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A Simple Pole in Ithaca, NY

In last words of this memorize the Author has written „The lives of my generation of Americans have been greatly enriched by the refugees about whom Laura Fermi wrote in her book *Illustrious Immigrants*. Very few mathematicians appear in her book, but Kac is one of them. She devotes only a sentence to him, but that sentence is a fitting tribute. What she says is something to the effect that Kac is the mathematician who is known by people who don't know any mathematicians. And, thirty years after his death, he still is."

Mark Kac was born on the eve of the First World War in the czarist Russian city of Krzemieniec i. His family were middle class Jews, and his father was a well educated intellectual with a Ph.D. in philosophy and a wide range of interests, including mathematics. At age 16, Kac found an elegant derivation of Cardano's formula for the solution to cubic equations, and this achievement won him recognition that launched his mathematical career[ii]. In 1931, he was admitted to The University of Lwów[iii], home of Poland's renowned school of analysis. The inter-bellum period from 1918 to 1939 was the golden era of Polish mathematics, and Lwów, together with Warsaw, were the centers most responsible for its success.

When Kac began his thesis program, he had a host of famous mathematicians to choose from. The one whom he chose was Hugo Steinhaus[iv]. Steinhaus was a gifted mathematician, but I do not think that was Kac's only reason for working with him. Instead, I believe that Kac was attracted by Steinhaus' remarkable combination of erudition, humor, and good sense. At the time, Steinhaus was trying to gain a deeper understanding of the concept of statistical independence. Kolmogorov's seminal treatise on the foundations probability theory had not appeared yet, and mathematicians were still undecided about the mathematical meaning of the subject. As one of the first people to recognize and exploit the power of Lebesgue's integration theory, Steinhaus, like Kolmogorov, understood the role that Lebesgue's theory should play in providing a

mathematical foundation for probability theory. However, unlike Kolmogorov, he adopted a more down to earth approach. Namely, rather than couch his ideas in terms of abstract measure spaces, he chose to do everything in terms of Lebesgue measure on the interval $[0, 1]$. From the perspective of people like me who learned their probability theory after Kolmogorov's approach had become dominant, Steinhaus's formulation seems awkward. On the other hand, its very awkwardness forced Steinhaus and Kac to think about questions that would not occur to disciples of Kolmogorov. It is not so easy to find infinite sequences of mutually independent random variables on the unit interval. Steinhaus may have been the first to realize that Rademacher functions provide an example and that Rademacher's theorem about the almost everywhere convergence of Rademacher series can be proved more easily when one takes this observation into account. Starting from the Rademacher functions, Steinhaus and Kac showed how to construct a host of other families of mutually independent random variables[1] I believe it was this background that led Kac to formulate the theorem, which appears in his famous paper with Erdös [1] about the statistics of prime divisors, and I am certain that his early search for mutually independent random variables contributed to the delight of his much loved monograph *Statistical Independence*.

As he makes clear in his autobiography, mathematics was not Kac's only activity in Lwów. He was young, and his infectious charm more than compensated for whatever he lacked in the way of physical attributes. The chapter he devotes to his early romantic experience is a beautiful portrayal of the miraculous capacity of human beings to find joy even in a world that was collapsing around them. Having grown up as a Jew in Eastern Europe, Kac did not expect life to be a bed of roses. His parents had experienced pogroms, and he frequently encountered antisemitism. However the cloud hanging over Poland threatened something more serious than a local pogrom or randomly inflicted cruelty. I don't know whether Kac anticipated the maniacal extermination campaign that the Nazis would visit on Poland, but he had no doubt that the world as he knew it was coming to an end. Nonetheless, he fell in love. He doesn't go into details, but he makes it clear that young love is possible and maybe more precious in dire circumstances. Kac

[1] I use the term *mutually independent* instead of the commonly used *independent* in honor of Kac. Kac's father had instilled in him a deep love and respect for language, and imprecision offended him. As he pointed out to me, saying that random variables are *independent* is meaningless unless one specifies that what they are independent of is one another.

may have been in love, but he also had strong survival instincts. Thus he made repeated applications for traveling fellowships, and when he received an offer of one from the Rockefeller Foundation, he did not hesitate before accepting it. In 1938, Kac left Poland, and by the time that he returned more than twenty years later, his parents had been slaughtered, Lwów had been given to Stalin in return for Wrocław[v], and Poland's former Jewish population had all but disappeared. Kac would spend the rest of his life trying to reconcile his gratitude for having been rescued with his feelings of guilt about having survived.

