $2x-1<5 \Leftrightarrow$

$x>-2$ and $x<3$

Answer: $-2<x<3$

Item 75. Given a problem: Solve $|2x-1|>5$. Which solution do you like better?

Solution a.

$|2x-1|>5$

$2x-1=5$ neg. $2x-1=-5$

$x=3$ $x=-2$

Answer: $3>x>-2$

Solution b.

We use the theorem: $|a|>b \Leftrightarrow a<-b$ or $a>b$

$|2x-1|>5 \Leftrightarrow 2x-1<-5$ or $2x-1>5 \Leftrightarrow x<-2$ or $x>3$

Answer: $x<-2$ or $x>3$

Solutions "b" in items 74 and 75 made reference to properties of absolute value and were based on logical deduction. Solutions "a" resembled the "two-column" procedure, commonly taught in secondary school, which reduces solving an inequality to solving two equations and then following rules for writing the inequalities in the answer. These rules are many and complicated, so students often just make a guess and then verify if they got it right by substitution of concrete values from the range in the answer into the inequality. Mistakes are frequent because of poor understanding of the notion of solution to an inequality. Item 75 presents this kind of mistake in solution "a". In item 74 both answers are correct.

We chose inequalities with absolute value because, in our experience of teaching PMC (especially Course 6, Algebra and functions), some students showed remarkable resistance[5] to adopting the theoretical approach, loudly protesting and arguing for the procedural one, even when they were obtaining incorrect solutions with it.

[5]This attachment to familiar concepts and strategies remembered (rightly or wrongly) as effective in the past is a well-known phenomenon in studies on adults' learning (Knox, 1978).

The incorrect solution in item 75 was meant to probe students' sensitivity to truth in mathematics. This issue was, for us, related with the students' control over the validity of their solutions, addressed in item 56. *I need the teacher to tell me if I am right or wrong.*

Students' responses to items on absolute value and item 56 (above) play a major role in (Sierpinska et al., in press), devoted to sources of students' frustration.

4.5 Procedures used in analyzing responses

Responses to the questionnaire were organized into a single Excel file, with respondents' names listed vertically and possible responses to questionnaire items listed horizontally. Respondents were sorted into "mature" and " "non-mature" and given number codes; respondents 1-63 were mature students, those numbered 64-96 were non-mature students. If an item had three possible responses, Agree, Disagree, Neutral, three columns were devoted to this item (Table 2). Number 1 in the box corresponding to respondent no. x and response "agree" to item y, meant that this is the answer the respondent checked in the questionnaire. Number 0 in a box meant that the respondent did not choose the answer. Essay responses and commentaries were reproduced in full in the same file.

Respondent	...	Item n Agree	Item n Disagree	Item n Neutral	...	Item 76 Math is...
1		1	0	0		[full text of response]
...						
96		0	1	0		[full text of response]

Table 2. Structure of the database

According to the example in Table 2, the first respondent agreed with the statement expressed in the n'th item of the questionnaire while the 96th respondent disagreed with it.

The zeros and one's in columns corresponding to closed items were added to give three numbers: the number of all responses of a given kind in the whole group of 96 respondents, the number of such responses among the mature respondents (i.e. respondents #1-63), and the number of such responses among the non-mature respondents (i.e. respondents #64-96). Corresponding percentages were calculated, representing the frequencies of a given kind of response in the whole group, the ms group and the nms group.

Analysis of the responses required taking intersections and unions of sets of respondents who answered in a certain way to two or more items. Intersections

were identified by taking the products of numbers corresponding to the same row (i.e. same respondent). The product was 1 for a person only if all responses of a certain kind to the chosen items were coded 1. For the identification of a union, the numbers in rows were added; if the sum was greater than 0, number 1 was assigned.

