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GEOMETRY OF THE FREE PART OF THE SHELL OF AN AIR SPRING

1. Introduction. Air springs are widely applied in motor-cars and railroad vehicles (see [1], [3], [5], [6], [9], [10], [14], [16]-[18]). They are used in buses and big lorries and in the passenger railroad vehicles as well. In Poland, the buses "Jelcz—Berliet" are equipped with air springs of type "sleeve with cover" (Fig. 1f). The reason for such a large propagation of air springs is that they have many advantages as compared with gum and steel springs. Their main advantages are the following:

it is easy to obtain the required characteristic (1) which may be regulated; for instance, the load being changed, the self-active return of the car body to the given height may be assured;

it is possible to disjoint constructing the springs and constructing the conduct of wheels;

large changes of shape yield small tensions in the shell;

the acoustic isolation is given as an additional effect;

the pressure in the air spring may be used to measure the load of the vehicle for controlling the brake system.

By a *spring* we mean in the sequel the system consisting of:

(i) a given quantity of gas whose pressure is greater than 1 atm. and which may be compressed and decompressed;

(ii) a vessel containing this gas and having a flexible non-expanding shell as an essential part.

The typical air springs are shown in Fig. 1. They are all circle-shaped except for the bag (Fig. 1g,h) whose shape is a lengthened one.

The energy of decompressing the gas contained in the spring from

(1) The *characteristic of a spring* is a function which describes the dependence between bending and load.

the volume v to the volume v_1 is given by the formula

$$L = \int_{v}^{v_1} p\,dv. \tag{1}$$

It is equivalent to the energy of compressing which consists of the work L_P done by the external force P and of the energy produced by the atmospheric pressure p_a:

$$L = -\int_{h_1}^{h} P\,dh - \int_{v_1}^{v} p_a\,dv. \tag{2}$$

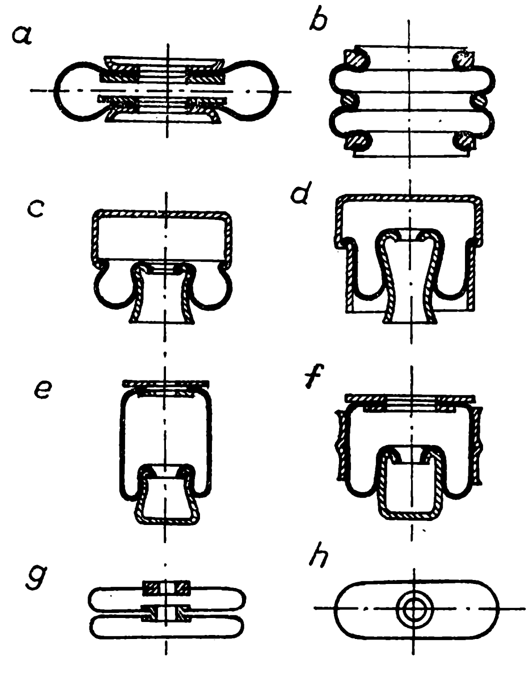

Fig. 1

Comparing (1) and (2) we obtain the following formula for the value of the work done by the external force:

$$L_P = \int_{v}^{v_1} p\,dv - \int_{v}^{v_1} p_a\,dv.$$

The derivation of L_P with respect to the height h of the spring yields

$$\frac{dL_P}{dh} = \frac{dL_P}{dv}\,\frac{dv}{dh} = -(p-p_a)\,\frac{dv}{dh}.$$

Introducing the bending $f = h_1 - h$, we obtain

$$P = \frac{dL_P}{dh} = (p - p_a)\frac{dv}{df}.$$

The form of the function $p(v)$ follows from the well-known principles of thermodynamics. The process in gas connected with the change of the spring shape is in general a polytropic one with variable exponent. It is close to the adiabatic one if this change is quick and to the isothermic one in the case of a slow change. If the increments of the volume are small relatively to the global volume of the air, this process may be approximately treated as an isobaric one. The form of the function $f(v)$ is connected with the shape of a spring and with the way of its deformation. A most frequently used model is the piston in a cylinder with a supplementary volume v_z. Then $v = Sh + v_z$, where S denotes the area of the piston. The domain of validity of this computing model has been treated in [12]. A more exact model of an air spring or of its part is a solid of revolution, consisting of a flexible non-expanding ring-shaped fold and of two rigid plane targets (Fig. 2). The parallels of the fold are circles

