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A tribute to Juliusz Schauder[1]

Dedicated to Wojciech Kryszewski for his sixtieth birthday anniversary

Abstract. An analysis of the mathematical contributions of Juliusz Schauder through the writings of contemporary mathematicians, and in particular Leray, Hadamard, Banach, Randolph, Federer, Lorentz, Ladyzhenskaya, Ural'tseva, Orlicz, Gårding and Pietsch.

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1. Introduction

I feel as happy and honored here than embarrassed. Some of the work of Juliusz Schauder has been of such fundamental importance in my mathematical life, that I should be awarding a medal to Schauder, instead of receiving the one bearing his name.

I feel embarrassed to have enjoyed an already long and eventless life, when Schauder lived only forty-four years, including two World Wars, two changes of citizenship, twelve years of publication, several years of clandestinity and a racist assassination.

During his tragic and short destiny, Schauder has always given much more than received. He has left to the mathematical community a deep and wide legacy, which produces new fruits every day and everywhere.

The mathematical community of his time, although aware of his genius and of the deepness of his contributions, and his own country, although aware of the patriotism shown after World War One, remained unable, in the political and the cultural environment of the time, to provide him an academic position. Ironically, Schauder had to wait

[1]Expanded version of the lecture given at Nicolas Copernicus University at Torún, On May xx 2013, when receiving the Schauder Medal

to see his region invaded by Soviet troops and annexed to the Ukrainian Republic to become a university professor for a couple of years.

When an infamous pact came to an end, SCHAUDER was expelled, round up and assassinated by the Nazis. He was even deprived of a grave, and of a precise date of death.

Within the emotion linked to this testimony of esteem and friendship from the Polish mathematicians in general, and the ones of Toruń in particular, I hope you will forgive me not giving a talk about my research, but using the opportunity to speak about SCHAUDER and his achievements.

Polish mathematicians have cultivated with care and talent the history of mathematics in Poland [2,3,9,10,34,47]. Several biographical articles like [5,8,24,33,41] have been published, so that a strictly biographical lecture would be inappropriate. Detailed analysis of SCHAUDER's work are available as well, like [23, 29, 30]. Hence I restrain myself to a somewhat impressionist tribute, and leave mostly the words to some of the greatest mathematicians of the XXth century, to show the high rank they all give to SCHAUDER's contributions.

2. Leray's appreciation of Schauder

2.1. A letter

Nobody will be surprised if JEAN LERAY first enters on stage. He was a close collaborator and a friend of SCHAUDER. His life was not eventless, but exceptionally long and mathematically fruitful. His mathematical spectrum is unusually broad. However, a letter he wrote to me on June 12, 1981 starts as follows:

> I am very moved by your so warm letter, giving to the memory of Jules Schauder a so much deserved homage, and manifesting so much enthusiasm for what is, maybe, the best part of my writings.[2]

This short sentence reveals both the deep friendship and admiration of LERAY for SCHAUDER, and the extreme value LERAY attributes, within his monumental mathematical production, to his joint work with SCHAUDER [29].

LERAY's appreciation for his friend was not limited to some sentences of private letters. He has expressed it *urbi et orbi* and in many different

[2] Je suis très touché de votre lettre si chaleureuse, rendant à la mémoire de Jules Schauder un hommage tellement mérité et manifestant tant d'enthousiasme pour ce qui est, peut-être, la meilleure part de mes écrits.

ways.

2.2. Leray's notice of candidature to the Academy of Science of Paris

In this *Notice sur les travaux scientifiques*, written to emphasize his own scientific contributions, LERAY insisted on the importance and value of SCHAUDER's work, collaboration and inspiration on his research:

> Under its first form, my theory of equations [...] appeared at the very moment where a remarkable Polish mathematician, who was, alas ! victim of nazi racism, published his memoir. [...] He used there algebraic topology, that I was soon hoping to apply to equations with possibly several solutions. I made it in collaboration with Schauder. [...] To this purely mathematical theory of functional equations, it was important to give purely mathematical applications. [...] J. Schauder, who had remarkably simplified and clarified the beginning of the works of S. Bernstein [on nonlinear Dirichlet problem], had just discovered, in their last part, an error explaining why their computation did not have the necessary geometrical invariance. [...] [The theory of functional equations] borrowed to the work of J. Schauder the method consisting in starting from the topology of Euclidian spaces and to go by continuity from an Euclidian space to a space having an infinity of dimensions.[3]

2.3. Leray's analysis in Schauder's 'Oeuvres' LERAY's analysis of SCHAUDER's work [23] published in the *Oeuvres* [44] of the Polish mathematician published in 1978, starts as follows:

[3]Sous sa première forme, ma théorie des équations [...] parut au moment-même où un remarquable mathématicien polonais, qui fut, hélas ! victime du racisme nazi, J. Schauder, publiait son Mémoire [où il] [...] employait la topologie algébrique, que j'espérai aussitôt appliquer aux équations susceptibles d'avoir plusieurs solutions. Je le fis en collaboration avec J. Schauder. [...] À cette théorie purement mathématique des équations fonctionnelles il importait de donner des applications purement mathématiques. [...] J. Schauder, qui avait remarquablement simplifié et clarifié le début des travaux de S. Bernstein, venait de découvrir, dans leur dernière partie, une erreur, qui expliquait pourquoi leurs conclusions n'avaient pas les propriétés d'invariance géométrique nécessaires. [...] [La théorie des équations fonctionnelles] empruntait aux travaux de J. Schauder la méthode qui consiste à partir de la topologie des espaces euclidiens et à passer par continuité d'un espace euclidien à un espace ayant une infinité de dimensions.

> This work looks diversified: it brings often fundamental contributions to four branches of mathematics, usually studied independently one from each other. Indeed this work has a great continuity: Juliusz Schauder was not a specialist; he returned to the origin of the problems; he realized that they required completely different approaches from the ones usually applied; his thought progressed, without caring about the partition-works that arbitrarily divide science, and that really original minds always had to turn upside down.[4]

Leray insists also on Schauder's personality and motivations:

> His intellectual power was allied to the most beautiful qualities of character: he was a hard worker, very scrupulous. He studied the 'problems that set up and not the ones that one sets up'; his work was not a brilliant game, but a harsh labour, which ended in creating the method whose elegant perfection will direct the future.[5]

The four branches mentioned by Leray are integration theory, algebraic topology of Banach spaces, second order elliptic partial differential equations of elliptic type, and second order hyperbolic partial differential equations. Leray's description of Schauder's achievements, with precise references to his various papers, is a masterpiece of science and friendship. He never mentions his own name when dealing with a joint contribution with Schauder.

