

BEATRIZ PORRAS POMARES* (Madrid)

A note on the representing measures of operators defined on spaces of vector-valued continuous functions

Abstract. We prove that when E^* is the dual of a Banach space which does not contain a subspace isomorphic to l_1 , then for any compact Hausdorff space K a subset M of Borel regular countably additive bounded variation E^* -valued measures on K ($M \subset \text{rcabv}(\beta(K), E^*)$) is weakly conditionally compact if and only if it is uniformly strongly bounded. Using this result we study some properties of the representing measures of linear bounded operators defined on spaces of vector-valued continuous functions.

Let K be a compact Hausdorff space, and let E, F be Banach spaces. We denote by $C(K, E)$ the space of all continuous E -valued functions, defined on K , with the supremum norm (we shall denote by $C(K)$ the space $C(K, \mathbf{K})$, when E is the scalar field \mathbf{K}); $\beta(K)$ is the Borel σ -algebra of K , and we denote by $B(\beta(K), E)$ the space of those functions which are uniform limits of E -valued measurable simple functions, with the supremum norm (and we shall denote it by $B(\beta(K))$ if E is the scalar field \mathbf{K}).

Given a compact space K , $\text{cabv}(\beta(K))$ will be the space of countably additive scalar measures of bounded variation defined on $\beta(K)$; and for a Banach space E , $\text{rcabv}(\beta(K), E)$ will be the space of regular countably additive E -valued measures of bounded variation, defined on $\beta(K)$.

It is well known that every (continuous linear) operator T from $C(K, E)$ to F has a finitely additive representing measure m , of bounded semivariation $|m|$, defined on the Borel σ -algebra of K , and with values in $\mathcal{L}(E, F^{**})$ (where F^{**} is the second dual of F), in such a way that

$$T(f) = \int f dm \quad \text{for each } f \in C(K, E).$$

One of the topics in the study of operators defined on spaces of vector-valued continuous functions is to characterize classes of operators by means of the properties of their representing measures.

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Many of the properties of an operator T from $C(K, E)$ to F are shared by its representing measure, in the sense that $T \in \Psi(C(K, E), F)$ for an operator ideal Ψ implies that its representing measure m satisfies $m(A) \in \Psi(E, F)$ for all $A \in \beta(K)$.

For many operator ideals it is also true that T belonging to $\Psi(C(K, E), F)$ implies that its representing measure is continuous at \emptyset (or equivalently, m is s -bounded) (see for instance [1], [2], [4]).

Conversely, if K is a metrizable non-dispersed compact space, these two properties characterize operators belonging to $\Psi(C(K, E), F)$ when Ψ is the ideal of weakly compact operators, unconditionally convergent operators, Dunford–Pettis operators, Dieudonné operators, strictly singular operators, and some others (see [5]). But this is not true for all compact spaces ([3]).

Trying to characterize these operator ideals independently of the compact space K , for all the cases mentioned above one can reduce the scope to operators defined on spaces $C(K, E)$ with K a metrizable compact space: for those ideals, $T \in \Psi(C(K, E), F)$ if and only if for every metrizable quotient \hat{K} of K , the induced operator $\hat{T} = T \circ \hat{\pi}$ belongs to $\Psi(C(\hat{K}, E), F)$ ($\hat{\pi}$ is defined by $\hat{\pi}(f) = f \circ \pi$, where π is the quotient map from K to \hat{K}).

Thus, in order to obtain a complete characterization of those classes of operators $\Psi(C(K, E), F)$, for all compact spaces K , in terms of their representing measures, it remains to study the case when K is a non-dispersed compact space.

It is a classic result that for every non-dispersed compact space K there is an onto map from K to the unit interval $[0, 1]$. In this case, for every Banach space E , $C([0, 1], E)$ is a closed subspace of $C(K, E)$. In [6] Bombal and Rodríguez-Salinas studied the general case, when K and S are compact spaces such that there is an onto map from K to S , obtaining some results about the relations between the representing measures of operators defined on $C(K, E)$ and the induced operators on $C(S, E)$, and some results about the extensions of operators defined on $C(S, E)$ to the whole $C(K, E)$ with similar properties.

Another classical result of Milyutin asserts that for every metrizable non-dispersed compact space K , $C(K)$ is isomorphic to $C([0, 1])$. In this note we study the general case, when K and S are two compact spaces such that $C(K)$ and $C(S)$ are isomorphic.