We did not use any statistics more sophisticated than that. Simple frequencies were calculated, just to give us an idea of the relative importance of types of responses. As we said, the sample was opportunistic, based on individuals' good will and interest. Therefore, our results should not be generalized to speak about all PMC students, even at the time when the 96 were kind enough to answer the questionnaire. The outcomes of the exercise of computing frequencies is to pose conjectures which could sensitize mathematics educators to notice PMC students' problems and think about institutional, pedagogical, didactic and humane ways of addressing them.

Essay responses and interviews required interpretation; this work was done in groups of 2-6 experienced researchers and graduate students, in several rounds of discussions about the choice of categories and then seeking consensus in attributing the categories to the responses. In particular, the categorization of responses to item 76. *Complete the sentence, "Math is"*, according to positions (of Person, Learner, Student, Client) and emotive coloring (positive, negative, ambivalent and neutral), was decided in a group of 3 mature researchers and 3 graduate students. This paper will present the results of this categorization.

Analysis of the interviews was assisted by the use of a software for qualitative document analysis called "Atlas-ti". Themes of discussion, interviewees' diagnoses of the main sources of students' frustration have thus been identified.

In the next section we present some results of our analyses of the responses.

5 Results on relations among emotions, positions and achievement

The following conventions will be used in the presentation of the results of the study. The frequencies of types of responses in the questionnaire will be given in the form of a triplet of pairs of numbers:

A% (A1), B% (B1), C% (C1),

where A is the percentage of all 96 respondents to the questionnaire who gave the answer of a given type; A1 is the number of these respondents; B is the percentage of the 63 mature respondents who gave this type of answer and B1 is the number of these respondents; C is the percentage of the 33

non-mature respondents who gave this type of answer and C1 is the number of these respondents. A1 is the sum of B1 and C1.

Citations of students' comments or essay-type answers will be given in smaller font, and will be pre-fixed by "#n", which gives the code of the student in our data base. For n between 1 and 63, #n represents a mature student ("ms"). For n between 64 and 96, #n represents a non-mature student ("nms").

We first estimate the magnitude of frustration among the 96 respondents, 63 mature and 33 non-mature, and then its relationship with (a) positions and (b) achievement. We chose achievement, since this was an important outcome of participation in PMC for many respondents (about 2/3 of all; see responses to item 65).

5.1 General estimation of the level of frustration among respondents

This estimation is based on the group of respondents who said they were enthusiastic about coming back to school (Item 9) and, at the same time, expressed negative emotions in responses to some other questionnaire items (namely items 10, 12, 14, 46, 47, 66, 68, 76; see frequencies in Table 3). The intersection of these sets counted 70% (67), 78% (49), 55% (18) students: thus, many students appeared to have had their initial enthusiasm frustrated in the course of their studies in PMC.

As we can see from Table 3, many respondents agreed with Item 9. *I was enthusiastic about going back to school*: 84% (81), 95% (60), 64% (21). The relatively small number of nms can be explained by the fact that item 9 was addressed to ms, not nms: it was not about enthusiasm for studying in general, but about returning to study. Still, quite a few nms responded positively to this item (interpreting it, maybe, as "going back to school" after holidays, or after having completed another type of school).

There were some striking differences between ms and nms. In particular, nms were more likely than ms to complain about teachers not being sympathetic to their difficulty of managing life and school duties. In fact, in interviews with instructors, two out of the four expressed the belief that nms have no major "life duties" and if they don't study, miss classes, don't come to see teachers during their office hours, it is because they are "lazy" or "not serious" about their studies. They could find excuses for ms but not for nms. But almost half of our younger respondents suggested that their "life duties" do compete with school duties for priority in their lives. Another example of noticeable difference between ms and nms is the expression of negative affect

in completing the sentence "Math is" (item 76). Nms were almost twice as likely as ms to express negative affect in relation to mathematics.

