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CERTAIN TYPES OF AFFINE MOTION IN A FINSLER MANIFOLD. I

BY

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1. Introduction. Takano [20], [21] studied certain types of affine motion generated by contra, concurrent, special concircular, recurrent, concircular, torse forming, and birecurrent vectors in a non-Riemannian manifold of recurrent curvature. Following Takano, the authors Sinha [19], Misra [5]-[7], Misra and Meher [8]-[10], Meher [4], and Kumar [1]-[3] studied the above-mentioned types of affine motion in a Finsler manifold of recurrent curvature and in some special Finsler manifolds [5], [10]. In spite of their efforts, none of the above authors could succeed in finding the necessary and sufficient conditions satisfied by the above vectors to generate affine motion even in case of the above particular types of Finsler manifolds. The aim of this paper is to obtain the necessary and sufficient conditions satisfied by those vectors to generate affine motion in a general Finsler manifold. The notation of this paper differs slightly from that of [18].

2. Preliminaries. Let $F_n(F, g, G)$ be an n-dimensional Finsler manifold of class at least C^7 equipped with a metric function F([1]) satisfying the required conditions [18], the corresponding symmetric metric tensor g, and the Berwald's connection G. The coefficients of Berwald's connection G, denoted by G^i_{jk}, satisfy

$$\text{(2.1)} \qquad \text{(a) } G^i_{jk} = G^i_{kj}, \quad \text{(b) } G^i_{jk}\dot{x}^k = G^i_j, \quad \text{(c) } \dot{\partial}_k G^i_j = G^i_{jk},$$

where $\dot{\partial}_k$ stands for partial differentiation with respect to $\dot{x}^k$. The partial derivatives $G^i_{jkh} \overset{\text{def}}{=} \dot{\partial}_h G^i_{jk}$ of the connection parameters G^i_{jk} constitute a tensor symmetric in all its lower indices and satisfy

$$\text{(2.2)} \qquad G^i_{jkh}\dot{x}^h = 0.$$

The covariant derivative of a tensor T^i_j for these connection parameters is

([1]) Unless otherwise stated, all the geometric objects are supposed to be functions of the line elements $(x^i, \dot{x}^i)$. The indices $i, j, k, \ldots$ take positive integer values from 1 to n.

given by

$$\mathscr{B}_k T^i_j = \partial_k T^i_j - (\hat{\partial}_r T^i_j) G^r_k + T^r_j G^i_{rk} - T^i_r G^r_{jk}, \tag{2.3}$$

where $\partial_k \equiv \partial/\partial x^k$. The commutation formulae for the differential operators $\hat{\partial}_k$ and $\mathscr{B}_k$ are

$$(\hat{\partial}\, \mathscr{B}_k - \mathscr{B}_k \hat{\partial}_j) X^i = G^i_{jkh} X^h \tag{2.4}$$

$$(\mathscr{B}_j \mathscr{B}_k - \mathscr{B}_k \mathscr{B}_j) X^i = H^i_{jkh} X^h - (\hat{\partial}_h X^i) H^h_{jk}, \tag{2.5}$$

where H^i_{jkh} [2] constitute Berwald's curvature tensor. This tensor is skew-symmetric in first two lower indices and positively homogeneous of degree zero in $\dot{x}^i$'s. The tensor H^i_{jk} appearing in (2.5) is connected with the curvature tensor by the relations

$$\text{(a) } H^i_{jkh} \dot{x}^h = H^i_{jk}, \quad \text{(b) } \hat{\partial}_h H^i_{jk} = H^i_{jkh}. \tag{2.6}$$

The transvection of the tensor H^i_{jk} by $\dot{x}^k$ yields a tensor H^i_j, called the *deviation tensor*. This tensor satisfies

$$\text{(a) } H^i_{jk} \dot{x}^k = H^i_j, \quad \text{(b) } \tfrac{1}{3}(\hat{\partial}_k H^i_j - \hat{\partial}_j H^i_k) = H^i_{jk}. \tag{2.7}$$

The associate vector y_i of $\dot{x}^i$ satisfies

$$\text{(a) } y_i \dot{x}^i = F^2, \quad \text{(b) } \hat{\partial}_i y_j = g_{ij}, \tag{2.8}$$

$$\text{(c) } y_i H^i_{jk} = 0, \quad \text{(d) } y_i H^i_j = 0, \quad \text{(e) } g_{ij} H^i_k = g_{ik} H^i_j$$

(see [17]), where g_{ij} are components of the metric tensor g.