In his conclusion, Leray locates Schauder within a dynasty of most prestigious mathematicians:

> To found algebraic topology of Banach space, to reduce the classical problems of the theory of partial differential equations to

[4]Cette œuvre est en apparence diverse: elle apporte des contributions souvent fondamentales à quatre branches des mathématiques, qu'on a coutume d'étudier indépendamment l'une l'autre. En fait, c'est une œuvre d'une grande continuité: Jules Schauder n'est pas un spécialiste; il remonte à l'origine des problèmes; il s'aperçoit qu'ils exigent de tout autres procédés que ceux qu'on a pris l'habitude de leur appliquer; sa pensée progresse, sans souci des cloisonnements qui compartimentent arbitrairement la science et qu'on dû toujours bousculer les esprits vraiment originaux.

[5]Sa puissance intellectuelle s'allie aux plus belles qualités de caractère: il est un grand travailleur, très scrupuleux. Il étudie "les problèmes qui se posent et non ceux qu'on se pose"; son œuvre n'est pas un jeu brillant, mais un rude labeur, qui finit par créer la méthode dont l'élégante perfection orientera l'avenir.

> proving that some linear mappings on function spaces have a finite norm means to be both a great precursor and a disciple of Stefan Banach. [...] Juliusz Schauder has got other influences: this is because S. Bernstein and J. Hadamard impassioned him that he created the methods passing their ones. And mostly, through L.E.J. Brouwer, G.D. Birkhoff and O.D. Kellogg, he was the disciple of H. Poincaré, who wrote in the Notice about his works: 'For me, all the various ways where I entered successively led me to Analysis situs'; Juliusz Schauder was led there through a completely new road.[6]

3. Schauder and Leray: the steps of a collaboration going beyond death

3.1. Schauder and Leray first meeting in Paris

The meeting, organized by Hans Lewy, took place during Spring of 1933 in a restaurant of Soufflot street. The story, beautifully told by Leray himself in [24], needs not to be repeated here. Leray said to Schauder:

> I have read your paper on the relationship between existence and uniqueness of the solution of a nonlinear equation. I know now that existence is independent of uniqueness. I admire your topological methods. In my opinion they ought to be useful for establishing an existence theorem independent of any uniqueness and assuming only some a priori estimates.

Schauder answered:

> That would be a theorem ![7]

It became 'ein Satz', written two weeks later in the Jardin du Luxembourg and in Meudon, announced in a note to the *Comptes Rendus de*

[6]Fonder la topologie algébrique des espaces de Banach, réduire les problèmes classiques de la théorie des équations aux dérivées partielles à la preuve que certaines applications linéaires d'espaces fonctionnels ont une norme finie, c'est être à la fois un grand précurseur et un disciple de Stefan Banach. [...] Jules Schauder a subi d'autres influences; c'est parce que S. Bernstein et J. Hadamard le passionnaient qu'il créa des méthodes surclassant les leurs. Et surtout, par L.E.J. Brouwer, G.D. Birkhoff et O.D. Kellogg, il fut le disciple de H. Poincaré, qui déclare dans sa Notice sur ses travaux, „Quant à moi, toutes les voies diverses où je l'étais engagé successivement me conduisaient à l'Analysis situs"; Jules Schauder y fut conduit par une voie toute nouvelle.

[7]Das wäre ein Satz!

l'Académie des Sciences de Paris of July 10, 1933 [25], and published in the *Annales de l'École Normale Supérieure* in 1934 [26]. A tentative dedication to their common friend Leon Lichtenstein, who died in August 1933, was refused by Émile Picard, the main editor of the journal! One can hope that anti-semitism had nothing to do with the decision.

3.2. Leray's formula on the degree of composed mappings

Leray's note to the *Comptes Rendus* of 1935 [12], containing the formula for the degree for composed mappings and its topological applications, namely the Alexandroff and invariance of domain theorems, starts as follows:

> One can bring the following complements to our work done in collaboration with M.J. Schauder.[8]

Schauder's inspiration is emphasized inside the paper:

> M. Schauder has established this theorem [invariance of domain] under slightly stricter hypotheses in 1932; the proof that I present here has the narrowest relations with his proof. M. Schauder furthermore has shown how this theorem found interesting applications in the theory of partial differential equations.[9]

Much more can be learned from personal memories of Leray and extracts of Schauder's letters to his French friend described in [24]. Indeed both discussed about the invariance of domain theorem for general Banach spaces when they met in Paris in 1933, and Schauder thought that it should be true, but requiring a difficult proof.

In preparing his lectures at the *Cours Peccot* in the *Collège de France* in 1935, Leray found the formula for the degree of composed mappings, and the application to the Alexandroff theorem (equality of the number of components of complements of two homeomorphic closed sets). Schauder was asked to scrutinize it and wrote in a letter to Leray, that it was superfluous to mention that he had conjectured the result and added:

[8] On peut apporter les compléments ci-dessous à notre travail fait en commun avec M.J. Schauder.

[9] M. Schauder a établi ce théorème sous des hypothèses un peu plus strictes en 1932; la démonstration que j'expose ici a les rapports les plus étroits avec la sienne. M. Schauder a en outre montré comment ce théorème trouvait des applications intéressantes dans la théorie des équations aux dérivées partielles.

> It gives me great pleasure that you can generalize and complete the theory in such a beautiful and elegant manner. The various sentences where you compare your results with the old ones bear witness of your far too great modesty. [...] It is really amazing that one can obtain the Alexandroff theorem in such an elegant and easy manner, and – if I must be sincere – it is even disquieting for me; but I am not able to find mistakes.

