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On some oscillation problems for the equation

$$\Delta^{(n)} u - f(r) u = 0 \text{ in a three-dimensional space}$$

The purpose of this paper is to give some theorems concerning the existence of oscillatory solutions of the equation

$$\Delta^{(n)} u - f(\sqrt{x^2 + y^2 + z^2}) u = 0$$

where $\Delta^{(n)}$ denotes the $n$-times iterated Laplace operator. Our consideration will be based on results contained in [1] and [2]. We give now definitions and theorem which will be used in the sequel.

**Definition 1.** A solution $u(x, y, z) = 0$ of equation (1), defined and of class $C^2$ in an unbounded domain $D$, whose complement $\mathcal{C}D$ is compact, will be called oscillatory if the exterior of every sphere contains a zero of $u$ and the set of the zeros of $u$ has no interior points.

**Definition 2.** A bounded domain $G$ will be called a proper knot domain of the function $u$ if $u(x, y, z) = 0$ in the interior of $G$ and $u(x, y, z) = 0$ on the boundary $F(G)$ of $G$.

**Definition 3.** A solution $y(t) \neq 0$ of the equation

$$\frac{d^n y}{dt^n} + \varphi(t)y = 0, \quad t \geq a > 0,$$

defined and of class $C^n$ in $(a, \infty)$ will be called oscillatory if for every $b > a$ there is at least one zero of $y(t)$ in $(b, \infty)$ and the set of the zeros of $y(t)$ has no accumulation points in $(a, \infty)$.

**Theorem of Ananeva and Balaganskii [1].** If $\varphi(t)$ in equation (2) is positive, if $n$ is even and if

$$\int_a^{\infty} t^{n-2} \varphi(t) dt = \infty,$$

then every solution $y(t) \neq 0$ of equation (2) is oscillatory.

**Theorem of Kondratev [2].** If $n$ is even and $m$ is a positive integer then there exist a function $\varphi(t)$ and two non-trivial solutions $y_1(t)$ and
$y_2(t)$ of the corresponding equation (2) such that between every two successive zeros of $y_1(t)$ there are at least $m$ zeros of $y_2(t)$.

We shall prove the following

**Theorem 1.** If the coefficient $f(r)$ in (1) is positive and if

$$\int_a^\infty r^{2n-2}f(r)dr = \infty,$$

then there exists an oscillatory solution of equation (1).

**Proof.** Let

$$u(x, y, z) = u(\sqrt{x^2 + y^2 + z^2})$$

be a non-constant solution of equation (1) which depends only on $r = \sqrt{x^2 + y^2 + z^2}$. We put $v(r) = ru(r)$. A simple computation shows that the function $u(r)$ satisfies the equation

$$u^{(2n)}(r) + \frac{2n}{r} u^{(2n-1)}(r) + f(r)u(r) = 0,$$

whence it follows that $v(r)$ satisfies the equation

$$v^{(2n)}(r) + f(r)v(r) = 0.$$

According to the theorem of Ananeva and Balaganskii every non-trivial solution $v(r)$ of (4) is oscillatory. Thus the function

$$u(r) = r^{-1}v(r)$$

is an oscillatory solution of equation (1).

**Corollary.** If $r_1 < r_2 < \ldots < r_v < \ldots$ is the sequence of zeros of $v(r)$, then the knot domains of the function $u(r)$ are the spherical rings

$$r_v < r < r_{v+1}, \quad v = 1, 2, \ldots$$

**Theorem 2.** For every positive integer $m$ there exist a coefficient $f(r)$ and two oscillatory solutions $u_1$ and $u_2$ of the corresponding equation (1) such that every knot domain of $u_1$ contains at least $m$ knot domains of $u_2$.

**Proof.** We make the substitution applied in the proof of Theorem 1, and we obtain our result by the theorem of Kondratev.

**References**
