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UNSTEADY STRATIFIED FLOW PAST A CYLINDER

1. Introduction.

The main work in inviscid, non-diffusive, stratified fluid flow has been carried out by Long [3], Yih [7], Kao [2] and Debler [1]. All these are based on the steady non-linear equations of motion which, with the assumption of suitable upstream conditions, reduce to a linear equation. The solutions obtained suffer from the disadvantage that they violate the assumption of the perturbed motion to vanish at a sufficiently great distance upstream.

Kathleen Trustrum [5] has shown that the assumption of a uniform undisturbed upstream flow which has been basic to most theories is not valid. Thus the steady-state equations which take the linear form with the above assumption become more difficult to deal with, as one cannot envisage what corresponding conditions should be taken for the upstream flow.

In her paper she has pointed out the qualitative resemblances of the equations governing rotating and stratified flows past an obstacle. For a perturbation introduced on the plane $x = 0$ as one Fourier component she has studied the upstream and downstream solutions for $R \gtrless 1$ (R being the Froude number for stratified fluids and the Rossby number in the case of rotating fluids). In the upstream flow for $R > 1$ the perturbation flow decays exponentially as $x \to -\infty$ and has an irrotational character, but for $R < 1$ the solution is independent of x and describes a one-dimensional flow extending to upstream infinity. In the downstream flow for $R > 1$ the solution has an irrotational character and for $R < 1$ it gives rise to waves in the downstream. From the experiments on rotating fluids [4] it is well known that for low Rossby numbers the wave motion is observed only downstream.

In this paper we consider the unsteady two-dimensional inviscid non-diffusive stratified fluid flow past a circular cylinder for small values of the Froude number. The flow considered here is that which is due to the slow uniform motion, started impulsively from relative rest, of a circular cylinder along the horizontal axis of it. The equations are written in non-

-dimensional form with $F = U(gl)^{-1/2}$, the Froude number, as a dimensionless parameter (U is the characteristic velocity, l — the radius of the cylinder). After taking the Laplace transforms the governing equations have been reduced to a single linear equation in $\overline{N}$, where $\overline{N} = \overline{p} - \chi, \overline{p}$ being the transform of non-dimensional pressure and χ the velocity potential of the initial irrotational flow. The flow pattern is discussed for various regions for small and large times. It has been shown that the flow will never become stready on the cylinder. In general the flow is of oscillatory type whose amplitude of oscillation decreases to zero as time progresses, so that the ultimate motion tends to a steady state everywhere except on the cylinder and on the axis of the cylinder where the perturbation velocities continue to oscillate indefinitely with small amplitude.

2. Governing equations and solution.

The Euler equations of two-dimensional flow of a stratified fluid which is assumed to be incompressible, inviscid and non-diffusive are [5]:

$$\varrho' \frac{\partial u'}{\partial t'} + \varrho' u' \frac{\partial u'}{\partial x'} + \varrho' w' \frac{\partial u'}{\partial z'} = - \frac{\partial p'}{\partial x'}, \tag{2.1}$$

$$\varrho' \frac{\partial w'}{\partial t'} + \varrho' u' \frac{\partial w'}{\partial x'} + \varrho' w' \frac{\partial w'}{\partial z'} = - \varrho' g - \frac{\partial p'}{\partial z'}, \tag{2.2}$$

in which u', w' are the velocity components parallel to Ox' and Oz', respectively, where z' is measured in a direction opposing gravity. Since the fluid is incompressible and non-diffusive,

$$\frac{\partial \varrho'}{\partial t'} + u' \frac{\partial \varrho'}{\partial x'} + w' \frac{\partial \varrho'}{\partial z'} = 0 \tag{2.3}$$

and the continuity equation reduces to

$$\frac{\partial u'}{\partial x'} + \frac{\partial w'}{\partial z'} = 0. \tag{2.4}$$

Let us consider the initial stratification of the fluid to be $\varrho_0' - \beta z'$, where ϱ_0' corresponds to the characteristic density and β is the stratification constant.