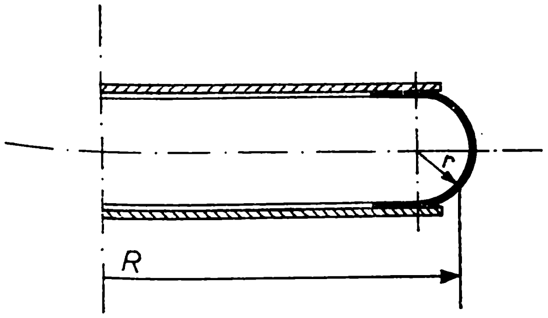

Fig. 2

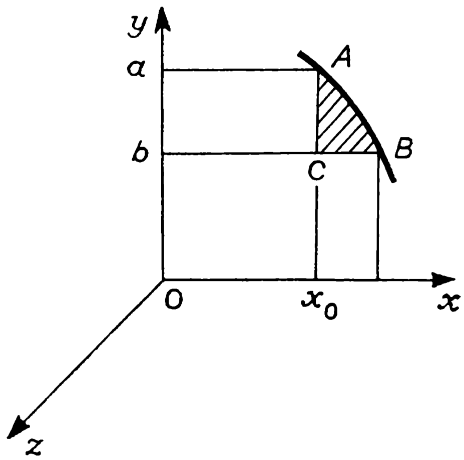

Fig. 3

because such a shape has been given to it. Usually, the meridians are also treated as circles [7], [8], [13], but generally there is no reason for making such an assumption. A more detailed study of a meridian section of the fold is the purpose of this paper.

2. Formulation of the problem. It is known that the gas compressed in a vessel tends to have the maximal volume. Accordingly, the shell tends to have such a shape which assures the maximal volume of the solid with given area of its surface. Therefore, to define the meridian section of the fold, we are to solve the following problem (Fig. 3):

Suppose that two points A, B of the (x, y)-plane are joined by a line such that the surface obtained by its rotation about the y-axis has a given area. Choose this line in such a way that the volume of the solid obtained by rotation of the figure $aABb$ is a maximal one.

In such a formulation of the problem the fold has to be convex. Therefore, the point of the line which is the nearest to the axis of rotation should be one of the points A and B. Accordingly to Fig. 3, let A be such a point. Our problem can be formulated equivalently if we subtract from the volume of the solid the volume of a cylinder of radius Aa and height ab. Such a cavity of the solid makes it easier to compare its volumes corresponding to different choices of the shapes of its surface.

The formulated here "space" problem may be replaced by a "plane" one if we use the well-known theorems due to Guldin (see [11]). They state that the ratio of the area of a surface of revolution to the statical moment of the line circumscribing this surface and the ratio of the volume of the solid of revolution to the statical moment of the plane section of this solid are both constant and equal to 2π. Therefore, our problem may be reformulated as follows (Fig. 3):

Given a statical moment of a line AB with respect to the y-axis, we seek such a form of this line for which the statical moment of the surface ACB with respect to the y-axis is a maximal one. Such a "plane" formulation of the problem is very convenient because it makes easy the verification of the obtained results if we know the length of the line, the area of the surface, and the position of its centres of gravity.

3. Derivation of the equation of the line. The statical moment of the plane domain with respect to the y-axis (Fig. 3) is given by the formula

$$U_y = \iint_S x\,dS = \int_b^a \Big(\int_{x_0}^{x} \xi\,d\xi\Big)\,dy,$$

whence

$$U_y = \frac{1}{2}\int_b^a (x^2 - x_0^2)\,dy.$$

The statical moment of the line with respect to the y-axis is defined as

$$T_y = \int_{AB} x(y)\,dl = \int_b^a x\sqrt{1+x'^2}\,dy.$$

We try to solve our problem using the well-known Lagrange method. So we are looking for the extrema [2] of the functional

$$P = U_y + \lambda T_y = \int_b^a \Big(\frac{1}{2}x^2 - \frac{1}{2}x_0^2 + \lambda x\sqrt{1+x'^2}\Big)\,dy, \tag{3}$$

where λ is a constant. As the integrand in (3) does not depend on y, denoting it by F we get the first integral of the Euler-Lagrange equation in the form

$$F - x'F'_{x'} = C_1$$

or, after evaluating the derivative $F'_{x'}$, in the form

$$\frac{1}{2}x^2 - \frac{1}{2}x_0^2 + \lambda x\sqrt{1+x'^2} - \frac{\lambda x x'^2}{\sqrt{1+x'^2}} = C_1. \tag{4}$$