Leray's note ends with a small overlooked section entitled 'Generalization' showing that Leray and Schauder already thought, in 1935, about generalizing their theory to nonlinear spaces:

> One can extend this study to abstract nonlinear spaces $\mathcal{E}$: it suffices to substitute to the translations the homeomorphisms of $\mathcal{E}$. A few assumptions, concerning the group of those homeomorphisms, are necessary; we will explicit them later.[10]

3.3. Leray and Schauder in Geneva, and Schauder in Moscow

One of the *Conférences internationales des Sciences mathématiques*, organized from June 17 till June 20 1935 by the University of Geneva, on *Partial differential equations. Proper conditions determining the solutions*, gave a second occasion for Schauder and Leray to meet. On June 19, Schauder surveyed his work on linear elliptic equations [42], and Leray [13] described their joint work [25,26] on abstract functional equations and nonlinear elliptic problems, and his own contributions to Navier-Stokes equations.

In the historical and biographical notes of his lecture, Leray insisted upon the fact that:

> M. Schauder was the first one to discover that some theorems of combinatorial topology remain true in Banach spaces, when one takes the essential precaution to replace the notion of continuous transformation by that of of transformation of the type $x+\mathcal{F}(x)$ (where $\mathcal{F}(x)$ is completely continuous).[11]

[10] On peut étendre cette étude à des espaces abstraits $\mathcal{E}$ non linéaires: il suffit de substituer aux translations les homéomorphies de $\mathcal{E}$. Quelques hypothèses, concernant le groupe de ces homéomorphies, sont nécessaires; nous les expliciterons ultérieurement.

[11] M. Schauder, le premier, découvrit que des théorèmes de Topologie combinatoire valent encore dans les espaces de Banach, quand on prend la précaution essentielle de subtiliser,à la notion de transformation continue quelconque celle de transformation de type $x+\mathcal{F}(x)$ (où $\mathcal{F}(x)$ est complètement continue.

He also insisted on SCHAUDER's results on elliptic equations:

> Mr. Schauder, using theorems of topology generalized to abstract spaces, has proved results that the method of successive approximations cannot reach. [...] M. Schauder has revisited the majoration of the second derivatives, exhibiting the fact that the six quoted pages [of Bernstein] constitute the essential part of the resolution of the quasi-linear equation without right-hand member.[12]

As told in [24], SCHAUDER presented LERAY's result on the degree of composed mappings at the *International Conference on Topology* held in Moscow from September 4 till September 10, 1935. He wrote to LERAY:

> I met all leading topologists, told them your beautiful result; one of them we revere the most admitted to be ashamed not to have proved it for the finite-dimensional case.

He added:

> The studies [in Moscow] are not geared to applications, and the mathematicians present were only interested in topology as such. I have always stressed that I am not a topologist. [...] I am, like you, a man of applications.

SCHAUDER's contribution to the Moscow conference [43] contains a number of interesting remarks and conjectures about possible developments of Leray-Schauder's theory. The first direction concerns the generalization to infinite dimensions of other invariants than degree:

> In a way similar to the one made here for the degree of an application, it will be possible to define other topological invariants (Betti groups, numbers, etc.) (as Mr LERAY recently briefly announced to me) for the infinite dimensional spaces. However,

[12] M. Schauder, en s'appuyant sur des théorèmes de Topologie généralisée aux espaces abstraits, a établi des résultats que ne peut atteindre la méthode des approximations successives. [...] M. Schauder a repris la majoration des dérivées secondes en mettant bien en évidence que les six pages citées [de Bernstein] constituent la partie essentielle de la résolution de l'équation quasi-linéaire sans second membre.

> neither Mr. Leray nor me see for the moment significant applications of those other invariants in the case of function spaces of analysis.[13]

The second direction is the extension to more general classes of spaces:

> On the other side, I can also develop topological degree when one deals with more general spaces, namely linear metric spaces in which arbitrarily small convex neighborhoods of the origin are given (A joint work of Mr. Leray and myself on various topological invariants in spaces as general as possible will appear). Also, nonlinear spaces can be treated, namely metric spaces whose neighborhoods are homeomorphic to Banach spaces.[14]

The announced joint work never appeared, and the extensions to locally convex topological vector spaces and to some nonlinear spaces were concretized by Leray alone during and after the Second World War.

3.4. Schauder on hyperbolic equations and Leray on fully nonlinear elliptic equations

In the second half of the years nineteen thirty, Schauder devoted his time and energy to the Cauchy and mixed problems for linear and quasilinear hyperbolic equations of the second order.

On the other hand, Leray extended his results with Schauder on quasilinear elliptic equations in the plane to the case of fully nonlinear equations, already considered in the last section of their celebrated joint paper [26]. In 1938, Leray [14] proved that for a fully nonlinear elliptic equations $f(r, s, t, p, q, z, x, y) = 0$ on a bounded open planar domain with smooth boundary γ, one can obtain an a priori estimate for the absolute values of the second derivatives of a solution z in terms of a bound on the norm of the gradient of z, the local behavior of γ and the local behavior of f. He noticed in the introduction:

[13] Auf ähnliche Weise, wie hier für den Abbildungsgrad dargelegt wurde, lassen sich auch weitere topologische Invarianten (Bettische Gruppen, Zahlen usw.) (Wir mir soeben Herr Leray brieflich ankündigt) für den unendlichdimensionalen Raum definieren. Doch weder Herr Leray, noch ich sehen vorläufig irgendeine Anwendung dieser anderen Invarianten für den Fall des Funktionalraumen in der Analysis.

[14] Anderseits kann ich eine ähnliche Theorie des Abbildungsgrades auch dann entwickeln, wenn es sich um allgemeinere Räume handelt, etwa um lineare metrische Räume, in welchen es beliebig kleine, konvex Umbegungen der Null gibt (Eine gemeinsame Arbeit von Herrn Leray und mir über verschiedene topologische Invarianten in möglichst allgemeinen Räumen wird erscheinen). Auch nichtlineare Räume könnten betrachtet werden (Etwa metrische Räume, deren Umbebungen, z.B. den Umgebungen in Banachschen Räumen homöomorph sind).