Item number	Statement	Frequency of agreement		
		%All (N=96)	%ms (N=63)	%nms (N=33)
	POSITIVE AFFECT			
9	I was enthusiastic about coming back to school	**84% (81)**	**95% (60)**	**64% (21)**
	NEGATIVE AFFECT			
	Union of sets of students who expressed negative affect in items 10, 12, 14, 46, 47, 66, 68, 76)	**83% (80)**	**83% (52)**	**85% (28)**
	Intersection of the sets of students who agreed with item 9 and expressed negative affect in other items	**70% (67)**	**83% (49)**	**55% (18)**
	Details			
10	I was terrified of having to study math again	40% (38)	35% (22)	49% (16)
12	I wasn't happy with my results in the course	43% (41)	52% (33)	24% (8)
14	I have sometimes been angry at the teacher in the course	33% (32)	38% (24)	24% (8)
46	In the course, the teacher did not encourage us	30% (29)	27% (17)	36% (12)
47	In the course, the teacher did not understand my difficulty to manage life and school duties	38% (36)	32% (20)	49% (16)
66	I don't like math	30% (29)	27% (17)	36% (12)
66	I like math [but sometimes have negative feelings about it]	1% (1)	2% (1)	0% (0)
68	I didn't do as well as I had hoped in the course, because... [negative affect expressed]	20% (19)	21% (13)	18% (6)
76	Math is... [negative affect expressed]	15% (14)	11% (7)	21% (7)

Table 3. Disappointment. Initial enthusiasm frustrated in the clash with reality.

In the next section we look at the association between expression of negative affect and positions from which students made their statements.

5.2 Association between emotions and positions

Our estimation of association between emotions and positions was based on responses to item 76. *Complete the sentence, 'Math is'.*

In their responses students wrote not only about mathematics as an area of scholarly knowledge, but also as a school subject, or as a course in the PMC institution. The statements could be characterized by the position they represented (Person, Learner, Student or Client) and their "emotive coloring" (positive, negative, ambivalent or neutral).

For example, the response, "Math is interesting" (#1) was classified as representing the position of Learner and a positive emotive coloring. On the other hand, the following response was classified into the Student / ambivalent category.

> #11 I took math in high school and did very poorly which gave me a fear of having to take another math course. Last semester I tried taking [Course 2] but dropped it because I found it too difficult. This summer, I took [Course 0] and found it to be better than I thought. I ended up with an A but I worked really hard for it. This class gave me more confidence in my math skills. The professor was understanding of our situation, that being that there were many mature students in the class.

"Math is a practical and objective way of thinking" was classified into the Person / neutral category. "Math is very hard and time-consuming" — into the Learner / negative category. The following three long responses were considered to represent the Client / negative category:

> #17 Obviously Math can be an emotional course to take and our self-perception plays an important role in how well we do. However, I have to admit that this was the worst academic experience I had at [The University]. I first enrolled in the course last winter and dropped it. I enrolled a second time during the summer and to my surprise I got the same instructor. She was very kind and pleasant, however, had poor teaching and language skills, which made it very difficult to obtain answers to precise questions in class. I often saw the instructor out of class to clarify any questions. I ended up having to hire a math tutor whose skills, patience and knowledge saved my semester. I wish I had done it earlier during the course, maybe I would have gotten a better mark than a D+, which brought my [grade average too low] to be accepted into the Psychology program. So today, I guess, I am a little resentful of [The University's] Math Department and their lack of accountability towards the students.
>
> #21 Math is a complete waste of time unless it is specifically geared towards the field you are going into. Please know, I have nothing against

people who have made math their career and life choice, but please understand that for some of us, the subject gives us nightmares, heart palpitations and cold sweats just thinking about it.

#39 Math is extremely discouraging when you are forced to take it as a prerequisite. If I were going into an Arts & Science program with a major in math, then I would understand that the course is necessary. However, in the Commerce program, there is nowhere near as much or as difficult math as I have just taken. I also have another year of math prerequisites to take in order to get into the program I want. If I fail math, I don't get into commerce. So I feel math is the only thing that's stopping me from getting into the program I want. I can't imagine the number of people who have dropped out of their dream just because of this one subject that got into their way. I am currently spending 20 hours of studying outside of class and I got 30% on my midterm. I am starting to think that I'm the problem and that's very discouraging.