Kac's fellowship was for a one-year post-doctoral position at Johns Hopkins University in Baltimore. He knew essentially no English. In fact, when he submitted his first paper written in English to the Journal of the London Mathematical Society, G. H. Hardy accepted it on the condition that he resubmit it in French. Thus, much of his first year was spent in mastering the English language, and master it he did. Indeed, there are few mathematicians who have the command of any language that Kac eventually had of English. His major mathematical accomplishment during that year was the one in the paper with Erdös about prime divisors. The following year he moved to Cornell University in Ithaca NY, which would be his home university for the next twenty two years. During the years that he spent there, Cornell would become the center of probability theory in America. At various times, Kac would be joined there first by William Feller and Gilbert Hunt, both of whom were subsequently stolen by Princeton, and later by Frank Spitzer and Kac's student Harry Kesten. However, when Kac arrived, the colleague who most fascinated him was not a mathematician but instead the physicist Richard Feynman. Feynman had been asked to give some lectures on his pathspace approach to solving the Schrödinger equation. From a rigorous mathematical standpoint, much of what Feynman said was, and still is, nonsense. Nonsense it might be, but Kac recognized that there was genius behind it. In particular, he realized that if one replaces Schrödinger's equation with the heat equation, then what Feynman was doing could be made rigorous. Thus was born what is now called the *Feynman–Kac formula*. Excited to test the formula, Kac at first applied it to computations that he could check by other means. Checking them turned out to involve the theory of special functions and required him to delve into Watson's book on Bessel functions ([5]). Kac once told me that he spent a miserable night because a formula in Watson's book seemed to contradict his own computation. Only after hours of painstaking work did he realize that Watson had

failed to specify the lower limit of integration in his formula and that the contradiction disappeared when the lower limit was the one that Kac had incorrectly assumed it was.

Having never been at Cornell when Kac was there, my impression of his years in Ithaca is based entirely on what he and others have told me. Cornell and the town of Ithaca are friendly communities that have served as a refuge for a host of immigrants escaping Hitler. By the end of the WWII, Kac knew that he had no living relatives in Poland, and he was a man who craved human contact. In Ithaca, Kac found a wife, raised two children, and became a much admired figure. So far as I know, Kac's marriage was a success, the only bone of contention being that his wife was a better bridge player than he. Life at the university was satisfying both intellectually and socially. Evidence of the collegiality in the mathematics department can be found in Feller's famous probability book ([2]). According to the index, there is a reference to Kac on page 55, but page 55 turns out to contain only exercises and no obvious reference to Kac. However, on closer inspection, one realizes that problem 13 is about Kac. That problem describes a Cornell professor who received 12 parking tickets and was trying to determine whether it would cost him more to continue parking illegally or to rent a parking space. Kac was also a beloved teacher who is responsible for deflecting the paths of several students from medicine or law into mathematics. His standards were high, but he never used his own mathematical abilities to bludgeon others. For example, the title of this talk derives from a story I heard about an oral exam in complex analysis that Kac gave to a weak graduate student. Kac asked the student to describe the singularity of a certain function at the origin. The student hemmed and hawed, but could not come up with the answer. Taking mercy, Kac asked him „what am I?", the correct answer being „a simple pole."

Sometime in the 1950's, Kac's interests began to shift away from mathematics for its own sake toward mathematical physics. Not surprisingly, the topic there that attracted him most was statistical mechanics. Initially he worked on generalizations of the Ising model, but his most famous contribution to the field was what is called the *Kac gas*, a model that Kac showed is exactly solvable. Like other mathematically rigorous models of gases, this one is physically unrealistic. On the other hand, unlike most mathematical models, it is sufficiently realistic that physicists put some credence in it.

In 1961, Kac was asked to create a mathematics department at The

Rockefeller Institute. The offer appealed to him for many reasons. From a scientific standpoint, it would bring him into immediate contact with several physicists, like George Uhlenbeck and T.H. Berlin, who were doing work related to his own. But that is not the only, and maybe not even the most important, reason why he took the job. Ithaca is often called the „the most centrally located, isolated spot on earth". My son is a Cornell professor now, and I can attest that Ithaca is between four and six hours from everywhere else. Kac was restive. He was a gifted expositor an loved the adulation that his gift won him, and as long as he stayed in Ithaca, he would never satisfy his craving for that adulation. New York is a city to which people who need celebrity gravitate.