Let a vector $v^i(x^j)$ generate the infinitesimal transformation

$$\bar{x}^i = x^i + \varepsilon v^i, \tag{2.9}$$

where ε is an infinitesimal constant. Denoting by £ the operator of Lie differentiation with respect to the above transformation, we have

$$\pounds T^i_j = v^r \mathscr{B}_r T^i_j - T^r_j \mathscr{B}_r v^i + T^i_r \mathscr{B}_j v^r + (\hat{\partial}_r T^i_j) \mathscr{B}_s v^r \cdot \dot{x}^s \tag{2.10}$$

and

$$\pounds G^i_{jk} = \mathscr{B}_j \mathscr{B}_k v^i + H^i_{hjk} v^h + G^i_{jkh} \mathscr{B}_r v^h \dot{x}^r, \tag{2.11}$$

where T^i_j is any tensor [22]. The infinitesimal transformation (2.9) is called an *affine motion* if

$$\pounds G^i_{jk} = 0. \tag{2.12}$$

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[2] The symbols H^i_{jkh} used here for Berwald's curvature tensor coincide with H^i_{hkj} of [18], equation (4.6.7).

The vector v^i is called *contra, concurrent, special concircular, recurrent, concircular, torse forming,* and *birecurrent* according as it satisfies (see [20])

(2.13)
(a) $\mathscr{B}_k v^i = 0$,
(b) $\mathscr{B}_k v^i = C\delta^i_k$, C being a constant,
(c) $\mathscr{B}_k v^i = \varrho\delta^i_k$, ϱ is not a constant,
(d) $\mathscr{B}_k v^i = \mu_k v^i$,
(e) $\mathscr{B}_k v^i = \mu_k v^i + \varrho\delta^i_k$, $\mathscr{B}_j \mu_k = \mathscr{B}_k \mu_j$,
(f) $\mathscr{B}_k v^i = \mu_k v^i + \varrho\delta^i_k$,
(g) $\mathscr{B}_j \mathscr{B}_k v^i = \varphi_{jk} v^i$,

respectively. The affine motion generated by the above vectors is called a *contra affine motion,* a *concurrent affine motion,* a *special concircular affine motion,* a *recurrent affine motion,* a *concircular affine motion,* a *torse forming affine motion,* and a *birecurrent affine motion,* respectively.

3. Some lemmas. In this section, we shall prove some lemmas which will be helpful in the discussion of the next sections.

LEMMA 3.1. *For every vector* $v^i(x^j)$, *the following two conditions are equivalent*:

(3.1) (a) $H^i_{jkh} v^h = 0$, (b) $H^i_{hjk} v^h = 0$.

Proof. Let $v^i(x^j)$ be any vector satisfying (3.1a). Transvecting the Bianchi identity [18]

(3.2) $$H^i_{jkh} + H^i_{hjk} + H^i_{khj} = 0$$

by v^h and using (3.1a), we have

(3.3) $$H^i_{hjk} v^h + H^i_{khj} v^h = 0.$$

By the skew-symmetric property of the curvature tensor in first two of its lower indices, (3.3) takes the form

(3.4) $$H^i_{hjk} v^h = H^i_{hkj} v^h.$$

Transvecting (3.4) by $\dot{x}^k$ and using (2.6a), (2.6b), and (2.7a), we have

(3.5) $$2H^i_{hj} v^h = \hat{\partial}_j H^i_h v^h.$$

Transvecting (3.5) by y_i and using (2.8c), we get

(3.6) $$y_i \hat{\partial}_j H^i_h v^h = 0,$$

which, by (2.8b) and (2.8d), reduces to

(3.7) $$g_{ij} H^i_h v^h = 0.$$

Transvecting (3.7) by g^{jm} and using $g_{ij} g^{jm} = \delta^m_i$, we get

(3.8) $$H^i_h v^h = 0.$$

Consequently, (3.5) reduces to

$$H^i_{hj}\, v^h = 0. \tag{3.9}$$

Differentiating (3.9) partially with respect to $\dot{x}^k$ and using (2.6b), we get (3.1b). Conversely, if the vector $v^i(x^j)$ satisfies (3.1b), the transvection of the Bianchi identity (3.2) by v^h gives (3.1a). Hence (3.1a) and (3.1b) are equivalent.