Let a cylinder impulsively start from rest and move along the x-axis at $t' = 0$ with a uniform velocity U. If we choose the origin of coordinates to be in the body, we have in effect superposed a uniform velocity $-U$ on the system and brought the body to rest. Let the subsequent velocity components parallel to the x' and z' axes be (u', w'), the density be

$\varrho_0'-\beta z'+\varrho''$ and the pressure be p_0+p'', where u', w', p', ϱ'' are assumed to be small and where

$$\frac{dp_0}{dz'} = -(\varrho_0'-\beta z')g.$$

Introducing the non-dimensional variables as

$$u = \frac{u'}{U}, \quad w = \frac{w'}{U}, \quad x = \frac{x'}{l}, \quad z = \frac{z'}{l}, \quad p = \frac{p''}{v^2\varrho_0'}, \quad t = \frac{t'U}{l}$$

and if the equations (2.1)-(2.4) are written in non-dimensional form we find that U^2/gl, the Froude number, is a dimensionaless parameter of the problem. Here l is the characteristic length (radius of the cylinder) and U the characteristic velocity of the problem.

Taking slow motion into consideration for small F, the linearized equations of motion in the non-dimensional form will be

(2.5) $$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x},$$

(2.6) $$\frac{\partial w}{\partial t} = -\frac{gl}{U^2}\varrho - \frac{\partial p}{\partial z},$$

(2.7) $$\frac{\partial \varrho}{\partial t} = \frac{\beta l}{\varrho_0'} w,$$

(2.8) $$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0.$$

The boundary conditions are that

(2.9) $u \to -1, \quad w \to 0 \quad$ as $\quad x \to -\infty$ for fixed $\quad z, t,$

and, on the body, that

(2.10) the component of the fluid velocity normal to the body is zero.

Let the velocity potential of the initial perturbed motion be

$$\Phi(x, z) = -x+\chi(x, z)$$

(for an impulsive start the initial motion will be irrotational).

We have at $t = 0$

$$u = -1+\frac{\partial \chi}{\partial x}, \quad w = \frac{\partial \chi}{\partial z}.$$

Let the Laplace transforms of u, w, p be $\bar{u}, \bar{w}, \bar{p}$, respectively, i. e.

$$\bar{u} = \int_0^\infty u(x, z) e^{-pt} dt, \qquad \text{etc.}$$

Now, taking Laplace transforms and introducing a function [6] $\bar{N} = \bar{p} - \chi$, the equations (2.5)-(2.8) can be reduced (by substituting the values for $\bar{u}$ and $\bar{w}$ in the transformed continuity equation) to

$$\frac{\partial^2 \bar{N}}{\partial x^2} + \frac{1}{1+a'^2} \frac{\partial^2 \bar{N}}{\partial z^2} = 0, \tag{2.11}$$

where $a' = 1/Rp$, R being the modified Froude number $U(g\beta/p_0 l^2)^{-1/2}$. The boundary condition (2.9) becomes

$$\bar{u} \to -\frac{1}{p}, \qquad \bar{w} \to 0 \quad \text{as} \quad x \to \infty. \tag{2.12}$$

Taking the transformation $X = x$, $Z = z\sqrt{1+a'^2}$ equation (2.11) reduces to

$$\frac{\partial^2 \bar{N}}{\partial X^2} + \frac{\partial^2 \bar{N}}{\partial Z^2} = 0. \tag{2.13}$$

In the transformed coordinates the boundary condition (2.12) becomes

$$\frac{\partial \bar{N}}{\partial X} \to 0 \quad \text{as} \quad x \to \infty \tag{2.14}$$

and the condition on the body (2.10) will reduce to

$$X \frac{\partial \bar{N}}{\partial X} + \frac{Z}{1+a'^2} \frac{\partial \bar{N}}{\partial Z} = -X. \tag{2.15}$$

(The circular cylinder $x'^2 + z'^2 = l^2$ will become an elliptic cylinder $X^2 + Z^2/(1+a'^2) = 1$ in the transformed coordinates.)

Now the equation (2.13) is to be solved with boundary conditions (2.14) and (2.15).

We introduce new coordinates ξ, η defined by

$$X = c\xi\eta, \qquad Z = c(1+\xi^2)^{1/2}(1-\eta^2)^{1/2}.$$

(In the (ξ, η)-plane the ellipse $X^2 + \dfrac{Z^2}{1+a'^2} = 1$ will be given by $\xi = Rp$ (constant).)