Whatever were the reasons for Kac going there, I am eternally grateful that he did. I went to Rockefeller in the fall of 1962 as a graduate student in neurophysiology. My decision was an act of pure hubris. I had not taken a biology class since high school. I had no reason to believe that I was cut out to be a biologist, and, as I soon found out, I was not. For a year and a half, I attempted, without success, to insert a microelectrode into the optic nerve of a crab. This would have been a devastating experience if I had not discovered that Rockefeller had acquired a mathematician. Every Friday afternoon, I and another student (who has since become a respected biologist) went to Kac's office for a tutorial. As an undergraduate, I had taken quite a few mathematics courses at Harvard. I got a good deal out of those courses, but they had left me with the impression that mathematics, like music, is something that only the truly gifted should attempt. Under Kac's influence, I began to believe that even someone with my modest talent could do mathematics worth doing. Thus, when it became clear that I would never become a biologist, I asked Kac if he would take me on as a student. I could tell that Kac was not pleased. Not having to take responsibility for graduate students was one of the reasons for his decision to leave Cornell, and the prospect of taking on a reject from biology was not one that he welcomed. Fortunately for me, Kac did not feel that he could refuse someone with whom he had been meeting regularly for more than a year. Even so, he made it clear that I was not to become a burden on him. During the following year I saw him less regularly than I had before I became his student. Occasionally I'd see him in the reading room, where he would browse through the recently arrived journals, less to read the articles than to see the names of the authors. He payed special attention to the Polish journals, and I once heard him mutter „peasants" as he scanned the list of contrib-

utors. I also saw him at seminars. The one that I remember best was given by Klaus Krickeberg. Krickeberg was to become a champion of liberal causes, but he was then still one's gestalt of a stiff German Herr Professor. Kac introduced him by saying „and now wewill hear from Krickeberg von Heidelberg, or is it Heidelberg von Krickeberg?" Krickeberg was not amused. This is not the only time that Kac revealed his enmity for Germany. I was not there, but I was told that after a lecture he delivered at an early Oberwolfach conference Kac was asked why he had not given his talk in German. He replied, „in this case, I prefer to speak the language of the victors."

To Kac's great relief, after a year I had begun to make some progress. He had given me a set of notes containing his own approach to potential theory. As I came to understand, these notes were more interesting for the insight they gave into Kac's prejudices than for their mathematical contribution. Kac had an aversion to measure theoretic gymnastics. In particular, he never came to terms with Doob's theory of martingales and stopping times. Instead, he based his approach to potential theory on the Feynman–Kac formula. He knew that there were deficiencies in his results, but he was not sure exactly where they lay. It was Zbigniew Ciesielski who first discovered the essential difference between Kac's theory and the one that Kakutani and Doob had developed. Kac met Ciesielski in Poland on his first visit there after the war. Ciesielski had been assigned to act as his guide, and the two of them became close friends. Shortly after his visit, Kac invited Ciesielski to spend a year at Cornell, and it was during that year that he became interested in Kac's potential theory.

Thinking that I would profit both mathematically and culturally from a visit to Poland, Kac arranged for me to spend the summer of 1965 with Ciesielski in Poznań. When I discussed this plan with my father, I recall his saying „Poznań, Poznań, Posen, that's the last place where my family lived before they left Europe." This came as a complete surprise to me since my father's family always thought of itself as members of the German Jewish community in New York, not part of the wave of Ostjuden who came to America later. In spite of their living in Poland for a couple of generations, my guess is that my ancestors never thought of themselves as Polish. They were one of the many German Jewish families, like the ones depicted by I. J. Singer in *The Brothers Ashkenazi*, who went to the Prussian part of the tri-partition to find employment but not to settle[vi]. Be that as it may, my wife and I spent two months in Poznań. Ciesielski went out of his way to be solicitous

but was distracted by problems that his wife was having with her pregnancy. As a consequence, I had lots of time to work on mathematics, and it was in our rented room in Poznań that I had the idea on which my thesis is based. I explained my idea to Ciesielski who immediately understood its implications, but, ironically, it was not until I returned to the United States that it became clear how well my ideas meshed with his own.

Back at Rockefeller, Kac's demeanor toward me underwent an abrupt change. Ciesielski had written him that I had the makings of a thesis, and so I was no longer likely to become a drag on his time. Thinking of me more as a junior colleague than a nuisance, Kac invited me to attend the daily lunches that the mathematics and physics faculty had in the common room. These were congenial events at which Kac presided. He was a gifted raconteur, but he lacked discipline. My wife and I once drove him to his home in Scarsdale, a trip that should take half an hour but, because of traffic, lasted an hour and half, every minute of which Kac regaled us with a steady stream of stories. Often his monologue was sprinkled with bits of wisdom. For example, once when I expressed my admiration for the work of Gödel, he said that there are three types of people in the world. There are those who do not believe in God, and they are the happiest; there are those who believe in God and think that God loves them, and they too are happy; and there are those who believe in God and think that God hates them, and they are logicians. Only George Uhlenbeck could shut him down. Resembling a friendly bear, Uhlenbeck, when he had finished his sandwich and was preparing to light his after-lunch Schimmelpfennig, would raise his paw-like hand and tell Kac to be quiet.

