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An addendum to the paper "Some Characterizations of Schur Matrices in Ultrametric Fields"

Abstract. In this short note, we add a few remarks in the context of [1].

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The present note is a continuation of [1]. We follow the notations and results of [1].

In Theorem 2.7 of [1], we have actually proved that if $A = (a_{nk})$ satisfies (4), (15) of [1] and $\lim_{n\to\infty} a_{nk}$ exists, $k = 0, 1, 2, \ldots$, then A is a Schur matrix. We note here that the converse holds too. To be more specific, we put the result as a theorem and prove it for the sake of completeness. This result gives one more characterization of Schur matrices in the ultrametric set up (see [1], Theorem 2.4).

Theorem 1 $A = (a_{nk})$ is a Schur matrix if and only if

$$\lim_{k\to\infty} a_{nk} = 0, n = 0, 1, 2, \ldots; \tag{1}$$

$$\lim_{n\to\infty} a_{nk} \text{ exists}, k = 0, 1, 2, \ldots; \tag{2}$$

and

$$\lim_{n,k\to\infty} (a_{n+1,k} - a_{nk}) = 0. \tag{3}$$

Proof Sufficiency. Let (1), (2), (3) hold. In view of (1), $(Ax)_n$ is defined for any $x = \{x_k\} \in \ell_\infty$, $n = 0, 1, 2, \ldots$ Again, using (1),

$$\lim_{k\to\infty} (a_{n+1,k} - a_{nk}) = 0, n = 0, 1, 2, \ldots. \tag{4}$$

(3) and (4) together imply that

$$\lim_{k\to\infty} \sup_{n\geqslant 0} |a_{n+1,k} - a_{nk}| = 0. \tag{5}$$

Using (2), we have,

$$\lim_{n\to\infty} (a_{n+1,k} - a_{nk}) = 0, k = 0, 1, 2, \dots . \tag{6}$$

Appealing to (5), (6) and Theorem 2.3 of [1], we have,

$$\lim_{n\to\infty} \{(Ax)_{n+1} - (Ax)_n\} = 0, \quad x = \{x_k\} \in \ell_\infty,$$

i.e., $\{(Ax)_n\}$ is a Cauchy sequence in K. Since K is complete, $\{(Ax)_n\} \in c$, $x = \{x_k\} \in \ell_\infty$, i.e., A is a Schur matrix.
Necessity. Let A be a Schur matrix. It is clear that (1) and (2) are necessary by considering the A-transforms of the sequences $\{1, 1, 1, \dots\}$ and $\{0, 0, \dots, 0, 1, 0, \dots\}$, 1 occurring in the kth place, $k = 0, 1, 2, \dots$. Since A is a Schur matrix,

$$\lim_{n+k\to\infty} (a_{n+1,k} - a_{nk}) = 0, \tag{7}$$

using Theorem 2.4 of [1]. Note that (7) implies (3), using Theorem 2.2 of [1] so that (3) is necessary too, completing the proof of the theorem. ∎

References

[1] P.N. Natarajan, *Some characterization of Schur matrices in ultrametric fields*, Comment. Math. Prace Mat. **52** (2012), 137–142.

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