> This theorem had only been stated and proved until now in the case where equation $f = 0$ is quasilinear. [...] The famous works of M.S. Bernstein contain this particular statement [...]; M.J. Schauder has made precise this reasoning of M. S. Bernstein. With the help of the finest theorems of M.J. Schauder on equations linear in r, s, t, p, q, z and in generalizing a lemma of M. H. Lebesgue [...], M.R. Caccioppoli has established the theorem above, f being quasilinear.[15]

Those estimates were used by Leray, one year later, together with Leray-Schauder theorem, to obtain existence results [15].

Leray visited Schauder in Lwów in 1938. It was their last meeting.

3.5. The International Malaxa Prize

Let us now come to the story of an event whose details do not seem to be well known. In Wladyslaw Orlicz's short biography of Schauder [33] written for his *Oeuvres*, one can read that:

> In 1938, Schauder and Leray obtained the International Metaxas Prize.

This is repeated by Orlicz in another paper on Polish mathematicians [34] and by Kazimierz Kuratowski in his book on the history of Polish mathematics in the XXth century [9]. Its origin may be a statement of Hugo Steinhaus in a biography of Stefan Banach [46], claiming:

> Among those of Banach's students who fell at the hands of murderers wearing uniforms adorned with the swastika, the most outstanding was undoubtly P.J. Schauder (sic), the laureate of the international Metaxas Prize, which was awarded to him and Leray ex aequo. It was Schauder who noticed how important were the Banach spaces for the boundary problems of partial differential equations. The difficulty lay in the selection of proper norms; it was overcome by Schauder and, thanks to that scientist, young at that time, the wreath of victory in such a classical

[15] Ce théorème n'avait été énoncé et démontré jusqu'à present que dans le cas où l'équation $f = 0$ est quasi-linéaire. [...] Les célèbres travaux de M. S. Bernstein contiennent cet énoncé particulier. [...] M. J. Schauder a précisé ce raisonnement de M. S. Bernstein. En s'aidant des théorèmes les plus fins de M. J. Schauder sur les équations linéaires en r, s, t, p, q, z et en généralisant un lemme de M.H. Lebesgue, M.R. Caccioppoli a établi le théorème ci-dessus, f étant quasi-linéaire.

> theory as partial differential equations was shared by France and Poland.

Metaxas is a famous brandy of Greek origin, still available now. Had it existed, a Metaxas Prize would have been more appropriate for BANACH...

LERAY's *Notice* [22] mentioned that he obtained in 1938 the *Malaxa Prize*. NICOLAE MALAXA was a Rumanian engineer and businessman who developed in his country, between the two World Wars, mechanical industry (locomotives, Diesel engines, steel industry, and later weapons), showing some sympathy with the Hitlerian regime. The real name of the Prize was the *International Malaxa Prize*. Here are details that I got recently with the help of my friend GEORGE DINCA.

MALAXA, in this time one of the richest men in Rumania, wanted to create an International Prize for Science, and the Rumanian mathematician OCTAV ONICESCU convinced him to attribute it to the most remarkable recent contribution to the theory of equations made by a young mathematician. A international jury was called, with TULLIO LEVI-CIVITA from Italy, HENRI VILLAT from France, WERNER HEISENBERG from Germany, and some Rumanian mathematicians. A final list of ten mathematicians was retained, and each member of the jury had to nominate three candidates. The ones who emerged were, in this order, LERAY and SCHAUDER. With MALAXA's approval, it was unanimously decided to attribute a Prize of 300.000 French Francs to LERAY and of 200.000 French Francs to SCHAUDER. According to ONICESCU [32]:

> The order was not so much determined by the intrinsic value of the obtained results, which were roughly equivalent, but mostly by the geometric interpretation that Leray had given to his results, which have shown to be, later, more suggestive.

ONICESCU commented as follows the reactions of the laureates:

> Leray was, I suppose, very satisfied, but showed a coldness that I only explained myself after I have known him personally, when visiting the Paris Academy of Science. [...] Schauder was deeply displeased, and considered that he had been harmed. He sent a fulminating letter that I did not dare to show to the sponsor, telling him that I had lost it.

One can easily imagine that LERAY's 'coldness' was his way to react to what he considered to be unfair: to have awarded different amounts to

Schauder and himself. It is easy to understand Schauder's reaction who, despite of his extreme value, was professionally discriminated in his country and discriminated, without any real reason, with respect to Leray in this contest. They were mostly crowned for their joint work and, in the period following their collaboration, it was really impossible to distinguish between Leray's subsequent applications of their joint paper to wakes and bows, and to fully nonlinear elliptic equations, and Schauder's pioneering work on linear and quasilinear hyperbolic equations.

3.6. The Second World War

In 1941, during the Second World War, the German occupation of Lwów definitely separated the destinies of Schauder and Leray. During the time where Schauder was hiding himself to escape to the Nazis, who finally assassinated him in Fall 1943, Leray was a war prisoner of the Germans and interned in a concentration camp for officers near Austerlitz [40, 45]. He organized there a University in captivity and, hiding his expertise in fluid mechanics to avoid being forced to collaborate to the German war effort, Leray concentrated his lectures on algebraic topology, a domain supposed to be harmless from the military viewpoint.

His main motivation was to extend the Leray-Schauder theory to some classes of nonlinear spaces (in order to include Lefschetz fixed point theorem) and to avoid the use of finite-dimensional approximations. In the notes sent in 1942 to the *Comptes Rendus* [16, 17], Leray wrote:

> This type encompasses the type of nonlinear equations in linear Banach spaces that M. Schauder and myself have studied. [...] Those two propositions [invariance of domain and Jordan-Alexandroff theorem] have been proved in the past in the case where the t are translations, X and Y being linear Banach spaces: the first one by M. Schauder, the second one by myself.[16]

Those notes were developed in a series of three large memoirs published in 1945 in the *Journal de mathématiques pures et appliquées*, just before Leray's return to France. They introduce new tools (spectral sequences and sheafs), which will revolutionate the treatment of algebraic topology

[16] Ce type englobe le type d'équations non linéaires des espaces linéaires de Banach que M. Schauder et moi-même avons étudié. [...] Ces deux propositions ont jadis été établies dans le cas où les t sont des translations, X et Y étant des espaces de Banach linéaires: la première par M. Schauder, la seconde par moi-même.

and of other parts of mathematics. In the third memoir [18], Leray, still unaware of the tragical death of his friend Schauder, wrote:

> Let us recall that it is M.J. Schauder (Math. Ann. t. 106, 1932, p. 661-721), who was the first to generalize the invariance of domain theorem to non Euclidian spaces, the Banach spaces, and to rattach to it Fredholm alternative.[17]

Remember that, in this time, an author was quoted just by his name if he was deceased, and by his name preceded by M. (Mr) if he was alive. It is not surprising that, in his *Offlag*, Leray ignored the tragic death of Schauder.