The frequencies of positions and emotive coloring separately are presented in Table 4.

	Positions				Emotive coloring			
	Person	Learner	Student	Client	Positive	Negative	Ambivalent	Neutral
% All (N=96)	25% (24)	37% (35)	23% (22)	12% (11)	30% (29)	23% (22)	28% (27)	15% (14)
% ms (N=63)	30% (19)	35% (22)	21% (13)	11% (7)	32% (20)	22% (14)	29% (18)	14% (9)
% nms (N=33)	15% (5)	40% (13)	27% (9)	12% (4)	27% (9)	24% (8)	27% (9)	15% (5)

Table 4. Frequencies of positions and emotive coloring represented in responses to item 76. Complete the sentence, 'Math is...'

Ms were much more likely to speak from the position of Person than nms. Perhaps the younger students did not see themselves yet as members of the grown-up society with the right to express their opinions about mathematics as it exists outside of school. They saw themselves mostly as Learners, learning knowledge from the more experienced elders, but not contributing yet to its development. However, the position of Learner was also prominent among ms; as PMC students, ms were humbled by being certified as ignorant in the domain of mathematics and thus obliged to take pre-university level math courses.

The position of Client was the least frequent in both groups.

The ratio of positive to negative emotional coloring was higher among ms (about 3:2) than among nms (about 1:1). As the instructor with 20 years experience was saying, nms are more likely to "whine" than ms.

The association between positions and emotive coloring is given in Table 5.

	EMOTION			
POSITION	Positive	Negative	Ambivalent	Neutral
Person	12%(11)	0%(0)	4%(4)	9%(9)
	14%(9)	0%(0)	6%(4)	10%(6)
	6%(2)	0%(0)	0%(0)	9%(3)
Learner	12%(11)	7%(7)	15%(14)	3%(3)
	11%(7)	6%(4)	12%(8)	5%(3)
	12%(4)	9%(3)	18%(6)	0%(0)
Student	7%(7)	8%(8)	7%(7)	0%(0)
	6%(4)	8%(5)	6%(4)	0%(0)
	9%(3)	9%(3)	9%(3)	0%(0)
Client	0%(0)	7%(7)	2%(2)	2%(2)
	0%(0)	8%(5)	3%(2)	0%(0)
	0%(0)	6%(2)	0%(0)	6%(2)

Table 5. ,Associations between positions and emotional coloring in responses to Item 76. Complete the sentence, 'Math is'.

If we look at the zero frequencies in Table 5, we can conclude that,

— as Persons, respondents were never negative;

— as Students, they were never neutral, and

— as Clients, they were never positive.

If we now look at the highest frequencies, we can conclude, that,

— positive emotions were expressed mostly from the positions of Person or Learner;

— negative emotions were equally likely to be expressed from the positions of Learner, Student or Client;

— ambivalent emotions were the most likely to be expressed from the position of Learner;

— neutral emotions were the most likely to be expressed from the position of the Person;

— from the position of the Person, respondents expressed mostly positive or neutral emotions;

— from the position of the Learner, emotions were mostly ambivalent or positive;

— from the position of the Student, respondents were almost equally likely to express positive, negative or ambivalent emotions;

— from the position of the Client, emotions were mostly negative.

5.3 Association between emotions and perceived achievement

We now look at the association between the emotive colorings of responses to Item 76. *Complete the sentence, 'Math is'*, and responses to Item 12. *I wasn't happy with my results in the course* (see Table 6 below).

Emotional coloring of response to 'Math is...'	AND response to item 12. I wasn't happy with my results	%All (N=96)	%ms (N=63)	%nms (N=33)
Positive	...and not happy	6	10	0
Positive	...and not unhappy	24	22	27
Negative	...and not happy	14	18	6
Negative	...and not unhappy	9	5	18
Ambivalent	...and not happy	16	21	6
Ambivalent	...and not unhappy	13	8	21
Neutral	...and not happy	4	3	6
Neutral	...and not unhappy	10	11	9
No answer		4	2	7
Totals		100	100	100

Table 6. Associations between satisfaction with one's achievement and feelings about math.