The equation (2.13) will take the form

$$(2.16)\qquad (1+\xi^2)\frac{\partial^2 \bar{N}}{\partial \xi^2}+(1-\eta^2)\frac{\partial^2 \bar{N}}{\partial \eta^2}+\xi\frac{\partial \bar{N}}{\partial \xi}-\eta\frac{\partial \bar{N}}{\partial \eta}=0.$$

The boundary condition (2.15) will become

$$(2.17)\qquad \frac{\partial \bar{N}}{\partial \xi}(\text{at}\,\xi = Rp) = -c\eta, \quad \text{where} \quad c=\frac{1}{Rp}.$$

In view of the above condition the general solution of the equation (2.16) can be written as

$$\bar{N} = (C_1\xi + C_2\sqrt{1+\xi^2})\eta,$$

C_1 and C_2 being arbitrary constants.

Using the boundary condition (2.14) we get $C_1 = -C_2$. By applying the condition (2.17) on the body, we find

$$C_1 = \frac{c\sqrt{c^2+1}}{1-\sqrt{c^2+1}}, \quad \text{where} \quad c = \frac{1}{Rp}.$$

Thus the appropriate solution of (2.16) is

$$(2.18)\qquad \bar{N} = \frac{c\sqrt{c^2+1}}{1-\sqrt{c^2+1}}(\xi-\sqrt{1+\xi^2})\eta.$$

Transferring back to the original coordinate system (x, z) we get

$$(2.19)\qquad \bar{N} = \frac{\sqrt{p^2+a^2}}{p-\sqrt{p^2+a^2}}\left(x-\frac{\sqrt{2(p^2+a^2)}}{n_1}xz\right),$$

$$(2.20)\qquad \bar{u} = -\frac{1}{p}-\frac{\sqrt{p^2+a^2}}{(p-\sqrt{p^2+a^2})p}+$$

$$+\frac{(p^2+a^2)n_2^2 z}{p(p-\sqrt{p^2+a^2}[((z^2-1)a^2+p^2(x^2+z^2))^2+4a^2p^2x^2]^{1/2}\times n_1},$$

$$(2.21)\quad \bar{w} = \frac{-\sqrt{2}a^2xpn_1}{(p-\sqrt{p^2+a^2})[((z^2-1)a^2+p^2(x^2+z^2))^2+4a^2p^2x^2]^{1/2}\times n_2^2},$$

where $a = 1/R$ and

$$n_1 = \{(z^2-x^2)p^2+(z^2-1)a^2+[((z^2-1)a^2+p^2(x^2+z^2))^2+4a^2p^2x^2]^{1/2}\}^{1/2},$$

$$n_2 = \{(z^2+x^2)p^2+(z^2-1)a^2+[((z^2-1)a^2+p^2(x^2+z^2))^2+4a^2p^2x^2]^{1/2}\}^{1/2}.$$

The velocity at any point in the fluid is given by the inverse transforms of $\bar{u}$ and $\bar{w}$.

3. General features of the flow.

3.1. On the cylinder.

$$\bar{N} = \frac{\sqrt{p^2+a^2}}{p}, \tag{3.1.1}$$

$$\bar{u} = -\frac{1}{p} - \frac{\sqrt{p^2+a^2}}{p^2} - \frac{(p^2+a^2)a^2x^2}{p^2(p-\sqrt{p^2+a^2})(p^2+a^2x^2)}, \tag{3.1.2}$$

$$\bar{w} = \frac{-a^2xz}{(p-\sqrt{p^2+a^2})(p^2+a^2x^2)}. \tag{3.1.3}$$

Taking the inverse transforms of the above we will get the expressions for the velocities on the cylinder as

$$u = -z^2(\cos axt - \frac{z}{x}\sin axt) + \frac{2}{\pi}\int_0^\infty \frac{\lambda(\mu'\cos at - \lambda'\sin at)}{\lambda'^2+\mu'^2} e^{-\lambda t} d\lambda +$$

$$+ \frac{2}{\pi}x^2\left\{\int_0^\infty \frac{\lambda^3[(\lambda'+4a^2)(\lambda'+a^2x^2)-4a^4]\sin at}{[(\lambda'+a^2x^2)^2+\mu'^2](\lambda'^2+\mu'^2)} e^{-\lambda t} d\lambda -\right.$$

$$\left. - \int_0^\infty \frac{2\lambda^2 a[\lambda'(\lambda^2+a^2)+\mu'^2+a^4x^2]\cos at}{[(\lambda'+a^2x^2)^2+\mu'^2](\lambda'^2+\mu'^2)} e^{-\lambda t} d\lambda\right\},$$