Indeed, the first papers of Leray published after learning Schauder's death and inspired by their joint work [19,20] both mention Schauder's assassination by the nazis. In [20], which extends Riesz-Fredholm-Schauder theory of compact linear operators from Banach to locally convex spaces, Leray recalls Schauder's contribution:

> Fredholm alternative will follows from a topological theorem, whose proof essentially uses nonlinear mappings: the invariance of domain theorem; Brouwer has proved it for Euclidean spaces; it has been extended to abstract spaces by the deep Polish mathematician J. Schauder, who was a victim of the nazi massacres.[18]

3.7. Leray's work on hyperbolic and holomorphic partial differential equations

But Schauder's source of inspiration for Leray did not end with the war, and with their joint work on nonlinear functional equations. After having completed, between 1946 and 1950, the algebraic topological consequences of his work written in the *Offlag*, Leray changed once more his interests and concentrated mostly (with a number of diversions) on hyperbolic and holomorphic partial differential equations.

This problem had occupied Schauder from 1935 and his influence has been recognized many times by Leray, for example in his famous Princeton lecture notes [21]:

[17] C'est M. J. Schauder (Math. Ann. t. 106, 1932, p. 661-721), rappelons-le, qui, le premier généralisa le théorème de l'invariance du domaine à des espaces non euclidiens, les espaces de Banach, et y rattacha l'alternative de Fredholm.

[18] L',,alternative de Fredholm" résultera d'un théorème de topologie, dont la preuve utilise essentiellement des applications non linéaires: le théorème de l'invariance du domaine; Brouwer l'établit pour les espaces euclidiens; il a été étendu aux espaces abstraits par le profond mathématicien polonais J. Schauder, qui fut victime des massacre nazis.

> In 1935, J. Schauder gave an easier method; the classical energy relation enabled him to extend the local Cauchy-Kowalewski's solution into a global one for the analytic and finally non analytic equations of the second order. Two years later, I. Petrowsky (Mat Sbornik 44, 1937, 815–866) defined the hyperbolic equations of order m ($m > 2$) and, using Schauder's process, proved the existence of the global solutions of Cauchy's problem [...] by a very complicated process; thanks to the weakness of the assumptions about the coefficients in the second part [...], we can simply use the method of successive approximations, that J. Schauder had applied to equations of the second order. [...] We do not solve the 'mixed boundary value problem'; this problem has been solved for the second order by J. Schauder and M. Krzyzański.

Leray's later work on various extensions of Cauchy-Kowalewsky's theorem also starts from Schauder's new proof and extension of this basic result.

4. Other opinions on Schauder's work

4.1. Hadamard on hyperbolic equations at Geneva Conference on PDE's

Jacques Hadamard chaired the 1935 Geneva's *International Conference on Partial Differential Equations* already mentioned, and delivered on June 17 the inaugural lecture on 'Partial differential equations. The conditions defined in general. The hyperbolic case' [7]. It is remarkable that the great French mathematician already reported upon the results of Schauder's first paper on quasilinear hyperbolic equations published the very same year:

> We will see, in the following lectures, and in particular in that of M. Schauder, that the [study of the linear case] has already started and is even remarkably advanced in the case of Dirichlet problem. It is also the case, again because of M. Schauder, in what concerns Cauchy's problem (and consequently the hyperbolic case) with $n > 2$ independent variables. [...] M. Schauder treats Cauchy's problem of the normal hyperbolic type [...] for a quasi-linear equation, [...] whose coefficients may depend, not only upon the independent variables, but also of u and its first derivatives. Such a generalization is far from trivial and easy.[19]

[19] On verra, dans les exposés suivants, particulièrement celui de M. Schauder, que

Hadamard even suggested some possible improvements:

> One should even search to improve in this respect the beautiful method of M. Schauder, because his analysis supposes the differentiability of the data at least until the order n, when the example of the linear case leads to assume that it is sufficient to go until the order $n/2$. The unavoidable role that the principles taken from the functional calculus play in this work is remarkable.[20]

This shows how fast the greatest expert in hyperbolic partial differential equations of the time and one of the founders of functional analysis understood the deepness and importance of Schauder's contributions.

4.2. Banach on nonlinear functional analysis at the Oslo ICM

Banach's plenary lecture at the *International Congres of Mathematicians* [1], held in Oslo in 1936, nicely described the recent results of Schauder:

> M.J. Schauder has proved the fixed point theorem for general spaces of type (B), and [...] solved in this way, for example, the Dirichlet problem for elliptic differential equations under the unique assumption of the continuity of the functions occurring in the data. M.J. Schauder has proved the theorem of the invariance of domain for some spaces of type (B) and one-to-one continuous mappings of a special type; [...] for general nonlinear elliptic differential equations, in some sense, the existence of solutions is a consequence of the unique solvability. M.J. Leray and J. Schauder have furthermore generalized the concept of degree of an application to the case of spaces of type (B) and deduced [...]

[l'étude du cas linéaire] est déjà entreprise et même remarquablement avancée en ce qui regarde le problème de Dirichlet. Elle l'est aussi, grâce également à M. Schauder, en ce qui regarde le problème de Cauchy (et, par conséquent le cas hyperbolique) à $n > 2$ variables indépendantes. [...] M. Schauder traite le problème de Cauchy du type hyperbolique normal [...] pour une équation quasi-linéaire, [...] les coefficients pouvant dépendre, non seulement des variables indépendantes, mais de u et de ses dérivées premières. Il s'en faut qu'une pareille généralisation soit banale et facile.