LEMMA 3.2. *There exists no non-zero vector $v^i(x^j)$ orthogonal to $\dot{x}^i$.*

Proof. If there exists a vector $v^i(x^j)$ orthogonal to $\dot{x}^i$, we have $g_{ij}\dot{x}^i v^j = 0$, which may be written as $y_j v^j = 0$. Differentiating $y_j v^j = 0$ partially with respect to $\dot{x}^i$ and using (2.8b), we get

$$g_{ij}\, v^j = 0. \tag{3.10}$$

Consequently, v^i is a zero vector. Thus, we see that if a vector $v^i(x^j)$ is orthogonal to $\dot{x}^i$, it is necessarily a zero vector. This completes the proof.

4. Contra affine motion. Let us consider an infinitesimal transformation generated by a contra vector $v^i(x^j)$ characterized by (2.13a). Differentiating (2.13a) covariantly with respect to x^j we get

$$\mathscr{B}_j \mathscr{B}_k v^i = 0. \tag{4.1}$$

Taking the skew-symmetric part of (4.1) and using (2.5), we have (3.1a), which, by Lemma 3.1, implies (3.1b). By (2.4), from (2.13a) we get

$$G^i_{jkr}\, v^r = 0. \tag{4.2}$$

Using (4.1), (4.2), and (3.1b) in (2.11), we get $\pounds G^i_{jk} = 0$; hence the infinitesimal transformation considered is an affine motion. Thus, we obtain

THEOREM 4.1. *Every contra vector generates an affine motion in a Finsler manifold.*

Since every affine motion is a projective motion and, by Theorem 4.1, every contra vector generates an affine motion, we come to the following conclusion:

COROLLARY 4.1. *Every contra vector generates a projective motion.*

Misra [7] and the author [12] were unaware of this fact. This is why they considered those contra transformations which were projective motions. Misra [7] concluded that a contra projective motion need not be an affine motion in a recurrent Finsler manifold([3]), which appears to be misleading in view of the facts proved above. Since the Lie derivative of the curvature tensor vanishes with respect to an affine motion, in this case we have

$$\pounds H^i_{jkh} = 0. \tag{4.3}$$

([3]) A recurrent Finsler manifold has been denoted by an HR-F_n in [7].

Sinha [19] proved that the recurrence vector λ_m of a recurrent Finsler manifold is Lie invariant with respect to an affine motion, i.e.,

$$\pounds\lambda_m = 0. \tag{4.4}$$

Misra [7] also proved that the Lie derivative of the curvature tensor with respect to a contra projective motion either vanishes or is proportional to itself, but here we observe (cf. equation (4.3)) that it necessarily vanishes.

COROLLARY 4.2. *If any contra vector $v^i(x^j)$ generates an infinitesimal transformation in a recurrent Finsler manifold, it must be orthogonal to the recurrence vector.*

Proof. Let us consider a recurrent Finsler manifold characterized by (see [2]-[4], [6]-[9], [15], [16], [19])

$$\mathscr{B}_m H^i_{jkh} = \lambda_m H^i_{jkh}, \qquad H^i_{jkh} \neq 0, \ \lambda_m \neq 0. \tag{4.5}$$

The non-zero vector λ_m appearing in (4.5) is called the *recurrence vector*. Suppose there is a contra vector $v^i(x^j)$ generating an infinitesimal transformation in the above manifold. Therefore, the curvature tensor satisfies (4.3). Expanding the left-hand side of (4.3) with the help of (2.10), using (2.13a) (the characterizing equation of a contra vector) and (4.5), we get $\lambda_m v^m H^i_{jkh} = 0$. Since $H^i_{jkh} \neq 0$, $v^m \lambda_m = 0$. Thus, we see that the contra vector v^i is orthogonal to the recurrence vector λ_m.