Consider two compact Hausdorff spaces K and S such that the spaces $C(K)$ and $C(S)$ are isomorphic. Let $I: C(K) \rightarrow C(S)$ be an isomorphism. For every Banach space E , I induces an isomorphism between the tensor products $C(K) \otimes E$ and $C(S) \otimes E$, which extends to an isomorphism \hat{I} between $C(K, E)$ and $C(S, E)$.

Now, any operator T from $C(K, E)$ to a Banach space F defines an operator T_1 from $C(S, E)$ to F by the formula $T_1 = T \circ \hat{I}^{-1}$. We study

the relations between the representing measures m and m_1 of T and T_1 respectively, focusing on the following properties: $|m|$ being continuous at \emptyset , and $m(A)$ belonging to $\Phi(E, F)$ for some operator ideal Φ and for every Borel set A in K .

In general we cannot say that m and m_1 have to enjoy similar properties, in the sense that m being of semivariation continuous at \emptyset implies that the semivariation of m_1 must be continuous at \emptyset , or that $m(A)$ being in $\Phi(E, F)$ for some operator ideal Φ and every Borel set A in K implies that $m_1(B)$ must be in $\Phi(E, F)$ for every Borel set B in S . But it is possible to have some results upon imposing some conditions on the spaces E or F .

The following theorem is a partial result towards a characterization of weakly conditionally compact subsets of $\text{rcabv}(\beta(K), E)$, the space of Borel regular countably additive E -valued measures on K , which generalizes to the vector case the result for scalar measures.

A subset M of a Banach space is *weakly conditionally compact* if every sequence in M has a weakly Cauchy subsequence ([9]). A subset M of $\text{rcabv}(\beta(K), E)$ is *uniformly strongly bounded* if for every sequence $\{A_n\}$ of pairwise disjoint Borel subsets of K , the sequence $\{m(A_n)\}$ tends to 0 uniformly in $m \in M$.

THEOREM 1. *Let K be a compact Hausdorff space. Let E^* be the dual of a Banach space E such that E^* does not contain a subspace isomorphic to l_1 . A subset M of $\text{rcabv}(\beta(K), E^*)$ is weakly conditionally compact if and only if it is uniformly strongly bounded.*

Proof. M being weakly conditionally compact always implies that M is uniformly strongly bounded (see [3]).

Thus, let $M \subset \text{rcabv}(\beta(K), E^*)$ be uniformly strongly bounded and suppose that M is not weakly conditionally compact. Rosenthal's Theorem in l_1 ensures that there is a sequence $\{m_n\}_{n \in \mathbb{N}}$ in M equivalent to the canonical basis of l_1 . Thus, there exist positive constants l, L such that for any finite family of scalars $\{a_1, \dots, a_n\}$ we have

$$(1) \quad l \sum_{i=1}^n |a_i| \geq \left\| \sum_{i=1}^n a_i m_i \right\| \geq L \sum_{i=1}^n |a_i|$$

$M \subset \text{rcabv}(\beta(K), E^*)$ being uniformly strongly bounded, there is a regular positive Borel measure μ in K such that M is uniformly μ -continuous. Hence, we can choose, for each $n \in \mathbb{N}$, a density function $g_n: K \rightarrow E^*$ for m_n such that

- $\langle x, g_n \rangle$ is μ -integrable, for all $x \in E$ and all $n \in \mathbb{N}$.
- $\langle x, m_n(A) \rangle = \int_A \langle x, g_n \rangle d\mu$ for all $x \in E$ and all $A \in \beta(K)$.
- $|m_n|(A) = \int_A \|g_n(t)\| d\mu(t)$ for all $A \in \beta(K)$.
- If ϱ is a lifting in $\mathcal{L}^\infty(\mu)$, for each μ -continuous measure in $\text{rcabv}(\beta(K), E^*)$,

we can choose the density g uniquely (almost everywhere) so that $\varrho[g] = g$ (see [10], Th. 13.5).