[20] Il y a même lieu de chercher à améliorer à ce point de vue la belle méthode de M. Schauder, car son analyse suppose la dérivabilité des données au moins jusqu'à l'ordre n, tandis que l'exemple du cas linéaire conduit à présumer qu'il suffit d'aller jusqu'à l'ordre $n/2$. Remarquable est le rôle indispensable que jouent, dans ce travail [...], les principes empruntés au Calcul fonctionnel.

> the existence in the large for elliptic differential equations, when no uniqueness is assumed.[21]

In addition, Banach announced the publication of a monograph by Schauder which never appeared:

> A book of M. J. Schauder: On partial differential equations of the second order of elliptic type is already planned in the already mentioned collection of monographs.[22]

Finally, Banach mentioned the often neglected Schauder's contributions, jointly with Stanislaw Mazur, to an abstract version of the direct method of the calculus of variations:

> The conditions for the semicontinuity of a convex functional in the spaces of type (B) allow to create a *new direct method of the calculus of variations*, [...] a simplification of the theory recently developed by M.L. Tonelli for a class of variational problems. With their help one can also solve some *more general variational problems*, either the ones involving a double integral in parameter form, or the ones where the derivatives of the searched functions occur in a higher order; also one can consider cases where the manifold of integration is closed. Those applications are due to M.S. Mazur and J. Schauder.[23]

[21]Herr J. Schauder [hat] den Fixpunktsatz für allgemeine Räume vom Typus (B) bewiesen, und [...] auf diese Weise z. B. das Dirichletsche Problem für elliptische Differentialgleichungen unter der einzigen Voraussetzung der Stetigkeit der in Betracht kommenden Funktionen gelöst hat. [...] Die Herren J. Leray und J. Schauder verallgemeinerten ferner den Begriff des Abbildungsgrades auf den Fall der Räume vom Typus (B), und erhielten daraus Existenzsätze im Grossen für elliptische Differentialgleichungen, wo keine Eindeutigkeit gefordert wird.

[22]Für die bereits erwähnte Monographieeinsammlung ist ein Buch von Herrn J. Schauder "Über partielle Differentialgleichungen zweiter Ordnung vom elliptischen Typus" bestimmt.

[23]Die Bedingungen für die Halbstetigkeit eines konvexen Funktionals in Räumen vom Typus (B) gestatten es eine neue direkte Methode in der Variationsrechnung zu schaffen., [...] eine Vereinfachung der vor kurzem vom Herrn L. Tonelli entwickelten Theorie einer Klasse von Variationsproblemen. Mit ihrer Hilfe kann man auch gewisse allgemeinere Variationsprobleme lösen, entweder solche bei welchen es sich um Doppelintegrale in Parameterform handelt oder solche bei welchen die Ableitungen der gesuchten Funktionen in beliebig hoher Ordnung vorkommen; auch darf die Integrationsmannigfaltigkeit geschlossen sein. Diese Anwendungen stammen von den Herren S. Mazur und J. Schauder.

The unique publication in this direction is the half page joint note of SCHAUDER and MAZUR in the Proceedings of the ICM in Oslo [31], not reproduced in SCHAUDER's *Œuvres*, but translated in Polish in the monograph [28].

4.3. Various comments on Schauder's surface integral

LERAY has described in [23] SCHAUDER's work on integration theory:

> His thesis (1926) studied, in the Euclidian space, the superficial measures defined by JANSEN and GROSS; in particular, he extended the validity of the Stokes formula which transforms a surface integral into a volume integral. He proved in 1929 the semicontinuity of those two superficial measures. In 1928, he made precise another Stokes formula, the one which, in the plane, transforms a simple integral into a double integral; he gave to this formula the level of generality that will be given, much later, by S. Sobolev's theory of generalized functions, L. Schwartz' theory of distributions, and finally G. de Rham's theory of currents; he applied it to the calculus of variations. The same year, he made a very fine analysis of mappings of the plane into itself, their functional determinants, inverse applications, change of variable in double integrals; he used the topological index.[24]

SCHAUDER's contributions have been used, quoted and appreciated by a number of experts in surface area and superficial measure. In his fundamental paper of 1928 on superficial measure [36], TIBOR RADÓ observed that:

> For the important special case, where the equations $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$ determine an invertible univocal relation between the points (u, v) and the points of the surface (x, y, z),

[24]Sa thèse (1926) étudie, dans l'espace euclidien, les mesures superficielles définies par Jansen et Gross; en particulier il étend la validité de la formule stokienne qui transforme une intégrale de surface en une intégrale de volume. Il prouve en 1929 la semi-continuité de ces deux mesures superficielles. En 1928, il précise une autre formule stokienne: celle qui, dans le plan, transforme une intégrale simple et une intégrale double; il lui donne le degré de généralité que devaient lui donner, fort ultérieurement, la théorie des fonctions généralisées de S. Sobolev, celle des distributions de L. Schwartz, enfin celle des courants de G. de Rham; il l'applique au calcul des variations. La même année, il fait une analyse très fine des applications du plan en lui-même, de leurs déterminants fonctionnels, des applications inverses, du chagement de variables dans les intégrales doubles; il utilise l'indice topologique.

> the invariant theorem in question also follows from the beautiful researches of Mr Schauder on Janzen superficial measure.[25]

When dealing with continuous planar transformations in 1936 [37, 38], the same author insisted on the inspiration found in SCHAUDER's papers:

> The purpose of this paper is to present certain remarks which occurred to me while studying the important papers of Schauder and Banach on continuous transformations. The method used in the present paper depends essentially upon certain results of Banach on functions of bounded variations. These results were first used by Schauder to control sequence of index-functions $n_j(x, y)$ with regard to term-wise integration.

When he developed SCHAUDER's ideas on Gauss-Green's lemma, JOHN R. RANDOLPH [39] wrote in 1935:

> Attacking the problem by radically different methods, J.P. Schauder obtained results for a class of boundaries with no order relation prescribed, [...] represented by two Lipschitzian functions.

In 1945, HERBERT FEDERER [4] insisted again on the pioneering aspect of SCHAUDER's work:

> A. P. Morse and J.R. Randolph have recently introduced and investigated the regularity of the boundary, [...] called Φ restrictness, [...] in the case of plane sets. The first attempt in this direction was made by J. Schauder (1926).