We expected a strong association between being not happy with achievement and negative emotive coloring of description of math. But, in fact, in the whole group, this association ranked only third. The strongest association was between being *not unhappy* with one's achievement ("disagree" or "neutral" in response to Item 12) and exhibiting positive emotions when speaking about math in Item 76.

If we look at the associations with "not happy", then we can see that ms were more often than nms not happy about their results. As could be expected, "not happy" was associated mostly with ambivalent or negative feelings about math in ms. Ms were also rarely neutral, which is also not surprising. But — and this is not so trivial — 1 in 10 ms was able to express positive emotions about math in spite of not being satisfied with the results. Nms not happy with their results were not capable of such magnanimity and never expressed positive emotions in responding to the "Math is" item. They were equally likely to express negative, ambivalent or neutral emotions in relation to math.

Looking now at associations with "not unhappy", the main difference is in the distribution of frequencies between negative and neutral emotive colorings of statements about math. For ms, the least frequently expressed coloring

was the negative one; among nms — the neutral. In both groups the not unhappy positive association was the most frequent. But it is amazing how often even the nms who were not unhappy with their achievement, could find negative things to say about mathematics. Ambivalent statements contained negative emotions as well as positive emotions, so if we add all statements in which negative emotions were expressed (whether accompanied by positive ones or not) by those who were not unhappy with the results,then, for nms the frequency was 39%, while among ms it was 13%. This is a rather disturbing result.

These differences between ms and nms confirm the opinion of the teacher with 20 years experience, who stressed many times during the interview that nms complain more, while ms try to be more objective in the way they assess their work; they are less likely than nms to blame the teacher, the choice of material or the institution in general, for their poor performance.

6 Conclusions

Our research was triggered by our experience as instructors or mentors of instructors of prerequisite mathematics courses, where, on several occasions, students expressed their frustration in strong terms. We felt that our reactions were not always appropriate. We thought that if only we understood better students' reasons for getting frustrated, we could undertake more effective pedagogical and didactic actions. Thus we decided to get some insight into the students' point of view by asking them a few questions and by interviewing some of them. We wanted to reach several hundred PMC students, but had to be content with responses from only 96. We also wanted to know if our impression of students' frustration was shared by other instructors and so we interviewed four instructors.

At this stage of our research, assuming that the 96 respondents are more or less representative of PMC students in general, *we are able to confirm our initial impression: many students of prerequisite mathematics courses are frustrated* (in the sense of being disappointed or deceived) with various aspects of these courses. Indeed, our analysis showed that many students, while initially enthusiastic about studying at the university, were either deceived in their expectations or experienced a disappointment as a result of not being able to complete the plans they had in mind at the outset of their studies.

By looking at the relationship between the positions from which students speak about mathematics, the emotional coloring of their assertions, and their assessment of their achievement, we were able to further nuance this gene-

ral conclusion and notice some differences between mature and non-mature students. In particular, the mature students in our study were more likely to take the position of Person than non-mature students; those speaking from the position of Person were less likely to express frustration than others; feelings about mathematics and achievement are related but in complex ways that differ for mature and non-mature students. Mature students were able to express positive feelings about mathematics even when they were not happy with their achievement. No non-mature student in our study displayed such "objectivity".

In evaluating the services of the university — that is, from the position of Clients — students never expressed positive emotions, while as Persons they never expressed negative feelings.

We also found that, when talking about their experience of learning mathematics (as Learners), students were more likely to express positive or ambivalent emotions. This suggests that by creating an atmosphere focused on learning (rather than passing exams) in our classes, we could perhaps reduce students' negative feelings about mathematics. This is a hope that has often been expressed by mathematics educators. Much effort has been spent on designing and experimenting classroom situations with the focus on mathematical thinking and problem solving and not on preparing for a final examination. Still, we must remain realistic about the goals of the majority of students forced to take prerequisite mathematics courses. Their interests are elsewhere and they are all adults who cannot afford spending their precious time engaging in what appears to them as esoteric intellectual activities. Achievement is very important for students in prerequisite mathematics courses and this must not be ignored in any further teaching proposals addressed to this particular "clientele"; they are and will be, in their majority, not only "learners", but mainly "students" and "clients" of a specific educational institution.