COROLLARY 4.3. *If a projective recurrent Finsler manifold admits an infinitesimal transformation generated by a contra vector $v^i(x^j)$, then the vector v^i is orthogonal to its recurrence vector.*

Proof. Let a projective recurrent Finsler manifold characterized by

$$\mathscr{B}_m W^i_{jkh} = \lambda_m W^i_{jkh}, \qquad W^i_{jkh} \neq 0, \ \lambda_m \neq 0, \tag{4.6}$$

where W^i_{jkh} is the projective curvature tensor, admit an infinitesimal transformation generated by a contra vector v^i. By Corollary 4.1, this transformation is a projective motion, and hence the Lie derivative of the projective curvature tensor vanishes [22], i.e., $\pounds W^i_{jkh} = 0$. Expanding the left-hand side of this equation and using (2.13a) and (4.6), we get $v^m \lambda_m W^i_{jkh} = 0$, which implies $v^m \lambda_m = 0$.

COROLLARY 4.4. *If a birecurrent Finsler manifold characterized by* (see [1], [5], [11])

$$\mathscr{B}_l \mathscr{B}_m H^i_{jkh} = a_{lm} H^i_{jkh}, \qquad a_{lm} \neq 0, \ H^i_{jkh} \neq 0, \tag{4.7}$$

admits an infinitesimal transformation generated by a contra vector $v^i(x^j)$, then the recurrence tensor a_{lm} satisfies

$$\text{(a) } a_{lm} v^m = 0, \quad \text{(b) } a_{ml} v^m = 0. \tag{4.8}$$

Proof. If a birecurrent Finsler manifold characterized by (4.7) admits an infinitesimal transformation generated by a contra vector v^i, then its curvature tensor satisfies (4.3). Expanding (4.3) with the help of (2.10), we have

$$v^m \mathcal{B}_m H^i_{jkh} = 0. \tag{4.9}$$

Differentiating (4.9) covariantly with respect to x^l and using (2.13a) and (4.7), we get (4.8a). Taking the skew-symmetric part of (4.7) with respect to the indices l and m and using (2.5), we get

$$H^r_{jkh} H^i_{lmr} - H^i_{rkh} H^r_{lmj} - H^i_{jrh} H^r_{lmk} - H^i_{jkr} H^r_{lmh} - (\hat{\partial}_r H^i_{jkh}) H^r_{lm} = (a_{lm} - a_{ml}) H^i_{jkh}. \tag{4.10}$$

While proving Theorem 4.1, we have seen that (3.1a) and (3.1b) hold. Transvecting (4.10) by v^m and using (3.1a), (3.1b), and (4.8a), we have (4.8b).

5. Concurrent affine motion. Let us consider a Finsler manifold admitting an infinitesimal transformation generated by a concurrent vector v^i characterized by (2.13b). Differentiating (2.13b) covariantly with respect to x^j, we have (4.1). Taking the skew-symmetric part of (4.1) and using (2.5), we get (3.1a), which, by Lemma 3.1, implies (3.1b). It follows from (2.11), (2.2), (3.1b), and (4.1) that the Lie derivative of the coefficients G^i_{jk} of connection vanishes. Hence, the infinitesimal transformation considered is an affine motion. Thus, we have

THEOREM 5.1. *If a Finsler manifold admits an infinitesimal transformation generated by a concurrent vector, then the transformation is necessarily an affine motion.*

Since the Lie derivative of the curvature tensor vanishes with respect to an affine motion, we have (4.3). Expanding (4.3) with the help of (2.10) and (2.13b), we get

$$v^m \mathcal{B}_m H^i_{jkh} + 2C H^i_{jkh} = 0, \tag{5.1}$$

which, after covariant differentiation, gives

$$v^m \mathcal{B}_l \mathcal{B}_m H^i_{jkh} + 3C \mathcal{B}_l H^i_{jkh} = 0. \tag{5.2}$$

Differentiating (3.1a) covariantly, we get

$$v^m \mathcal{B}_h H^i_{jkm} + C H^i_{jkh} = 0. \tag{5.3}$$

Thus, we obtain

THEOREM 5.2. *If a Finsler manifold admits an infinitesimal transformation generated by a concurrent vector characterized by* (2.13b), *then its curvature tensor satisfies* (5.1)-(5.3).