GEORGE G. LORENTZ [27] observed two years later that:

> The theorem of Schauder is proved in a very general form; it does not assume the existence of the normal and replaces $\cos(n, z)$ in the surface integral through the "extension coefficient" $D(X)$ at point X of the surface F, defined as follows. Let $U_r(X)$ be a r-neighborhood of the point X of the surface F, $V_r(X)$ its projection on the x, y-plane. When μ denotes the measure on the surface and m the two-dimensional Lebesgue measure on the x, y-plane, then by definition $D(X) = \lim_{r\to 0} \frac{mV_r(X)}{\mu U_r(X)}$.

[25] Für den wichtigen Sonderfall, wo die Gleichungen $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$, eine umkehrbar eindeutige Zuordnung zwischen den Punkten (u, v) und den Flächenpunkten (x, y, z) bestimmen, folgt der fragliche Invarianzsatz aus den schönen Untersuchungen von Herrn Schauder über das Janzensche Flächenmass.

> Schauder proves that $D(X)$ exists almost everyxwhere and that the [Gauss-Green formula] with $\pm D(X)$ instead of $\cos(n, z)$ holds.[26]

4.4. Ladyzhenskaya-Ural'tseva on second order elliptic partial differential equations

In their fundamental monograph on linear and quasilinear elliptic equations [11], Olga A. Ladyzhenskaya and Nina N. Ural'tseva insisted upon the importance of Schauder's contributions to linear elliptic partial differential equations of the second order, when using his ideas to deal with higher-order equations.

> During the past half century, linear second-order elliptic equations on bounded regions have been studied, if not exhaustively, at least with reasonable completeness and the fundamental questions concerning them have received rather simple solutions. In the work of Giraud and J. Schauder in the thirties, it was shown that the basic boundary-value problems are solvable for such equations under the assumption of sufficient smoothness of the coefficients and of the boundary of the region. [...] Out of all the previous results, for $n \geq 2$, there was only one that was impeccable in all respects. This was Schauder's theorem (supplementing Caccioppoli) regarding the solvability of the Dirichlet problem in the Hölder spaces $C_{l,\alpha}$, where $l \geq 2$, for equations with coefficients and inhomogeneous terms in $C_{l-2,\alpha}$. In this theorem, all the hypotheses were necessary because of the actual nature of the matter at hand. [...] Those basic results [...] have a character of finality: they cannot be extended within the spaces chosen here. [...] [The assumptions] cannot be weakened if we wish for all solutions of all such equations to belong to $C_{l,\alpha}$.

The same authors do not underestimate the importance of Schauder's contributions to nonlinear elliptic equations:

> In addition to the direct methods of investigating the solvability of boundary-value problems, mathematicians developed various

[26] In sehr allgemeiner Form ist der Satz von Schauder bewiesen worden; dieser setzt die Existenz der Normale nicht voraus und ersetzt *cos*(n, z) in dem Flächenintegral durch den ,“Dehnungskoeffizient” $D(X)$ im Punkte X der Fläche F. Dieses $D(X)$ wird wie folgt definiert: Es sei $U_r(X)$ eine r-Umgebung des Punktes X auf der Flache F, $V_r(X)$. ihre Projektion auf die x, y-Ebene. Wenn μ das Mass auf der Fläche und m das zweidimensionale Lebesguesche Mass auf der x, y-Ebene bedeutet, so ist $D(X) = \lim_{r \to 0} \frac{mV_r(X)}{\mu U_r(X)}$. Schauder beweist, dab $D(X)$ fast überall existiert und [Gauss-Green's formula] mit $\pm D(X)$ statt $\cos(n, z)$ richtig ist.

> topological methods. Of these methods, the most general and flexible, is the topological method of Leray and Schauder.

4.5. Orlicz on functional analysis

ORLICZ's description of the contributions of Polish mathematicians to functional analysis between 1919 and 1951 in his *Collected Papers* [34] provides informations about SCHAUDER and a warm appreciation of his work:

> Banach, and also Schauder, planned to investigate some questions of nonlinear functional analysis in the aspect of topological methods, with a view to a monographic presentation, but their object was never accomplished.

ORLICZ gives some information about the genesis of the open mapping theorem:

> One of the important results of the Lwów school of functional analysis was a theorem known today as the Interior Mapping Principle (Open Mapping Theorem): under a continuous linear mapping of one F-space onto another, the image of every open set is open. This theorem for B-spaces was announced explicitely in the paper of J.P. Schauder, *Über die Umkehrung linearer, stetiger Funktionaloperationen*, Studia Math. 2 (1930), [...] Using this fact, one can obtain two more important theorems: the Bounded Inverse Theorem (proved first for B-spaces by S. Banach and also by J.P. Schauder), [...] and the Closed Graph Theorem.

SCHAUDER's essential contribution to the concept of basis in a Banach space is mentioned as well:

> A sequence of elements (x_n) of a B-space X constitutes a basis for this space if, for every $x \in X$, there exists a uniquely determined sequence of scalars $(a_n(x))$ such that $x = \sum a_n(x)x_n$. [...] The notion of basis is due to J. Schauder, who introduced it in his paper *Zur Theorie stetiger Abbildungen in Funktionalräumen*, Math. Z. 26 (1927), in connection with the fixed point theorem. J. Schauder was also the first to give an example of a basis, namely, in the space C. He also showed that the Haar orthogonal system is a basis in L^p, $p \geq 1$.