At first Kac's decision to go to Rockefeller seemed to be a good one. It freed him of the mundane aspects of an ordinary academic appointment and provided him ample opportunity to exercise his expository skills. His services were in constant demand, and he did a great deal of traveling. The United States is a very large country, and Kac had a fear of airplanes. Thus, he ended up spending a lot of time on trains. I once went with him on The Twentieth Century Limited to a conference in Madison Wisconsin. He had booked state rooms for each of us, but, so far as I could tell, he never used his. Instead, he wiled away the night in the club car, downing martinis and relishing the chance to enthrall a new set listeners with his trove of stories.

Kac's ability to get pleasure out of life was one of his most endearing characteristics. I remember the time he was a member of a panel that

was asked to review the science curriculum at the University of Kentucky. It was a very distinguished panel, consisting of Lars Onsager, C.N. Yang, and Mark Kac. When they had made their report, the governor invited them to a banquet, and, after dinner, they were asked to stand. With great solemnity, the governor rose and dubbed them „Kentucky Colonels." This is an honor usually reserved for owners of race horses and defenders of the Confederacy, but it was just the sort of Americana that Kac loved, especially because it included a bottle of Kentucky bourbon with his name engraved on it. Feeling the need to respond and knowing that neither Onsager nor Yang was going to, Kac thanked the governor, saying that it was a particularly appropriate recognition since Onsager came from southern Norway, Yang from southern China, and he himself from southern Poland. (None of these assertions is completely accurate.) After he returned to Rockefeller, the monogrammed bottle of bourbon was prominently displayed on his desk, and he tried, without success, to get his colleagues to address him as Colonel Kac.

I was personally involved in one of the most successful of Kac's popular enterprises. The American Mathematics Society had a program to film lectures given by renowned mathematicians, and it asked Kac to participate. Kac chose to talk on a problem that came out of Planck's theory of black body radiation. Loosely speaking, the question is to what extent the geometry of a region is reflected in the spectrum of the associated Laplace operator. Hermann Weyl had solved the original problem using purely analytic techniques, but Kac knew a more intuitively appealing probabilistic argument that might lead to refinements of Weyl's result. When he described his ideas to his friend Lipman Bers, Bers immediately said that Kac was asking whether one can „hear the shape of a drum," and *Can One Hear the Shape of a Drum* became the title of Kac's movie ([4]). The movie was shot in June, and New York was already heating up. Day after day, Kac had to stand under glaring lamps wearing a cleanly pressed suit and tie. Normally when Kac lectured he would take off his jacket immediately after being introduced, explaining that he had worn it only to prove that he owned one, but this was not an option during the filming. Every half hour, he needed to take a break and cool off, and my role was to have a chilled martini ready for him.

I graduated from Rockefeller in the spring of 1966, the first, and very nearly the last, mathematics graduate of the institute. Kac had induced Gian-Caro Rota and Henry McKean to leave M.I.T. Rota was

there my final year, and McKean came the following fall. McKean had visited Rockefeller during the 1963-1964 academic year, the year when I switched to mathematics, and he, as much as Kac, is the person who taught me probability theory and guided my research. The institute was still committed to building its mathematics program further, and, with the arrival of McKean and Rota, it looked as if it would succeed. However that was not to be the case. Kac's attempts to recruit other mathematicians failed. At one faculty meeting, McKean expressed his frustration about the situation and Kac's frequent absences by suggesting that, rather than recruiting someone new, they should double Kac's salary and have him at the institute full time. For personal as well as professional reasons, Rota's decision to come to Rockefeller was a bad mistake, and he returned to M.I.T. after a couple of years. Shortly thereafter, the institute began having second thoughts about its mathematics and physics program. The idea of expanding the institute to include mathematics and physics had seemed reasonable when government funding for the sciences was plentiful, but even the Rockefellers could not afford to indefinitely support mathematicians and physicists whose grants did not cover their salaries. Seeing the handwriting on the wall, McKean resigned in 1970. McKean's departure was a terrible blow, only partially mitigated by the appointment of James Glimm a few years later.

