

# APPROXIMATION AND FUNCTION SPACES

*Editor of the Volume*

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## PREFACE

In the last twenty years the approximation theory became in the World an active field. In the socialist countries there were organized the following international conferences: Budapest 1969, 1976, 1980, Varna 1970, 1981, 1984, 1987, Poznań 1972, Kaluga 1975, Blagoevgrad 1977, Gdańsk 1979 and Havana 1987. In addition to the proceedings volumes of the listed conferences there have appeared as vol. 4 in the *Banach Center Publications* the proceedings of the VI Semester on *Approximation Theory* held in Banach Center during the Fall of 1975.

The XXVII Semester on *Approximation and Function Spaces* took place at the Stefan Banach International Mathematical Center in Warsaw during the period February 27 – May 27 of 1986. Its program was concentrated on the following topics: function spaces, interpolation and approximation by polynomials and splines, extreme problems of approximation theory, expansions of functions and extensions. There were held 203 hours of lectures. From 20 countries 162 mathematicians participated in the XXVII Semester.

This volume reports on the current research in the topics of the XXVII Semester. It should be useful not only to specialists in approximation theory but also to specialists who apply approximation theory techniques such as: spline and polynomial approximation, Fourier analysis, diameters, finite element methods etc.

As the chairman of the program committee I would like to thank for fruitful cooperation to all its members: S. M. Nikol'skii, A. Kufner, L. Leindler, G. Marinescu, Bl. Sendov, S. B. Stechkin, J. Szabados and H. Triebel. As the man responsible for organizing the XXVII Semester I am extending my warmest thanks to the Banach Center staff, in particular to Mrs. Grażyna Pieścik and to her office mates, for the splendid job they have done. Finally, as the editor of this volume I am indebted to Jerzy Trzeciak from the Polish Scientific Publishers for his translation of some of the articles from Russian into English.

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