Let the manifold considered be a recurrent manifold characterized by (4.5). By Theorem 5.2, it admits (5.3). In view of (3.1a) and (4.5), equation (5.3) gives $CH^i_{jkh} = 0$, which implies either $C = 0$ or $H^i_{jkh} = 0$. Both the conditions contradict our hypothesis. Hence, we obtain (see [7])

COROLLARY 5.1. *A recurrent Finsler manifold does not admit any infinitesimal transformation generated by a concurrent vector.*

If the manifold considered is a symmetric manifold characterized by

$$\mathcal{B}_m H^i_{jkh} = 0, \tag{5.4}$$

then equation (5.3) gives $CH^i_{jkh} = 0$, which implies $H^i_{jkh} = 0$ for $C \neq 0$. Thus, we see that a symmetric Finsler manifold admitting an infinitesimal transformation generated by a concurrent vector is necessarily flat([4]). Hence, we get

COROLLARY 5.2. *A non-flat symmetric Finsler manifold does not admit any infinitesimal transformation generated by a concurrent vector.*

If the manifold considered is a birecurrent manifold characterized by (4.7), it admits (5.3). Differentiating (5.3) covariantly with respect to x^l, we get

$$v^m \mathcal{B}_l \mathcal{B}_h H^i_{jkm} + C\mathcal{B}_h H^i_{jkl} + C\mathcal{B}_l H^i_{jkh} = 0. \tag{5.5}$$

Using (3.1a) and (4.7) in (5.5), we obtain

$$\mathcal{B}_h H^i_{jkl} + \mathcal{B}_l H^i_{jkh} = 0, \tag{5.6}$$

since $C \neq 0$. Transvecting (5.6) by v^l and using (5.1) and (5.3), we get $3CH^i_{jkh} = 0$, which implies $H^i_{jkh} = 0$, a contradiction. Thus, we have

COROLLARY 5.3. *A birecurrent Finsler manifold does not admit any infinitesimal transformation generated by a concurrent vector.*

This corollary generalizes the theorems of the author [14] and Misra [5].

Let us consider a bisymmetric Finsler manifold characterized by (see [13])

$$\mathcal{B}_l \mathcal{B}_m H^i_{jkh} = 0. \tag{5.7}$$

If this manifold admits an infinitesimal transformation generated by a concurrent vector v^i, then its curvature tensor satisfies (5.2) and (5.3). In view of (5.7), equation (5.2) reduces to $C\mathcal{B}_l H^i_{jkh} = 0$, which implies $\mathcal{B}_l H^i_{jkh} = 0$ as $C \neq 0$. Using $\mathcal{B}_l H^i_{jkh} = 0$ in (5.3), we get $H^i_{jkh} = 0$. Hence, a bisymmetric Finsler manifold admitting an infinitesimal transformation generated by a concurrent vector is flat. Thus, we have

COROLLARY 5.4. *A non-flat bisymmetric Finsler manifold does not admit any transformation generated by a concurrent vector.*

([4]) A Finsler manifold with vanishing curvature tensor is called *flat*.