Let $\{a_1, \dots, a_n\}$ be a finite family of scalars. The function $g = \sum_{i=1}^n a_i g_i$ satisfies

- $\langle x, g \rangle$ is μ -integrable.
- For every $x \in E$ and every $A \in \beta(K)$,

$$\langle x, \sum_{i=1}^n a_i m_i(A) \rangle = \sum_{i=1}^n a_i m_i(A)(x) = \sum_{i=1}^n \int a_i \langle x, g_i \rangle d\mu = \int \langle x, \sum_{i=1}^n a_i g_i \rangle d\mu.$$

- We have

$$\varrho[\sum_{i=1}^n a_i g_i] = \sum_{i=1}^n a_i \varrho[g_i] = \sum_{i=1}^n a_i g_i = g.$$

Thus, g is the density function of the measure $\sum_{i=1}^n a_i m_i$, and the inequality (1) takes the form

$$(2) \quad l \sum_{i=1}^n |a_i| \geq \int \left\| \sum_{i=1}^n a_i g_i(t) \right\| d\mu(t) \geq L \sum_{i=1}^n |a_i|.$$

On the other hand $\{g_n\}_{n \in \mathbb{N}}$ is uniformly integrable, since $\int_A \|g_n\| d\mu = |m_n|(A)$ for every $A \in \beta(K)$ and every $n \in \mathbb{N}$, and $\lim_{\mu(A) \rightarrow 0} |m_n|(A) = 0$ uniformly in $n \in \mathbb{N}$ (μ was a uniform control measure for $m \in M$). In particular,

$$\sup_{n \in \mathbb{N}} \left\{ \int_{\{t: \|g_n(t)\| \geq a\}} \|g_n(t)\| d\mu(t) \right\} \rightarrow 0 \quad \text{as } a \rightarrow \infty.$$

Thus, there exists $a \in \mathbb{R}$ such that for every $n \in \mathbb{N}$

$$\int_{\{t: \|g_n(t)\| \geq a\}} \|g_n(t)\| d\mu(t) \leq L/2.$$

We now define $h_n = g_n \chi_{\{t: \|g_n(t)\| < a\}}$; h_n is the weak density of the E^* -valued measure $m_{n,a}$ defined by $m_{n,a}(B) = m_n(B \cap \{t \in K, \|g_n(t)\| \leq a\})$ for every Borel set $B \in \beta(K)$. Then

- h_n are uniformly bounded when n runs over \mathbb{N} .
- For every finite sequence a_1, \dots, a_n of scalars, $\sum_{i=1}^n a_i h_i$ is the density function of the E^* -valued measure $\sum_{i=1}^n a_i m_{i,a}$.
- We have

$$(3) \quad \begin{aligned} \int_K \left\| \sum_{i=1}^n a_i h_i(t) \right\| d\mu &\leq \sum_{i=1}^n |a_i| \int_K \|h_i(t)\| d\mu \\ &= \sum_{i=1}^n |a_i| \int_{\{t: \|g_n(t)\| < a\}} \|g_i(t)\| d\mu \\ &\leq l \cdot \sum_{i=1}^n |a_i|. \end{aligned}$$

• In the other direction,

$$\begin{aligned}
 (4) \quad \int_K \left\| \sum_{i=1}^n a_i h_i(t) \right\| d\mu &= \int_K \left\| \sum_{i=1}^n a_i g_i(t) - \sum_{i=1}^n a_i (g_i(t) - h_i(t)) \right\| d\mu \\
 &\geq \int_K \left\| \sum_{i=1}^n a_i g_i(t) \right\| d\mu - \int_K \left\| \sum_{i=1}^n a_i (g_i(t) - h_i(t)) \right\| d\mu \\
 &\geq \int_K \left\| \sum_{i=1}^n a_i g_i(t) \right\| d\mu - \sum_{i=1}^n |a_i| \cdot \int_K \|g_i(t) - h_i(t)\| d\mu \\
 &= \int_K \left\| \sum_{i=1}^n a_i g_i(t) \right\| d\mu - \sum_{i=1}^n |a_i| \cdot \int_{\{t: \|g_i(t)\| \geq a\}} \|g_i(t)\| d\mu \\
 &\geq (L - L/2) \cdot \sum_{i=1}^n |a_i| = (L/2) \cdot \sum_{i=1}^n |a_i|
 \end{aligned}$$