After some calculations, (4) takes the form

$$\frac{2\lambda x}{\sqrt{1+x'^2}} = -x^2 + x_0^2 + 2C_1$$

or

$$\frac{2\lambda x}{\sqrt{1+x'^2}} = x_k^2 - x^2 \tag{5}$$

with

$$x_k^2 = x_0^2 + 2C_1 \tag{6}$$

if we choose the constant C_1 in such a way that the right-hand side of (6) is non-negative. Solving (5) with respect to x', we obtain

$$x' = \frac{\sqrt{-x^4 + 2x^2(x_k^2 + 2\lambda^2) - x_k^4}}{x_k^2 - x^2},$$

and integrating it with respect to x we get

$$y + C = \int \frac{(x_k^2 - x^2)\,dx}{\sqrt{w(x)}}, \tag{7}$$

where $w(x) = -x^4 + 2x^2(x_k^2 + 2\lambda^2) - x_k^4$.

Equation (7) describes the required line in the (x, y)-plane. This line will be called an *anti-ellipse* because, as will be seen below, it has the shape of an oval with some properties which are opposite to those characteristic of the ellipse.

4. The study of equation (7). In this section we study equation (7) more thoroughly and bring it to the form which will be more convenient in further investigations. We are going to express the constants λ and x_k by means of the roots of the polynomial $w(x)$ occurring in the denominator of the integrand in (7). Without loss of generality we may suppose λ to be non-negative because in (7) only λ^2 does occur. As will be seen in the sequel, the case $\lambda = 0$ may be neglected, so we assume $\lambda > 0$ from now on. From the equation

$$-(x^2)^2+2x^2(x_k^2+2\lambda^2)-(x_k^2)^2 = 0 \tag{8}$$

we obtain

$$(x^2)_1 = x_k^2+2\lambda^2-2\lambda\sqrt{x_k^2+\lambda^2},$$

$$(x^2)_2 = x_k^2+2\lambda^2+2\lambda\sqrt{x_k^2+\lambda^2}.$$

Thus (8) has four distinct real roots

$$x_1 = -\lambda+\sqrt{\lambda^2+x_k^2}, \quad x_2 = \lambda+\sqrt{\lambda^2+x_k^2},$$

$$x_3 = \lambda-\sqrt{\lambda^2+x_k^2}, \quad x_4 = -\lambda-\sqrt{\lambda^2+x_k^2}$$

and the following relations hold:

$$x_1 = -x_3, \quad x_1^2 = x_3^2, \quad x_2 = -x_4, \quad x_2^2 = x_4^2,$$

$$x_1x_2 = x_3x_4, \quad x_1+x_2 = -(x_3+x_4).$$

Obviously, $x_2 > x_1 > 0$ and

$$x_k^2 = x_1x_2, \quad \lambda = \tfrac{1}{2}(x_2-x_1). \tag{9}$$

We may suppose further that $x_k \geqslant 0$ because only x_k^2 occurs in our considerations. The denominator of the integrand in (7) may be written in the form $\sqrt{-(x^2-x_1^2)(x^2-x_2^2)}$, so it is real valued only for x belonging to the interval $[x_1^2, x_2^2]$. In other words, the right-hand side of (7) is real valued only for $[x_1, x_2]$ and $[x_4, x_3]$ lying on the non-negative and non-positive semi-axes, respectively. Thus the anti-ellipse consists of two branches. Let us consider first this one which corresponds to $x \geqslant 0$. It follows from our considerations that x_1 is the abscissa of this point of the line which is the nearest to the y-axis and x_2 is the abscissa of the second end-point. Changing the constant C in (7), we shift the anti-ellipse along the y-axis. This constant may be eliminated if we reformulate (7) using the definite integral. The following simple example shows that it is most convenient to integrate with x_2 as the lower bound, assuring in this way the condition $y(x_2) = 0$ to be satisfied. Let us consider namely the case where $x_1 = 0$, thus $x_k = 0$ and $x_2 = 2\lambda$. If we suppose that a positive value of the radical in (7) is taken, this equation is of the form