$$w = xz(\cos axt + \frac{z}{x}\sin axt) +$$

$$+ \frac{xz}{\pi}\left\{\int_0^\infty \frac{4a\lambda^2\cos at}{(\lambda'+a^2x^2)^2+\mu'^2} e^{-\lambda t} d\lambda + \int_0^\infty \frac{2\lambda(\lambda'+a^2x^2)\sin at}{(\lambda'+a^2x^2)^2+\mu'^2} e^{-\lambda t} d\lambda\right\},$$

where $\lambda' = \lambda^2 - a^2$, $\mu' = 2\lambda a$ and $a \neq 0$. (The integrals of the inverse Laplace transformation have been evaluated by inserting cuts in the p-plane from $p = +ia$ along lines on which the imaginary part of p is constant and the real part (λ) decreases. The path of integration may be replaced by a path round the infinite semi-circle $\mathrm{Re}(p) < 0$ and round the cuts.)

Thus we see that the flow does not become steady on the cylinder. At any time t the pressure on the cylinder will be

$$ax\int_0^t \frac{1}{t} J_1(at)\,dt.$$

Hence the drag on the cylinder will be

$$-\pi a l^2 \int_0^t \frac{1}{t} J_1(at)\,dt$$

(when $a = 0$, $u = -2z^2$, $w = 2xz$, which corresponds to the case of ordinary flow without stratification).

3.2. On the axis of the cylinder ($z = 0$).

(3.2.1) $$\bar{u} = -\frac{1}{p} - \frac{\sqrt{p^2+a^2}}{(p-\sqrt{p^2+a^2})p} + \frac{x\sqrt{p^2+a^2}}{(p-\sqrt{p^2+a^2})\sqrt{p^2x^2+a^2}}.$$

$\bar{u}$ should satisfy the boundary condition at infinity, namely that $\bar{u} \to -1$ as $x \to \infty$.

When x is large

(3.2.2) $$\bar{u} = -\frac{1}{p} - \frac{\sqrt{p^2+a^2}}{(p-\sqrt{p^2+a^2})p} + \frac{p\sqrt{p^2+a^2}}{(p-\sqrt{p^2+a^2})\left(p^2+\dfrac{a^2}{2x^2}\right)}.$$

Expanding the integrand (3.2.1) in a series of descending powers of p we get for small t after inversion

$$u = -\left(1-\frac{1}{x^2}\right) + \frac{a^2t^2}{4x^2} + \dots \quad \text{etc.}$$

(For $a = 0$ we have $u = -(1-1/x^2)$ which corresponds to the case of ordinary fluids).

Using the asymptotic expansion method to (3.2.2) we get for large times

$$u \sim -\frac{\sqrt{2x^2-1}}{2x^2}\left(\sqrt{2x^2-1}\cos\frac{at}{\sqrt{2}x} - \sin\frac{at}{\sqrt{2}x}\right) + \frac{4\sqrt{2}x^2\cos\left(at+\dfrac{\pi}{4}\right)\Gamma\left(\dfrac{3}{2}\right)}{a^{3/2}(2x^2-1)\pi t^{3/2}} - \frac{2\sqrt{2}\cos\left(at+\dfrac{\pi}{4}\right)\Gamma\left(\dfrac{3}{2}\right)}{a^{3/2}\pi t^{3/2}}.$$

Here $w = 0$, i. e. there is no velocity in the z direction for the fluid particles lying on the x-axis.

Thus the motion here is of an oscillatory type and the velocity will continue to oscillate indefinitely with small amplitude.

Similarly, we find the flow pattern on the planes $z = \pm 1$. It is found that on $z = \pm 1$ for small time

$$u = -\left(1 - \frac{x^2-1}{(x^2+1)^2}\right) + \frac{a^2x^2(x^6+4x^4+10x^3+3)}{(x^2+1)^4}\,\frac{t^2}{2} + \dots \quad \text{etc.}$$

$$w = \frac{2x}{(x^2+1)^2} + \frac{a^2x^3(x^2-2)}{(x^2+1)^4}\,t^2 + \dots \quad \text{etc.}$$

For large time the flow will be singular(¹).