Finally, SCHAUDER's fundamental work on nonlinear equations in abstract spaces is emphasized:

> The Polish mathematician who was the first to obtain important results in the topology of Banach spaces and who showed their significance in applications to the theory of differential equations was J.P. Schauder [...]. J. Schauder's results in the topology of infinite-dimensional spaces did not constitute for their author an objective of its own right, but were for him a tool for proving the existence of solutions of differential equations. However, it should be emphasized that in his works in this domain, J. Schauder used the conceptual apparatus of functional analysis not only in connection with the topological theorems, but also, for instance, in the so-called method of 'a priori estimates' of solutions of some partial differential equations of elliptic or hyperbolic type. The significance of J. Schauder's results for the theory of partial differential equations can be seen indirectly in the fact that J. Dieudonné, in his *Panorama des mathématiques pures, le choix bourbachique,* Paris, 1977, honored him by a mention among the five 'creators' of the theory of nonlinear equations. [...] In a series of Schauder's research papers devoted to the topology of infinite-dimensional function spaces, the results from the article by J. Leray and J. Schauder, *Topologie et équations fonctionnelles*, Ann. Ecole Norm. Sup. 51 (1934), are of particular importance. [...] The significance of these results lies in the fact that they permit to carry out proofs of existence (for partial differential equations, in hydrodynamics, etc.) even if no assumption which guarantee the uniqueness of solutions is made. The significance of J. Leray and J. Schauder's results was testified by the international Metaxas Prize which they were awarded in 1938.

4.6. Gårding on hyperbolic partial differential equations

The importance of SCHAUDER's papers on hyperbolic partial differential equations was recently emphasized by LARS GÅRDING [6]:

> The first existence proof [for 2^{nd} order strongly hyperbolic linear equations with variable coefficients] is due to Hadamard who constructed a parametrix for the fundamental solution. [...] A great step forward was taken by Schauder (1935,1936), who proved local existence of solutions of Cauchy's and mixed problem for

> quasilinear wave operators. The method is to use approximations starting from the case of analytic coefficients and analytic data. The success [...] depends on stable energy estimates [...] and the use of the fact that square integrable functions with square integrable derivatives up to order n form a ring under multiplication.

GÅRDING insisted on the crucial role played by SCHAUDER's method in IVAN G. PETROWSKY's contribution:

> Only a year after Schauder, Petrovsky (1937) extended his results for Cauchy's problem to strongly hyperbolic systems, in the simplest case, and the corresponding quasilinear versions. [...] The method is that of Schauder [...], but Petrovsky had to find his own energy estimate. For this, he used the Fourier transform, but the essential point is to be found in thirty rather impenetrable pages. [...] The intriguing paper (1937) by Petrovsky became the starting point for the development after 1950 of a general theory of hyperbolic differential operators by Leray and others.

And SCHAUDER's influence remains important nowadays:

> The subject of global solutions of semilinear wave equations got new life in the eighties. The impetus came from Fritz John's paper about life-time and blow-up of semilinear equations with small initial data. [...] The work done with these equations is ample confirmation of Schauder's remark that the solution of nonlinear equations means getting optimal bounds on solutions of linear equations. In particular, the improvement of the blow-up time with increasing n depends on the increasing dispersion of initial data for the linear equation.

Those comments confirm those made by LERAY in Section 3.7.

4.7. Pietsch on linear functional analysis and operator theory

In 2007, ALBRECHT PIETSCH published an impressive monograph on the history of Banach spaces and linear operators [35]. It gives an detailed palmares of SCHAUDER's important contributions in this area, which confirm and completes ORLICZ's description in Section 4.5:

> The concept of a **basis** was defined by Schauder (1927) to prove his fixed point theorem, seemingly just for this purpose. [...] Faber's construction was rediscovered by Schauder (1927) as a basis of $C[0,1]$; the name Faber-Schauder system is justified. [...] Schauder (1928) showed that Haar system is a basis in $L_p(0,1)$.
>
> Schauder (1930) found a significant simplification, based on Baire theorem, of the proof of **Banach bounded inverse theorem**. [...] The **open mapping theorem** is due to Schauder (1930).
>
> Banach (1929) and Schauder (1930) extended Riesz concept of **dual operator** to the abstract setting. Schauder (1930) added [...] duality to Riesz spectral theory of compact operators (**Riesz-Schauder theory**). Schauder proved that an operator is compact if and only if its adjoint is compact, that [...] the solution of $y = x + \zeta f(x)$ with f completely continuous is meromorphic in ζ.
>
> The definition of the **norm of a Hölder space** goes back to Schauder (1934). [...] Schauder's contributions to the theory of elliptic differential equations were the real starting point of all developments on **differential operators**.

4.8. A measure of Schauder's legacy

Another way to measure Schauder's legacy is to give a (non exhaustive) list of the concepts and results to which his name is attached today:

- **Linear functional analysis**: Banach-Schauder theorem, Faber-Schauder system, Riesz-Schauder theory, Schauder basis, Schauder frame, Schauder function, Schauder equivalence relations, Schauder decomposition, Schauder dimension.

- **Fixed point theory**: Schauder approximation theorem, Schauder projection, Schauder fixed point theorem, Leray-Schauder alternative, Leray-Schauder boundary condition, Leray-Schauder continuation theorem, Leray-Schauder fixed point theorem, Schauder conjecture.

- **Topology of Banach spaces**: Schauder open mapping theorem, Leray-Schauder reduction, Leray-Schauder degree, Leray-Schauder index formula, Leray-Schauder category, Leray-Schauder space.

- **Elliptic partial differential equations**: Schauder estimates, Schauder linearization method, Leray-Schauder theory for fully nonlinear elliptic boundary value problems.
- **Hyperbolic partial differential equations**: Schauder canonical form, Schauder energy method.
- **Surface area**: Schauder measure, Schauder transformation formula.

5. Conclusion

SCHAUDER's life resembles a Greek tragedy. Despite of his genius, he suffered discrimination, injustice, persecution, murdering. He did not deserve a place and a date for his death, he has no gravestone. He had to hide separate from his wife and his daughter Ewa, never knew that his wife was assassinated and that his daughter survived and finally committed suicide.

But he left to the mathematicians a wonderful and still alive legacy, and the least we can do is to cultivate the memory of his beautiful personality and the shine of his mathematics.

The *Schauder Center* at Nicolas Copernicus University is a wonderful initiative in this respect, and we all must be grateful for Copernicus University in Torún for creating, developing and hosting it.

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A tribute to Juliusz Schauder

Jean Mawhin

Streszczenie. Analiza matematycznego dorobku Juliusza Schaudera na podstawie artykułów współczesnych matematyków, a w szczególności Leray, Hadamarda, Banacha, Randolpha, Federera, Lorentza, Ladyzhenskaya, Ural'sesevy, Orlicza, Gårdinga i Pietscha.

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