We will now use a theorem due to Bourgain (Theorem 4 of [7]): Let X be the space c_{00} , and $\{e_n\}$ its canonical basis. Consider for each t in K the seminorm in X defined by

$$|x|_t = \left\| \sum_{i=1}^n x_i h_i(t) \right\|$$

where $x = (x_1, \dots, x_n, 0, \dots) \in c_{00}$. Then for all x in X , the function $t \rightarrow |x|_t$ is measurable, and

$$\sup_{i \in \mathbb{N}, t \in K} |e_i|_t = \sup_{i \in \mathbb{N}, t \in K} \|h_i(t)\| \leq a.$$

Also, if we consider in X the seminorm

$$|x| = \int_K |x|_t d\mu = \int_K \left\| \sum_{i=1}^n x_i h_i(t) \right\| d\mu$$

we deduce from (3) and (4) that $\{e_i\}$ is equivalent to the canonical basis of l_1 for the seminorm $|\cdot|$. Bourgain's theorem implies that there is an element t in K such that $\{e_i\}$ is equivalent to the canonical basis of l_1 for the seminorm $|\cdot|_t$. That is, there are t in K and positive constants r, R such that for any finite family of scalars $\{a_1, \dots, a_n\}$,

$$r \sum_{i=1}^n |a_i| \geq \left| \sum_{i=1}^n a_i e_i \right|_t = \left\| \sum_{i=1}^n a_i h_i(t) \right\| \geq R \sum_{i=1}^n |a_i|.$$

Thus, $\{h_i(t)\}_{i \in \mathbb{N}}$ in E^* is equivalent to the canonical basis of l_1 . This contradicts the hypothesis on E , and finishes the proof. ■

Remark. If E^* has the Radon–Nikodym property, the above result is a consequence of the criterion of weak compactness in $L^1(\mu, E^*)$ given by Bourgain (see [7]).

The James tree space JT provides an example of a Banach space such that every even dual has the Radon–Nikodym property, but no odd dual does. Thus, JT^* has neither the Radon–Nikodym property, nor a subspace isomorphic to l_1 (see [9]).

We shall use the theorem above and the following lemma to obtain some properties of the representing measures.

LEMMA 2. *Let K and S be compact spaces, and $I: C(K) \rightarrow C(S)$ an isomorphism. Let E and F be Banach spaces, and \hat{I} the isomorphism induced by I between $C(K, E)$ and $C(S, E)$. Let $T: C(K, E) \rightarrow F$ be an operator, $T_1 = T \circ \hat{I}^{-1}$, and let m, m_1 be the representing measures of T and T_1 respectively. Then, if m takes values in $\mathcal{L}(E, F)$, so does m_1 .*

Proof. Indeed, m takes values in $\mathcal{L}(E, F)$ if and only if for every $x \in E$ the operator $T_x: C(K) \rightarrow F$ defined by $T_x(\varphi) = T(\varphi \cdot x)$ for all $\varphi \in C(K)$ is weakly compact ([8], Th. 4.4). In our case, for every $x \in E$ the operator T_{1x} is defined by

$$T_{1x}(\varphi) = T_1(\varphi x) = T(\hat{I}^{-1}(\varphi x)) = T(I^{-1}(\varphi)x) = T_x(I^{-1}(\varphi)) = T_x \circ I^{-1}(\varphi)$$

and thus T_{1x} is also weakly compact. Hence, m_1 takes values in $\mathcal{L}(E, F)$. ■

THEOREM 3. *Let K, S, E, I, \hat{I} be as in the lemma. If E^* does not contain l_1 , then for every Banach space F and every operator T from $C(K, E)$ to F whose representing measure has semivariation continuous at \emptyset , the representing measure m_1 of the composed operator $T_1 = T \circ \hat{I}^{-1}$ has semivariation continuous at \emptyset .*

Proof. Let m be the representing measure of T and m_1 the representing measure of T_1 . $|m|$ is continuous at \emptyset if and only if (see [3]) m takes values in $\mathcal{L}(E, F)$, and the set $\{|m_{y^*}|: y^* \in B_{F^*}\}$ is uniformly strongly bounded in $\text{cabv}(\beta(K))$, or, equivalently, the set $\mathcal{M} = \{m_{y^*}: y^* \in B_{F^*}\}$ is uniformly strongly bounded in $\text{cabv}(\beta(K), E^*)$.

In this case, since E^* does not contain l_1 , thanks to Theorem 1 we know that the semivariation of m is continuous at \emptyset if and only if

- (i) m takes values in $\mathcal{L}(E, F)$, and
- (ii) $\mathcal{M} = \{m_{y^*}: y^* \in B_{F^*}\}$ is weakly conditionally compact,

and the analogous conditions are valid for m_1 .