3.3. Flow at any general point of the fluid. The velocity components at any point of the fluid are given by (2.20) and (2.21) as

$$u = -1 - \frac{1}{2\pi i}\int_{\nu-i\infty}^{\nu+i\infty} \frac{\sqrt{p^2+a^2}}{\sqrt{p-(p^2+a^2)}}\,\frac{e^{pt}}{p}\,dp +$$

$$+ \frac{z}{2\pi i}\int_{\nu-i\infty}^{\nu+i\infty} \frac{(p^2+a^2)n_2^2 e^{pt}\,dp}{p(p-\sqrt{(p^2+a^2)})\sqrt{[(z^2-1)a^2+p^2(x^2+z^2)]^2+4a^2p^2z^2}\times n_1},$$

$$w = -\frac{\sqrt{2}a^2x}{2\pi i}\int_{\nu-i\infty}^{\nu+i\infty} \frac{pn_1e^{pt}\,dp}{(p-\sqrt{p^2+a^2})\sqrt{[(z^2-1)a^2+p^2(x^2+z^2)]^2+4a^2p^2x^2}\times n_2^2}.$$

The above integrals are having $\pm ia$, $\pm ial_1$, $\pm ial_2$ as their branch points, where

$$l_j = \frac{x+(-1)^{j-1}z\sqrt{x^2+z^2+1}}{x^2+z^2} \quad \text{and} \quad a = \frac{1}{R}.$$

Thus for large values of t the integrals may be evaluated by inserting cuts in the p-plane from $p = \pm ial_2$, $p = \pm ial_1$ and $p = \pm ia$ along lines on which the imaginary part of p is constant and the real part decreases. The path of integration may now be replaced by a path rougd the infinite semi-circle $Re(p) < 0$ and round the six cuts. The contribution from each branch point can be evaluated separately.

(¹) „The longitudinal velocity u increases indefinitely with time and is of the order $O(t^{1/2})$, whereas the vertical component w tends to zero as t tends to infinity. Thus ultimately the parallel planes $z = \pm 1$ behave as singular surfaces separating the fluid into two regions wherein the fluid between the planes is completely blocked by the cylinder (the phenomenon of Taylor column) and the fluid outside the planes possesses a finite horizontal velocity."

For large time t the contribution from the branch points $p = \pm ial_2$ to u is found to be

$$\frac{z(l_2^2-1)[(z^2-1)-l_2^2(x^2+z^2)]\sin\left(\Phi - al_2t+\dfrac{\pi}{4}\right)}{a^{1/2}l_2^{3/2}(l_1^2-l_2^2)^{1/2}(x^2+z^2)[(z^2-1)-l_2^2(z^2-x^2)]^{1/2}} \times \frac{1}{\sqrt{\pi t}} + $$
$$+ (\text{terms of higher order in } 1/\sqrt{t}),$$

where $\Phi = \tan^{-1}\dfrac{l_2}{\sqrt{1-l_2^2}}$.

The contribution from the branch point $\pm ial_1$ to u will be

$$\frac{z(l_1^2-1)[(z^2-1)-l_1^2(x^2+z^2)]\sin\left(\Psi - al_1t+\dfrac{\pi}{4}\right)}{a^{1/2}l_1^{3/2}(l_2^2-l_1^2)^{1/2}(x^2+z^2)[(z^2-x^2)l_1^2-(z^2-1)]^{1/2}} \times \frac{1}{\sqrt{\pi t}} + $$
$$+ (\text{terms of higher order in } 1/\sqrt{t}),$$

where $\Psi = \tan^{-1}\dfrac{l_1}{\sqrt{1-l_1^2}}$.

The contribution from the branch point $\pm ia$ to u will be

$$\frac{-2\sqrt{2}\cos\left(at+\dfrac{\pi}{4}\right)}{a^{3/2}} \cdot \frac{\Gamma\left(\dfrac{3}{2}\right)}{\pi t^{3/2}} - \frac{4\sqrt{2}z\sin\left(at-\dfrac{\pi}{4}\right)}{a^{3/2}(x^2-1)^{3/2}} \cdot \frac{\Gamma\left(\dfrac{5}{2}\right)}{\pi t^{5/2}} + $$
$$+ (\text{terms of higher order in } 1/\sqrt{t})$$

when $|x| > 1$, and

$$\frac{-2\sqrt{2}\cos\left(at+\dfrac{\pi}{4}\right)}{a^{3/2}} \cdot \frac{\Gamma\left(\dfrac{3}{2}\right)}{\pi t^{3/2}} - \frac{2\sqrt{2}x\cos\left(at-\dfrac{\pi}{4}\right)}{a^{3/2}(1-x^2)^{1/2}} \cdot \frac{\Gamma\left(\dfrac{3}{2}\right)}{\pi t^{3/2}} + $$
$$+ (\text{terms of higher order in } 1/\sqrt{t})$$

when $|x| < 1$.

