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A Tauberian Theorem for Weighted Means in Non-Archimedean Fields - Revisited and Revised

Abstract. In this note, K denotes a complete, non-trivially valued, non-archimedean field. We correct a Tauberian theorem for weighted means in K proved earlier in [1].

2000 Mathematics Subject Classification: 40.

Key words and phrases: Non-archimedean field, weighted mean (or (\bar{N}, p_n) method), Tauberian theorem.

Throughout this short note, K denotes a complete, non-trivially valued, non-archimedean field. Sequences and series have entries in K . The present note is a sequel to [1]. In the proof of the main result, i.e., Theorem 4 of [1], there is an error towards the end of the proof. It seems that (2.1) is not sufficient to prove Theorem 4 of [1]. We have the following revised form of Theorem 4 of [1]. We suppose that (\bar{N}, p_n) is regular.

THEOREM 4 (revised form) *If $\sum_{k=0}^{\infty} a_k$ is (\bar{N}, p_n) summable to s and if*

$$(1) \quad a_n = O\left(\frac{p_n}{P_n}\right), \quad n \rightarrow \infty;$$

and

$$(2) \quad a_n \rightarrow \ell, \quad n \rightarrow \infty,$$

then $\sum_{k=0}^{\infty} a_k$ converges to s .

PROOF We can suppose that $s = 0$. We now claim that $\ell = 0$. Suppose not. Choose $\epsilon > 0$ such that $\epsilon < |\ell|$. We can now choose a positive integer N such that

$$(3) \quad |s_n| \left| \frac{p_n}{P_n} \right| < \epsilon, \quad n \geq N,$$

using Theorem 3 of [1] and

$$(4) \quad |a_n - \ell| < \epsilon, \quad n \geq N,$$

using (2). Now,

$$\begin{aligned} |a_n| &= |(a_n - \ell) + \ell| = \max(|a_n - \ell|, |\ell|) \\ &= |\ell|, \quad n \geq N. \end{aligned}$$

Also,

$$|a_n| \leq M \left| \frac{p_n}{P_n} \right|, \quad M > 0,$$

in view of (1). Thus, for $n \geq N$,

$$\begin{aligned} |\ell| &\leq M \left| \frac{p_n}{P_n} \right|, \\ \text{i.e., } \left| \frac{p_n}{P_n} \right| &\geq \frac{|\ell|}{M}. \end{aligned}$$

Using (3), for $n \geq N$,

$$\begin{aligned} \epsilon > |s_n| \left| \frac{p_n}{P_n} \right| &\geq |s_n| \frac{|\ell|}{M}, \\ \text{i.e., } |s_n| &< \frac{\epsilon M}{|\ell|}, \quad n \geq N. \end{aligned}$$

In other words, $s_n \rightarrow 0$, $n \rightarrow \infty$ so that $a_n \rightarrow 0$, $n \rightarrow \infty$. Thus $\ell = 0$, which is a contradiction. This contradiction leads to the fact that $\ell = 0$. Consequently $\sum_{k=0}^{\infty} a_k$ converges. Since (\bar{N}, p_n) is regular, $\sum_{k=0}^{\infty} a_k$ converges to 0, completing the proof. ■

REFERENCES

- [1] P.N. Natarajan, *A Tauberian theorem for weighted means in non-archimedean fields, p-adic Numbers*, Ultrametric Analysis and Applications, **1** (2009), 368–369.

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(Received: 7.08.2012)