$$y+C = -\int \frac{xdx}{\sqrt{4\lambda^2-x^2}}$$

or, after evaluating the integral,

$$y+C = \sqrt{4\lambda^2-x^2} \quad (x, y+C \geqslant 0). \tag{10}$$

The last equation describes a quadrant of the circle with radius 2λ and centre $(0, -C)$. The condition $y(x_2) = 0$ in (10) is equivalent to $C = 0$ and in this case the circle has the "best" position with respect to the coordinate system. According to these remarks we write equation (7) in the form

$$y = \int_{x_2}^{x} \frac{(x_k^2-x^2)\,dx}{\sqrt{-x^4+2x^2(x_k^2+2\lambda^2)-x_k^4}} \quad (x_1 \leqslant x \leqslant x_2) \tag{11}$$

or in the following two equivalent forms:

$$y = \int_{x_2}^{x} \frac{(x_1x_2-x^2)\,dx}{\sqrt{-x^4+x^2(x_1^2+x_2^2)-x_1^2x_2^2}} \quad (x_1 \leqslant x \leqslant x_2) \tag{12}$$

and

$$y = \int_{x_2}^{x} \frac{(x_1x_2-x^2)\,dx}{\sqrt{-(x_2^2-x^2)(x^2-x_1^2)}} \quad (x_1 \leqslant x \leqslant x_2). \tag{13}$$

Note that the right-hand side of (11) or, equivalently, of (12) and (13) is an improper integral. Its convergence follows easily from the form of (13). In our basic equation (7) the square root occurring in the integrand may take two values of opposite signs. Changing this sign in (12) we reflect the described line in the x-axis.

To consider the second branch of the anti-ellipse, corresponding to the interval $[x_4, x_3]$, we describe it formally by the equation

$$y = \int_{x_4}^{x} g(x)\,dx \quad (x_4 \leqslant x \leqslant x_3),$$

where $g(x)$ denotes the integrand in (7) and the square root takes also two values of opposite signs. As $g(x) = g(-x)$, it is readily seen that this second branch is symmetric, with respect to the y-axis, to that one described by (11). Since we have assumed that the shell of our air spring is the surface obtained by rotation of the required line about the y-axis, it is sufficient to consider only the first branch of the anti-ellipse, corresponding to non-negative x. In the sequel we suppose that the square root in (11), or in (12) and (13), has the positive sign, so this equation describes half the anti-ellipse lying upon the x-axis.

We introduce now some geometrical concepts concerning the anti-ellipse. We call namely the constant 2λ the *diameter* of the anti-ellipse.

The constant x_k, which according to (9) equals the geometrical mean of x_1 and x_2, may be called the *mean distance* of the anti-ellipse to the axis of rotation. It is readily seen from the form of the integrand in (11) or in (12) that the tangent to the anti-ellipse at the point $(x_2, 0)$ has the direction of the y-axis. The same holds at the point (x_1, y_1) if $x_1 \neq 0$. It is easy to verify also that $y(x)$ takes its maximum value for $x = x_k$.

5. The length of arc of the anti-ellipse and evaluation of the ordinates of its points. Denote by $l(x)$ the length of arc of the anti-ellipse from the point $(x_2, 0)$ to the variable point (x, y). Using the formula

$$l(x) = \int_{x_2}^{x} \sqrt{1+y'^2}\,dx,$$

we get, after calculating y' from (12),

$$l(x) = \frac{1}{2}(x_2 - x_1)\int_{x_2}^{x} \frac{2x\,dx}{\sqrt{-x^4+(x_1^2+x_2^2)x^2-x_1^2x_2^2}}$$

or, after evaluating the integral,

$$l(x) = (x_2 - x_1)\arctan\sqrt{\frac{x_2^2-x^2}{x^2-x_1^2}}. \tag{14}$$

Making use of the identity

$$\sin\varphi = \frac{\tan\varphi}{\sqrt{1+\tan^2\varphi}}$$

we bring (14) to the form

$$l(x) = (x_2 - x_1)\arcsin\sqrt{\frac{x_2^2-x^2}{x_2^2-x_1^2}}, \tag{15}$$

which will be more convenient in the sequel. It follows from (14) and (15) that

$$l(x_1) = \frac{\pi}{2}(x_2 - x_1) = \pi\lambda. \tag{16}$$

Therefore, the length of the anti-ellipse depends only on its diameter; particularly, it does not depend on x_1 and x_2.