References

B r o u s s e a u, G.: 1997, *Theory of Didactical Situations in Mathematics*, Dortrecht: Kluwer Academic Publishers.

C h e v a l l a r d, Y.: 1992, 'Fundamental concepts in didactics. Perspectives provided by an anthropological approach', in R. Douady & A. Mercier (eds), Research in Didactique of Mathematics, Selected Papers, extra issue of *Recherches en Didactique des Mathématiques*, Grenoble: La Pensée Sauvage editions, 131-167.

C h e v a l l a r d, Y.: 2002, 'Organiser l'étude 1. Structures et fonctions', in J.-L. Dorier et al. (eds), *Actes de la 11e Ecole d'Eté de Didactique des*

Mathématiques — Corps 21-30 Août 2001, Grenoble: La Pensée Sauvage editions, 3-22.

C o r n e l i u s, R. R.: 1996, *The Science of Emotion. Research and Tradition in the Psychology of Emotion*, Upper Saddle River, New Jersey: Prentice Hall.

C r o z i e r, M. & F r i e d b e r g, E.: 1977, *L'Acteur et le système. Les contraintes de l'action collective*, Paris: Editions du Seuil.

E v a n s, J.: 2000, *Adults' Mathematical Thinking and Emotions, A Study of Numerate Practices*, London and New York: Routledge/Falmer.

F i t z S i m o n s, G. E. & G o d d e n, G. L.: 2000, 'Review of research on adults mathematics learning', in D. Coben, J. O'Donoghue & G.E. FitzSimons (eds), *Perspectives on Adults Learning Mathematics*, Dortrecht: Kluwer Academic Publishers, 13-46.

H a n d a, Y.: 2003, 'A phenomenological exploration of mathematical engagement: approaching an old metaphor anew', *For the Learning of Mathematics* **23 (1)**, 22-29.

I l l e r i s, K.: 2004, *The Three Dimensions of Learning*, Malabar, Florida: Krieger Publishing Co. (translated from Danish. First published in 2002, by Roskilde University Press).

K n o x, A. B.:1978, *Adult Development and Learning*, San Francisco: Jossey-Bass Publishers.

M a n d l e r, G.: 1975, *Mind and Emotion*, New York: Wiley.

O s t r o m, E.: 2005, *Understanding Institutional Diversity*, Princeton, New Jersey: Princeton University Press.

R e y e s, L. H.: 1984: 'Affective variables and mathematics education', *Elementary School Journal* **84**, 558-581.

S c h e r e r, K. R.: (2000). 'Emotions as episodes of subsystem synchronization driven by non-linear appraisal processes'. In M. D. Lewis, & I. Granic (Eds.), *Emotion, development and self-organization. Dynamic systems approaches to emotional development* (pp. 70-99). Cambridge, UK: Cambridge University Press.

S i e r p i n s k a, A.: 2006, 'Sources of students' frustration in bridging mathematics courses', *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education, Prague, Czech Republic, July 16-21, 2006*, Vol. 5, 121-129.

S i e r p i n s k a, A., B o b o s, G. & K n i p p i n g, Ch. (in press), 'Sources of students' frustration in pre-university level, prerequisite mathematics courses', *Instructional Science*. The paper has been published OnlineFirst; it can be viewed at: http://dx.doi.org/10.1007/s11251-007-9033-6, and cited as DOI 10.1007/s11251-007-9033-6.