In spite of, or maybe in response to, the increasingly discouraging prospects for mathematics at Rockefeller, Kac's mathematical activity rejuvenated. The focus of his attention during this period was on completely integrable systems like the Toda lattice. I don't think that Kac himself made major original contributions to this subject, but he woke the interest of several people and set McKean on the course that he would follow for more than a decade. However, mathematical activity could not mask the fact that the program he had been hired to build would never take off. Uhlenbeck retired in 1971 and gave up his office a few years later, and Glimm left in 1974. Thus, aside from visitors, Kac was more and more isolated, a state that is intolerable for someone with his gregarious personality. When, around 1978, the University of Southern California offered him a special chair, he took it. I visited him there a couple of times. In some ways Los Angeles was an ideal place for him. Kac loved the country that had saved him from almost certain death, he savored the optimism and dynamism of America, and Los Angeles is America on steroids. He bought a house with a swimming pool, drove a large, air conditioned sedan to work on the congested

thruways, and basked in the respect accorded to him. His colleagues at USC did not provide him with a lot of intellectual stimulation, but he compensated for that by inviting Rota and others to visit him. On my first visit to him, I found a thoroughly happy man, but, tragically, time was running out for him. I am not sure exactly when Kac learned that he was doomed, but, as anyone who has read the second half of his autobiography knows, it had a devastating effect on him. On my second visit, he made a courageous effort not to be morose, but he had lost the ebullience that had been his hallmark. In the fall of 1984, I arranged a conference at M.I.T. to celebrate his 70th birthday. The night before the conference was to start, he called to say that he doubted if he would be able to attend. The day after, he was dead.

The lives of my generation of Americans have been greatly enriched by the refugees about whom Laura Fermi wrote in her book *Illustrious Immigrants* ([3]). Very few mathematicians appear in her book, but Kac is one of them. She devotes only a sentence to him, but that sentence is a fitting tribute. What she says is something to the effect that Kac is the mathematician who is known by people who don't know any mathematicians. And, thirty years after his death, he still is.

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A. Editors' note:

i. From 1382 to 1569 Krzemieniec was in the Grand Duchy of Lithuania, in 1569 becoming part of the Crown of Poland. From 1795 to 1917 it was in the Russian Empire.

ii. The proof was published in the journal „Młody Matematyk".

iii. In the years 1919-1939, the university bore the name of Jan Kazimierz University.

iv. In pre-war Poland there was no „thesis program", even though doctoral candidates were formally a category of university students. The course of studies could be followed by either the teacher's exam, or– in relatively few cases– by PhD proceedings, during which the candidate had to support himself/herself in some way while working on his/her thesis and preparing for exams. It was a professor who chose a PhD candidate (usually someone who already distinguished himself/herself), not the other way round. Recommendations of other instructors whose courses the student took often played a role. In his memoir *Enigmas of Chance* Mark Kac explicitly states: *It was Marceli Stark who had suggested to Steinhaus that he look after me* (cf. [EN1, p.54]). After Kac passed an exam with Steinhaus (as part of his coursework), he was admitted to an advanced seminar. And some time later Steinhaus posed the first problem to him.

v. Neither USSR nor the allied Western powers controlled Wrocław yet when they agreed in February 1945 that Stalin would keep the territories east of Bug river, including Lwów. And the Polish government in exile was not a party in these negotiations. Only in the symbolic sense did Poland „get Wrocław instead of Lwów"; the fact that many scientific and cultural Polish institutions relocated from Lwów to Wrocław after WWII lends some support to this view.

vi. The Jewish settlements in and around Poznań go back to the 14th century. In 1832, German became the official language of the Grand Duchy of Posen and other Polish territories incorporated by the Kingdom of Prussia. Speaking German offered an advantage for upward mobility. (Later the Bismarck government waged an active campaign against Polish language and culture in the occupied territories.) When Napoleon occupied Prussia, he granted Jews full civil rights. The law was upheld after Napoleonic wars, so many Jews moved to Prussia from other German states to enjoy rights they did not have at home. Thus Poznań Jews identified themselves with German language and culture, so much so that the majority of them left Poznań alongside Germans after it became Polish in 1919.

The analogy with „The Ashkenazi brothers" (see [EN2]) is tenuous. Łódź, where the main plot of the novel takes place, was located in the Russian-occupied Polish territories, and the twin heroes come from a Hassidic family in the Russian partition. While Łódź with its rapidly developing textile industry attracted workers and industrialists from all over Europe, including Southern Germany and German-occupied Silesia, the Grand Duchy of Posen could not offer work to many people from outside, as the economy there was based mainly on agriculture.

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