From (i) and Lemma 2, m_1 also takes values in $\mathcal{L}(E, F)$. So it remains to prove that \mathcal{M} being weakly conditionally compact in $\text{cabv}(\beta(K), E^*)$ implies that $\mathcal{M}_1 = \{m_{1y^*}: y^* \in B_{F^*}\}$ is weakly conditionally compact in $\text{cabv}(\beta(S), E^*)$.

For every $y^* \in B_{F^*}$, m_{1y^*} is the representing measure associated to $T_1^* y^*$ by the isometry defined between $C(S, E)^*$ and $\text{rcabv}(\beta(S), E^*)$, and thus it is enough to prove that $\{T_1^* y^*: y^* \in B_{F^*}\}$ is weakly conditionally compact. Since $T_1^* y^* = (\hat{I}^{-1})^* \circ T^*(y^*)$ for all $y^* \in F^*$, the result holds immediately from the same condition on m . ■

Remark. The condition “ E^* not containing l_1 ” transforms the problem from measure theory to topology, changing the condition of $|m|$ being continuous at \emptyset to a topological condition, more manageable from the point of view of the isomorphism between $C(K)$ and $C(S)$.

In any case, we have:

THEOREM 4. Let K, S, I be as in the lemma, E a Banach space, and let \hat{I} be the isomorphism from $C(K, E)$ to $C(S, E)$ induced by I . Let T be an operator from $C(K, E)$ to a Banach space F , and $T_1 = T \circ \hat{I}^{-1}$, and let m, m_1 be the representing measures of T and T_1 respectively. If m and m_1 have semivariation continuous at \emptyset , then, for every operator ideal Φ closed for the usual norm in $\mathcal{L}(E, F)$, the following assertions are equivalent:

- (i) For all A in $\beta(K)$, the operator $m(A)$ from E to F is in $\Phi(E, F)$.
- (ii) For all B in $\beta(S)$, the operator $m_1(B)$ from E to F is in $\Phi(E, F)$.

Proof. Suppose that $m(A) \in \Phi(E, F)$ for every $A \in \beta(K)$. Since m_1 has semivariation continuous at \emptyset , T_1 has a strong extension, in the sense that for every Borel set B in $\beta(S)$

$$m_1(B) = \lim_{G \in \Pi(B)} m_1(G) = \lim_{C \in \Pi_0(B)} m_1(C)$$

in the norm of $\mathcal{L}(E, F)$, where $\Pi(B)$ is the class of open sets containing B , and $\Pi_0(B)$ the class of compact sets contained in B (see, for instance, [1] and [3]).

At the same time,

$$m_1(C)(x) = \lim_{G \in \Pi(C)} T_1(\varphi_G \cdot x)$$

in the norm of F , uniformly in $x \in B_E$, where φ_G is a continuous function with support in G such that $\varphi_G(t) = 1$ for all $t \in C$, and $0 \leq \varphi_G(t) \leq 1$ for all t .

For each x in E , and each G in $\Pi(C)$,

$$\hat{I}^{-1}(x \cdot \varphi_G) = x \cdot I^{-1}(\varphi_G) \in C(K, E).$$

Consider a sequence $\{S_n\}$ of measurable simple functions such that $\|S_n - I^{-1}(\varphi_G)\| \rightarrow 0$ in $B(\beta(K))$ when $n \rightarrow \infty$. We have

$$T_1(x \cdot \varphi_G) = T(\hat{I}^{-1}(x \cdot \varphi_G)) = T(x \cdot I^{-1}(\varphi_G)) = \lim_n T^{**}(x \cdot S_n)$$

in the norm of F , and uniformly in $x \in B_E$. If we write $S_n = \sum_{i=1}^{k_n} a_i^n \chi_{B_i^n}$ with B_i^n in $\beta(K)$ for all $n \in \mathbb{N}$, and all $i \in \{1, \dots, k_n\}$, then

$$T_{S_n}^{**}(x) = T^{**}(x \cdot S_n) = \sum_{i=1}^{k_n} a_i^n m(B_i^n)(x) = \left(\sum_{i=1}^{k_n} a_i^n m(B_i^n) \right)(x)$$

and by hypothesis (i) the operator $\sum_{i=1}^{k_n} a_i^n m(B_i^n)$ is in $\Phi(E, F)$. Thus, the

operator

$$T_{1G}: E \rightarrow F, \quad x \rightarrow T_1(x \cdot \varphi_G),$$

which is the limit in norm of the operators $T_{S_n^{**}}$, must be in $\Phi(E, F)$. In the same way, $m_1(C) \in \Phi(E, F)$, since it is the limit in norm of the operators T_{1G} when G runs over $\Pi(C)$. Finally, $m_1(B) \in \Phi(E, F)$, since it is the limit of $m_1(C)$ when C runs over $\Pi_0(B)$. ■