Z badań nad frustracją kandydatów na studia uniwersyteckie uczestniczących w obowiązkowych kursach matematyki: emocje, pozycje i zadowolenie z wyników

S t r e s z c z e n i e

Artykuł opisuje fragment badań nad zjawiskiem frustracji wśród kandydatów na niektóre studia, inne niż matematyczne (psychologia, informatyka, handel), którym jako warunek przyjęcia stawia się zaliczenie pewnych kursów z zakresu matematyki szkoły średniej. Uniwersytet, z którego rekrutowali się badani, jest dużym północno-amerykańskim uniwersytetem. Uniwersytet ten oferuje wymagane kursy z zakresu szkoły średniej osobom dorosłym, które z różnych powodów nie zaliczyły tych kursów wcześniej. Osoby te mogły ukończyć szkołę średnią dawniej, kiedy te kursy nie były wymagane, bądź ukończyć ją w innym kraju, gdzie nie wchodziły one w zakres szkoły średniej, lub ukończyły szkołę średnią o profilu nie zawierającym bardziej zaawansowanych tematów z matematyki. Kandydaci są czasami zaskoczeni tym wymogiem lub rozczarowani kursami, które wydają się im nie związane z przyszłymi studiami lub zawodem. Nie jest łatwo być wykładowcą na tych kursach. Są to krótkie, najwyżej trzynastotygodniowe, intensywne zajęcia. Zaliczenie oparte jest w głównej mierze na pisemnym egzaminie końcowym. Na sali wykładowej panuje atmosfera pośpiechu, walki o punkty i stopnie, od których przecież tak wiele zależy. Studenci domagają się jasnych „reguł gry"; wyraźnego określenia zasad, na jakich będą oceniani. Studenci pilnują, aby wykładowca nie „odbiegał od tematu", nie uczył niczego więcej ponad to, co będzie wymagane na egzaminie końcowym. Twierdzenie jest ważne, o ile może być od razu przetworzone na metodę rozwiązywania zadania egzaminacyjnego. Ścisłe dowody nie są ważne. Liczą się przykłady, nieformalne ilustracje, i, przede wszystkim, przerabianie typowych zadań. Od czasu do czasu zdarza się głośny wybuch frustracji u któregoś ze studentów i wykład zostaje przerwany. Wykładowca musi uspokoić nerwy studentów (i swoje) zanim wróci do matematyki. Wydaje się, iż jego wysiłki byłyby skuteczniejsze, gdyby lepiej rozumiał odczucia studentów i głębiej znał przyczyny ich frustracji. Stąd pomysł podjęcia systematycznych badań nad zjawiskiem frustracji na tych kursach.

Artykuł przedstawia pewne aspekty podstaw teoretycznych badań, konstrukcję narzędzia badawczego (ankieta) oraz niektóre wyniki naszych badań. Podstawy teoretyczne badań zawierały, z jednej strony, uściślenie pojęcia frustracji, a z drugiej, ramy teoretyczne do badania zjawisk zinstytucjonalizowanego uczenia się i nauczania. W niniejszym artykule opisujemy bardziej szczegółowo pierwszy z tych aspektów, gdyż jest on bezpośrednio związany

z wynikami, które chcemy w nim przedstawić. Skupiamy się mianowicie na wynikach dotyczących ekspresji emocji w grupie respondentów.