6. Special concircular affine motion. Let us consider a Finsler manifold admitting an infinitesimal transformation generated by a special concircular vector characterized by (2.13c). If this transformation is an affine motion, we have $\pounds G^i_{jk} = 0$, which, by (2.11) and (2.13c), gives

$$\mathscr{B}_j \varrho \delta^i_k + H^i_{mjk} v^m = 0. \tag{6.1}$$

Transvecting (6.1) by $\dot{x}^k$ and using (2.6), we have

$$\mathscr{B}_j \varrho \dot{x}^i + H^i_{mj} v^m = 0. \tag{6.2}$$

Transvecting (6.2) by y_i and using (2.8a) and (2.8c), we have $F^2 \mathscr{B}_j \varrho = 0$, which implies $\mathscr{B}_j \varrho = 0$. Thus, we get a contradiction. Hence, we obtain

THEOREM 6.1. *An infinitesimal transformation generated by a special concircular vector cannot be an affine motion in a Finsler manifold. In other words, a Finsler manifold does not admit any special concircular affine motion.*

This theorem, on one hand, represents a generalization of the theorems of Sinha [19], Misra and Meher [10], Kumar [2], and the present author [14], while, on the other hand, contradicts the hypothesis of Misra [5].

7. Recurrent affine motion. Let us consider an infinitesimal transformation generated by a recurrent vector characterized by (2.13d). It follows from (2.11) and (2.13d) that the Lie derivatives of the coefficients of connection are determined by

$$\pounds G^i_{jk} = (\mathscr{B}_j \mu_k + \mu_j \mu_k) v^i + H^i_{mjk} v^m + \mu G^i_{jkr} v^r, \tag{7.1}$$

where $\mu \overset{\text{def}}{=} \mu_h \dot{x}^h$. By (2.4), the partial differentiation of (2.13d) with respect to $\dot{x}^j$ gives

$$G^i_{jkr} v^r = \dot{\partial}_j \mu_k v^i. \tag{7.2}$$

Transvecting (7.2) by $\dot{x}^k$ and using (2.2), we get

$$\mu_j = \dot{\partial}_j \mu. \tag{7.3}$$

Using (7.2) in (7.1), we have

$$\pounds G^i_{jk} = (\mathscr{B}_j \mu_k + \mu_j \mu_k + \mu \dot{\partial}_j \mu_k) v^i + H^i_{mjk} v^m. \tag{7.4}$$

If the above transformation is an affine motion, we have $\pounds G^i_{jk} = 0$. Hence, (7.4) reduces to

$$(\mathscr{B}_j \mu_k + \mu_j \mu_k + \mu \dot{\partial}_j \mu_k) v^i + H^i_{mjk} v^m = 0. \tag{7.5}$$

Transvecting (7.5) by $\dot{x}^k$ and using (2.6a), we get

$$(\mathscr{B}_j \mu + \mu \mu_j) v^i + H^i_{mj} v^m = 0. \tag{7.6}$$

Transvecting (7.6) by y_i and using (2.8c) and Lemma 3.2, we have

$$\mathscr{B}_j \mu + \mu \mu_j = 0. \tag{7.7}$$

From (7.6) and (7.7) we obtain $H^i_{mj} v^m = 0$, which by partial differentiation gives $H^i_{mjk} v^m = 0$. Using this result in (7.5), we get

$$\mathscr{B}_j \mu_k + \mu_j \mu_k + \mu \mathring{\partial}_j \mu_k = 0. \tag{7.8}$$

Thus, we see that condition (7.8) is a necessary consequence of a recurrent affine motion. Now we shall establish that condition (7.8) is sufficient for an infinitesimal transformation generated by a recurrent vector characterized by (2.13d) to be an affine motion. To prove this, let us assume that condition (7.8) holds. Taking a skew-symmetric part of (7.8), we have

$$\mathscr{B}_j \mu_k - \mathscr{B}_k \mu_j = 0. \tag{7.9}$$

Differentiating (2.13d) covariantly with respect to x^j, and then taking a skew-symmetric part, we have $H^i_{jkm} v^m = 0$ (here we have used (2.5) and (7.9)). By Lemma 3.1, $H^i_{jkm} v^m = 0$ implies $H^i_{mjk} v^m = 0$. Using this equation and (7.8) in (7.4), we get $\pounds G^i_{jk} = 0$. Hence the transformation considered is an affine motion. Thus, we obtain

THEOREM 7.1. *Condition* (7.8) *is necessary and sufficient for an infinitesimal transformation generated by a recurrent vector* v^i *characterized by* (2.13d) *to be an affine motion.*