Similarly, for large time the contribution from the branch point $p = \pm ial_2$ to w is found to be

$$\frac{2x[(z^2-1)-l_2^2(z^2-x^2)]^{1/2}l_2^{1/2}\cos\left(\Phi + al_2t+\dfrac{\pi}{4}\right)}{a^{1/2}(l_1^2-l_2^2)^{1/2}(x^2+z^2)[(z^2-1)-l_2^2(x^2+z^2)]} \times \frac{1}{\sqrt{\pi t}} + $$
$$+ (\text{terms of higher order in } 1/\sqrt{t}).$$

The contribution from the branch point $\pm i a l_1$ will be

$$\frac{2x[l_1^2(z^2-x^2)-(z^2-1)]^{1/2} l_1^{1/2} \cos\left(\Psi + a l_1 t + \frac{\pi}{4}\right)}{a^{1/2}(l_1^2-l_2^2)^{1/2}(x^2+z^2)[(z^2-1)-l_1^2(x^2+z^2)]} \times \frac{1}{\sqrt{\pi t}} +$$

$$+ (\text{terms of higher order in } 1/\sqrt{t}),$$

and the contribution from $\pm ia$ to w will be

$$\frac{2x\cos\left(at-\frac{\pi}{4}\right)}{a^{3/2}(x^2-1)^{1/2}} \cdot \frac{\Gamma\left(\frac{3}{2}\right)}{\pi t^{3/2}} + (\text{terms of higher order in } 1/\sqrt{t})$$

when $|x| > 1$, and

$$\frac{2\sqrt{2} z \sin\left(at+\frac{\pi}{4}\right)}{a^{1/2}(1-x^2)^{3/2}} \cdot \frac{\Gamma\left(\frac{3}{2}\right)}{\pi t^{3/2}} + (\text{terms of higher order in } 1/\sqrt{t})$$

when $|x| < 1$. Φ and Ψ have the same meaning as before.

For small t, formulae for u and w can be found by expanding the integrals into a series of descending powers of p, as in section 3.2.

Thus we find that, in general, the amplitude of the oscillatory motion decreases to zero so that the ultimate flow is steady except in the regions considered in sections 3.1 and 3.2.

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NIESTACJONARNE PRZEPŁYWY WOKÓŁ WALCA PORUSZAJĄCEGO SIĘ W CIECZY

STRESZCZENIE

W pracy badane są za pomocą transformaty Laplace'a niestacjonarne przepływy wywołane ruchem walca kołowego w cieczy o gęstości zależnej od współrzędnej z. Stwierdzono istnienie dwu wyróżnionych płaszczyzn stycznych do walca, wzdłuż których składowe prędkości stają się nieskończone, oraz niestacjonarność przepływu na powierzchni walca. Wynika stąd wniosek, że przepływ w tych rejonach nie dopuszcza linearyzacji odpowiednich równań rządzących przepływem. Lepszą aproksymację przepływu można uzyskać, uwzględniając w równaniach przepływu, przynajmniej częściowo, nieliniowe wyrazy inercjalne.

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НЕСТАЦИОНАРНЫЕ ПОТОКИ ВОКРУГ ЦИЛИНДРА ДВИЖУЩЕГОСЯ В ЖИДКОСТИ

РЕЗЮМЕ

В статьи рассматриваются с помощью преобразования Лапласа нестационарные потоки, вызванные движением круглого цилиндра в жидкости с плотностью зависимой от координаты z. Доказано существование двух сингулярных плоскостей, касающихся цилиндра, на которых компоненты скорости стремятся к бесконечности, а также нестационарность потока на поверхности цилиндра. Отсюда следует, что поток в этих областях не допускает линеаризации соответствующих уравнений, определяющих потоки. Лучшую аппроксимацию можно получить, учитывая в уравнениях потока, хотябы частично, нелинейные инерциальные члены.