Rozróżniamy dwie podgrupy respondentów. Pierwsza to „studenci dojrzali" (mature students), a druga - „studenci niedojrzali" (non-mature students). Do grupy studentów dojrzałych uniwersytet zalicza osoby majace 21 lub więcej lat, które przerwały uprzednio naukę na przynajmniej jeden rok. Grupa respondentów liczyla 96 osób, 63 studentów dojrzałych i 33 studentów niedojrzałych. W artykule najpierw opisujemy ogólny poziom frustracji w grupie respondentów. W całej grupie bylo 70 procent osób, które z entuzjazmem wracały na studia, aby następnie przeżywać frustrację w toku zaliczania wymaganych kursów z matematyki. W grupie studentów dojrzałych dotyczyło to 83 procent respondentów; w grupie studentów niedojrzałych - 55 procent. Następnie przedstawiamy wyniki dotyczące zwiazków między emocjami w stosunku do matematyki, „pozycjami" wobec wymaganych kursów i poziomem oceny własnych osiągnięć. Ostatnie pytanie ankiety brzmiało: Dokończ zdanie, „Matematyka jest...". Respondenci wykorzystali to pytanie do otwartego wyrażania swoich odczuć wobec matematyki i wymaganych kursów, i opowiedzenia tego ze swych doświadczeń, co nie mieściło się w innych pytaniach ankiety. Analizując emocje wyrażone w tych wypowiedziach, podzieliłyśmy je na reprezentujące emocje o zabarwieniu pozytywnym, negatywnym, ambiwalentnym lub neutralnym. W swoich wypowiedziach respondenci przyjmowali też różne „pozycje" wobec wymaganych kursów. Jedni pisali je z pozycji „klienta" instytucji uniwersyteckiej, który płaci za studia i uważa, że ma prawo do oceny jej usług. Inni przyjmowali pozycję „uczącego się", a więc pozycję podmiotu poznawczego wobec matematyki i pisali o swoich trudnościach z tym przedmiotem, lub o radości intelektualnej odczuwanej, na przykład, gdy udało się rozwiązać zadanie matematyczne. Jeszcze inni przyjmowali pozycję „studenta", czyli podmiotu instytucji edukacyjnej, który nie tylko poznaje matematykę, ale musi się swą wiedzą wykazać i zostaje za to przez tę instytucję oceniony. Jeszcze inni przyjmowali pozycję „osoby", która musi godzić naukę z życiem codziennym (praca, rodzina), stosuje lub ma zamiar stosować matematykę w pracy zawodowej i komentuje o miejscu matematyki w świecie kultury, nauki i techniki.

Okazało się, że istnieją związki między zabarwieniem emocji a pozycją, oraz między pozycją a oceną własnych osiągnięć na wymaganych kursach matematyki. Jako „osoby", respondenci nigdy nie wyrażali negatywnych emocji w swoim uzupełnieniu zdania „Matematyka jest...". Jako „studenci", nigdy nie pozostawali neutralni, a jako „klienci" nigdy nie wyrażali emocji pozytywnych. Emocje pozytywne wyrażane były głównie z pozycji „osoby" lub „uczącego się". Emocje ambiwalentne najczęściej wyrażane były z pozycji „uczącego

się". Studenci dojrzali wyrażali się znacznie częściej z pozycji „osoby" niż studenci niedojrzali, którzy widzieli swoją sytuację głównie z pozycji „uczącego się".

Oczekiwałyśmy silnego związku między niezadowoleniem ze swoich wyników na wymaganych kursach matematyki a negatywnym zabarwieniem wypowiedzi na temat matematyki. Jednak w całej grupie ten związek uplasował się dopiero na trzecim miejscu, jeśli chodzi o częstość. Najsilniejszy związek zachodził między „nie jestem niezadowolony(a) ze swych wyników" a pozytywnym zabarwieniem wypowiedzi na temat matematyki. Studenci dojrzali byli częściej niezadowoleni ze swoich wyników niż pozostali studenci. Byli oni również rzadko neutralni. Ponadto, jeden na dziesięciu studentów dojrzałych potrafił zdobyć się na pozytywne emocje wobec matematyki pomimo niezadowalenia z wyników na kursach. Studenci niedojrzali niezadowoleni ze swoich wyników nigdy nie wyrażali się pozytywnie o matematyce.

Dość niepokojącym wynikiem naszych badań był brak zainteresowania dużej liczby badanych prawdą matematyczną i rozumowaniem. Piszemy o tym w oddzielnym artykule (w druku w czasopiśmie *Instructional Science*), poświęconym źródłom frustracji u studentów wymaganych kursów, a więc racjonalizacji emocji u studentów. W obecnym artykule skupiamy się na ekspresji emocji i tylko sygnalizujemy wyniki dotyczące źródeł frustracji.