Equation (15) may be used to the approximate calculation of the ordinates y of the points of the anti-ellipse. We divide the interval $[x_1, x_2]$ into n parts by means of the points

$$x_1 = \xi_n < \xi_{n-1} < \ldots < \xi_0 = x_2$$

and we put $\Delta\xi_i = \xi_i - \xi_{i-1}$ $(i = 1, 2, \ldots, n)$ and $\Delta l_i = l(\xi_{i-1}) - l(\xi_i)$, where the values $l(\xi_j)$ are obtained from (15). Identifying Δl_i with the length of the chord, we obtain the following approximate formula for $\Delta y_i = y_i - y_{i-1}$:

(17) $$|\Delta y_i| \approx \sqrt{(\Delta l_i)^2 - (\Delta\xi_i)^2}.$$

Therefore, the ordinate y_i may be approximately evaluated after summing some number of the right-hand side expressions in (17) under the assumption that the increment Δy_i is positive for $\xi_i > x_k$ and negative for $\xi_{i-1} < x_k$ (Fig. 4).

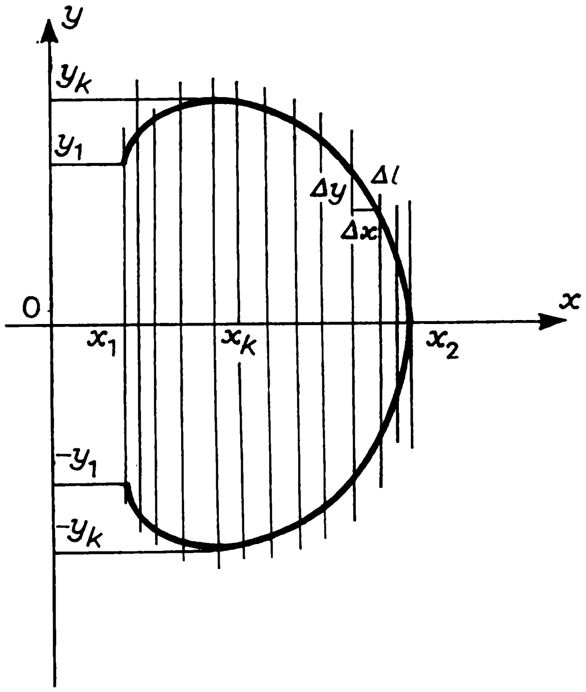

Fig. 4

Let us suppose now that we have changed the scale on the x-axis putting $x = \tau\hat{x}$ $(\tau > 0)$ and, in the same manner, $x_1 = \tau\hat{x}_1$, $x_2 = \tau\hat{x}_2$. Therefore, by (9), $x_k = \tau\hat{x}_k$ and $\lambda = \tau\hat{\lambda}$. Transforming the integral in (12) we obtain

(18) $$y(x) = \tau\hat{y}(\hat{x}),$$

where $\hat{y}$ denotes the right-hand side of (12) expressed in terms of new parameters. In other words, multiplying the parameters x_k and λ by a constant factor yields a similarity transformation of the anti-ellipse. Consequently, two anti-ellipses having the same ratio x_k/λ will be called *similar*. The values of $\hat{y}(\hat{x})$ can be simply obtained from (18) if the values $y(x)$ are just known.

6. The evaluation of $y(x)$ by using the tables of elliptic integrals. Equation (13) may be written in the equivalent form

$$(19)\qquad y = \int\limits_x^{x_2} \frac{x^2\,dx}{\sqrt{(x^2-x_1^2)(x_2^2-x^2)}} - x_1 x_2 \int\limits_x^{x_2} \frac{dx}{\sqrt{(x^2-x_1^2)(x_2^2-x^2)}}.$$

Using the known methods [4] we transform the last identity into

$$y = x_2 E(\varphi, k) - x_1 F(\varphi, k),$$

where

$$k = \sin\alpha = \frac{\sqrt{x_2^2-x_1^2}}{x_2} \quad \text{and} \quad \varphi(x) = \arcsin\sqrt{\frac{x_2^2-x^2}{x_2^2-x_1^2}}.$$

By (15) we have

$$\varphi(x) = \frac{l(x)}{x_2-x_1} = \frac{l(x)}{2\lambda}.$$

The numerical values of functions E and F (in terms of the independent variables α and φ) are to be found in the tables of elliptic integrals [4]. Note that this method of evaluating $y(x)$ is less convenient than that described in Section 5, which may be realized using the computers.

