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# Anti-self dual groups

Let $G$ be a commutative topological group and $\hat{G}$ its dual (the group of all characters of $G$ with compact open topology). $G$ is called *anti-self dual* if the only continuous homomorphism of $G$ into $\hat{G}$ is the constant homomorphism and conversely. The main theorem of Armacoast [1] is that each LCA group $G$ is not anti-self dual unless $G$ is a singleton. We give here an easy proof of this result.

THEOREM. *An LCA group $G$ which is not a singleton is not anti-self dual.*

Proof. Let the dual of $G$ be $\hat{G}$. Let 0 denote the 0-elements of $G$ and $\hat{G}$ and also the constant homomorphisms between any two groups. If $G = G_1 \times R^n$ with $n \neq 0$ and $G_1$ LCA group, then $\hat{G} = \hat{G}_1 \times R^n$ and so the map $0 \times \mathrm{id}\colon G_1 \times R^n \to \hat{G}_1 \times R^n$ is a non-zero continuous homomorphism of $G$ into $\hat{G}$. So we can assume that $G$ contains a compact open subgroup $H$. Let $G_0$ be connected component of 0 in $G$ and $L = G_0^{\perp} \subset \hat{G}$. Let $G_0 \neq \{0\}$. Then $\exists\, x_0 \in G$ of infinite order and $\hat{G}_0/L$ contains an element $y_0$ of $\infty$ order. So there is an isomorphism $\varphi\colon [x_0] \to [y_0]$, where $[a]$ denotes the cyclic group generated by $'a'$. Since $G_0$ is divisible $\varphi$ extends to a homomorphism $\varphi_1\colon \hat{G}_0/L \to G_0$. So $\varphi_1 \circ \lambda\colon \hat{G} \to G$ is a non-zero continuous homomorphism from $\hat{G} \to G$, where $\lambda\colon \hat{G} \to \hat{G}/L$ is canonical map. So the problem reduces to the case when both $G$ and $\hat{G}$ are totally disconnected. Let $\hat{G}$ have an element $\chi$ of finite order $n \neq 1$. Let $H \subset G$ be the kernel of $\chi$. Then there is an isomorphism $T\colon G/H \to [\chi]$. So $T \circ \lambda\colon G \to \hat{G}$ is a non-zero continuous homomorphism, where $\lambda\colon G \to G/H$ is the canonical map. So the problem reduces to the case when both $G$ and $\hat{G}$ are totally disconnected and torsion free. Then $G = J_p \times G_1$ for some LCA group $G_1$ and for some prime $p$, where $J_p$ denotes the LCA group of $p$-adic numbers (see Theorem 12 of [3] and Lemma 7 of [4]). Then $\hat{G} = J_p \times \hat{G}_1$ and so the map $\mathrm{id} \times 0\colon J_p \times G_1 \to J_p \times \hat{G}_1$ is a non-zero continuous homomorphism. This proves the theorem.

**References**

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