As a consequence, we have the following corollary which generalizes in some way the results known for weakly compact and Dunford–Pettis operators (proved by P. Cembranos in her PhD thesis *Algunas propiedades del espacio de Banach $C(K, E)$* , Universidad Complutense de Madrid, 1984).

COROLLARY 5. *Let Ψ be an operator ideal satisfying:*

- Ψ is closed for the usual operator norm.
- For every compact space K , and for every pair of Banach spaces E, F , an operator T from $C(K, E)$ to F belongs to $\Psi(C(K, E), F)$ if and only if for any metrizable quotient \hat{K} of K the induced operator \hat{T} from $C(\hat{K}, E)$ to F belongs to $\Psi(C(\hat{K}, E), F)$.

Then, if E^* does not contain l_1 or F does not contain c_0 , the following assertions are equivalent:

(i) Every operator $T: C([0, 1], E) \rightarrow F$ whose representing measure m satisfies

- (a) $|m|$ is continuous at \emptyset ,
- (b) for all $A \in \beta([0, 1])$ the operator $m(A)$ belongs to $\Psi(E, F)$,

belongs to $\Psi(C([0, 1], E), F)$.

(ii) For every compact space K , every operator $T: C(K, E) \rightarrow F$ whose representing measure m satisfies

- (a) $|m|$ is continuous at \emptyset ,
- (b) for all $A \in \beta(K)$ the operator $m(A)$ belongs to $\Psi(E, F)$,

belongs to $\Psi(C(K, E), F)$.

Proof. Obviously, (ii) implies (i). Now, suppose (i) and let K be a compact space, and T an operator from $C(K, E)$ to F satisfying (ii)(a) and (ii)(b). Let \hat{K} be a metrizable quotient of K , and π the quotient map.

If \hat{K} is not dispersed $C(\hat{K}, E)$ is isomorphic to $C([0, 1], E)$, and if \hat{T} is the induced operator in $C(\hat{K}, E)$, $\hat{T} = T \circ \pi$, and \hat{T}_1 the operator induced by \hat{T} and the isomorphism between $C(\hat{K}, E)$ and $C([0, 1], E)$ in this space, we have:

- If F does not contain c_0 , the representing measures \hat{m} and \hat{m}_1 of \hat{T} and \hat{T}_1 respectively both have semivariation continuous at \emptyset .
- If E^* does not contain l_1 , $|m|$ being continuous at \emptyset implies that $|\hat{m}|$ is

continuous at \emptyset (since $\hat{m}(A) = m(\pi^{-1}(A))$) (see for instance [4]), and by Theorem 3, $|\hat{m}_1|$ is also continuous at \emptyset .

In any case, we can apply Theorem 4, and (ii)(b) implies (i)(b).

Now, (i) implies that \hat{T}_1 belongs to $\Psi(C([0, 1], E), F)$, and then \hat{T} belongs to $\Psi(C(\hat{K}, E), F)$, since Ψ is an operator ideal.

If \hat{K} is dispersed, it is an immediate consequence of Th. 1.8 of [5] that \hat{T} belongs to $\Psi(C(\hat{K}, E), F)$. The hypothesis on Ψ implies that T belongs to $\Psi(C(K, E), F)$, which finishes the proof. ■

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DEPARTAMENTO DE ANÁLISIS MATEMÁTICO, UNIVERSIDAD COMPLUTENSE DE MADRID
MADRID, SPAIN

Current address:

DEPARTAMENTO DE MATEMÁTICAS, ESTADÍSTICA Y COMPUTACIÓN
FACULTAD DE CIENCIAS
UNIVERSIDAD DE CANTABRIA
AVENIDA DE LOS CASTROS S/N
39071 CANTABRIA, SPAIN