If the manifold considered is a Landsberg manifold characterized by

$$y_i G^i_{jkh} = 0, \tag{7.10}$$

the transvection of (7.2) by y_i and the application of (7.10) and Lemma 3.2 give $\mathring{\partial}_j \mu_k = 0$. Hence, in this case, condition (7.8) takes the form

$$\mathscr{B}_j \mu_k + \mu_j \mu_k = 0. \tag{7.11}$$

Thus, we obtain

COROLLARY 7.1. *Condition* (7.11) *is necessary and sufficient for an infinitesimal transformation generated by a recurrent vector to be an affine motion in a Landsberg manifold.*

Since an affinely connected Finsler manifold is a Landsberg manifold, Corollary 7.1 is also true in this case.

Note. The rest three types of affine motion will be discussed in the next paper.

REFERENCES

[1] A. Kumar, *On some type of affine motion in birecurrent Finsler spaces II*, Indian Journal of Pure and Applied Mathematics 8 (1977), p. 505-513.

[2] – *Some theorems on affine motion in a recurrent Finsler space. IV*, ibidem 8 (1977), p. 672-684.

[3] – *On the existence of affine motion in a recurrent Finsler space*, ibidem 8 (1977), p. 791-800.

[4] F. M. Meher, *An SHR-F_n admitting an affine motion. II*, Tensor (New Series) 27 (1973), p. 208-210.
[5] R. B. Misra, *A birecurrent Finsler manifold with affine motion*, Indian Journal of Pure and Applied Mathematics 6 (1975), p. 1441-1448.
[6] – *A turning point in the theory of recurrent Finsler manifolds*, Journal of the South Gujarat University 6 (1977), p. 72-96.
[7] – *A turning point in the theory of recurrent Finsler manifolds. II: certain types of projective motions*, Bolletino della Unione Matematica Italiana (5) 16-B (1979), p. 32-53.
[8] – and F. M. Meher, *An SHR-F_n admitting an affine motion*, Acta Mathematica Academiae Scientiarum Hungaricae 22 (1971), p. 423-429.
[9] – *On the existence of affine motion in an HR-F_n*, Indian Journal of Pure and Applied Mathematics 3 (1972), p. 219-225.
[10] – *CA-motion in a PS-F_n*, ibidem 6 (1975), p. 522-526.
[11] A. Moór, *Über Finslerräume von zweifach rekurrenter Krümmung*, Acta Mathematica Academiae Scientiarum Hungaricae 22 (3-4) (1971), p. 453-465.
[12] P. N. Pandey, *Contra projective motion in a Finsler manifold*, The Mathematics Education 11 (2) (1977), p. 25-29.
[13] – *On bisymmetric Finsler manifolds*, ibidem 11 (4) (1977), p. 77-80.
[14] – *CA-collineation in a birecurrent Finsler manifold*, Tamkang Journal of Mathematics 9 (1978), p. 79-81.
[15] – *A recurrent Finsler manifold with a concircular vector field*, Acta Mathematica Academiae Scientiarum Hungaricae 35 (3-4) (1980), p. 465-466.
[16] – *A note on recurrence vector*, Proceedings of the National Academy of Sciences of India, Section A, 50 (1980), p. 6-8.
[17] – *On decomposability of curvature tensor of a Finsler manifold*, Acta Mathematica Academiae Scientiarum Hungaricae 38 (1981), p. 109-116.
[18] H. Rund, *The differential geometry of Finsler spaces*, Springer-Verlag, 1959.
[19] R. S. Sinha, *Affine motion in recurrent Finsler spaces*, Tensor (New Series) 20 (1969), p. 261-264.
[20] K. Takano, *Affine motion in non-Riemannian K*-spaces, I, II, III* (with M. Okumura), *IV, V*, ibidem 11 (1961), p. 137-143, 161-173, 174-181, 245-253, 270-278.
[21] – *On the existence of affine motion in the space with recurrent curvature*, ibidem 17 (1966), p. 68-73, 212-216.
[22] K. Yano, *The theory of Lie derivatives and its applications*, Amsterdam 1957.

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