It should be noticed that the usually used methods of approximate integration, as trapezium method or Simpson's method [15], cannot be applied to evaluating the integrals occurring on the right-hand side of equation (13) because its derivative becomes infinite at the points x_1 and x_2.

7. The radius of curvature of the anti-ellipse. Using the well-known formula for the radius of curvature

$$R = \frac{(1+y'^2)^{3/2}}{y''},$$

we get, by (12),

$$(20)\qquad R(x) = \frac{x^2(x_2-x_1)}{x^2+x_1x_2}$$

or, equivalently, after introducing constants λ and x_k,

$$(21)\qquad R(x) = \frac{2\lambda x^2}{x_k^2+x^2}.$$

Particularly, we have

$$R(x_j) = \frac{2\lambda x_j}{x_1+x_2} \quad (j = 1, 2) \quad \text{and} \quad R(x_k) = \lambda$$

if $x_k \neq 0$. Assuming $x_1 = 0$ and, therefore, $x_k = 0$, by (10) we get $R = 2\lambda$.

We shall now see how $R(x)$ changes if x_k grows to infinity but the diameter 2λ of the anti-ellipse remains constant. From (21) and (9) we obtain

$$\frac{x_1^2}{x_1x_2+x_2^2} \leqslant \frac{R(x)}{2\lambda} \leqslant \frac{x_2^2}{x_1x_2+x_1^2}. \tag{22}$$

As $x_k < x_2$ and $x_1 = x_2 - 2\lambda$, both x_1 and x_2 also tend to infinity. But in this case, as is easy to verify, the first and the third terms in (22) have the same limit 1/2. Therefore,

$$\lim_{x_k \to \infty} R(x) = \lambda. \tag{23}$$

We can see now how the shape of the anti-ellipse changes when x_k runs over the interval $[0, \infty]$. For $x_k = 0$ the anti-ellipse is the semi-circle of radius 2λ, for $x_k > 0$ it is an oval with two end-points $(x_1, y(x_1))$ and $(x_1, -y(x_1))$. It follows from (23) that for $x_k \to \infty$ the anti-ellipse becomes a part of the circle with radius λ. But according to (16) this part must coincide with the whole circle (Fig. 5).

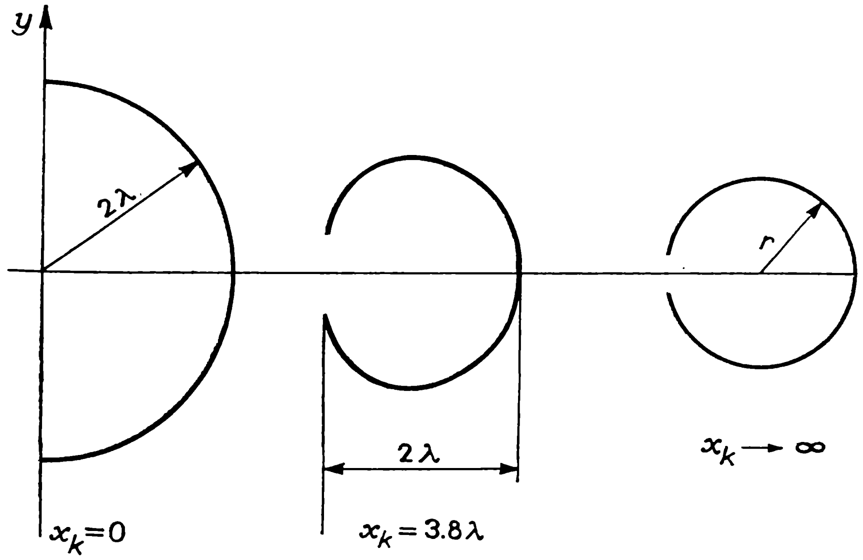

Fig. 5

The knowledge of numerical values of the radius of curvature, which may be found by (20) or (21), yields the method of approximate construction of the anti-ellipse (Fig. 6). We divide the interval $[x_1, x_2]$ as follows:

$$x_1 = \xi_{2n} < \xi_{2n-1} < \ldots < \xi_1 < \xi_0 = x_2.$$

As we have seen in Section 4, the x-axis has the normal direction to the anti-ellipse at the point $(x_2, 0)$. Thus the centre of curvature S_0 at this point lies on the x-axis and has the abscissa $\xi_0 - R(\xi_0)$. Setting S_0 to be the centre, we draw an arc of the circle with radius $R(\xi_0)$, which meets the line $x = \xi_1$ at the point Q_1. On the segment S_0Q_1 we choose a point S_1 such that its distance to Q_1 equals $R(\xi_1)$. Now, taking S_1 as the centre, we draw an arc of the circle with radius $R(\xi_1)$, which meets the line $x = \xi_2$ at the point Q_2. Iterating this procedure we obtain the approximation of the anti-ellipse in the form of a line with a constant tangent. Joining the points $S_0, S_1, S_2, \ldots$ by the segments of straight lines, we obtain the approximate shape of the evolvent (Fig. 6).

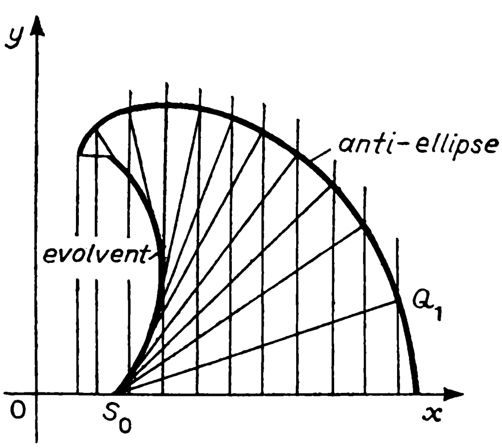

Fig. 6

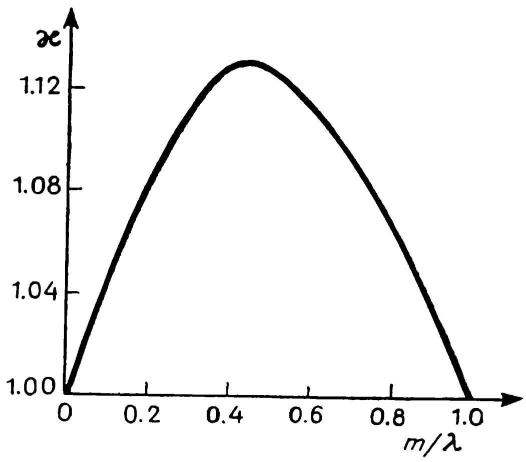

Fig. 7

8. Final remarks. The plane line studied in this paper, called by us an *anti-ellipse*, has been obtained as an integral curve of the Euler-Lagrange equation which, as is well known, gives only a necessary condition for the extremum of a functional. So it remains to prove that the anti-ellipse is indeed the solution of the variation problem posed in Section 2. This proof, based on certain numerical investigations, will be published in a forthcoming paper.

To complete our considerations let us introduce the difference

$$m = \frac{x_1 + x_2}{2} - \sqrt{x_1 x_2}$$

which will be called the *eccentricity* of the anti-ellipse. For $x_1 = 0$ both ends of the anti-ellipse are on the axis of rotation and we have $m = x_2/2$. It is easy to verify by simple calculations that m tends to zero if x_k grows to infinity and the diameter 2λ remains constant. We have seen that in the two "extreme" positions, i.e. $x_k \to \infty$ and $x_k = 0$, the anti-ellipse is a circle or a part of a circle. In the remaining situations, where $x_k \in (0, \infty)$, the anti-ellipse may be considered as a deformated circle and the ratio $\varkappa = y(x_k)/(x_2 - x_k)$ gives a measure of this deformation (note that $\varkappa = 1$ in the two "extreme" cases described above). The dependence of $\varkappa$ on the ratio m/λ is shown in Fig. 7. Numerical calculations prove that the maximal value of $\varkappa$ is close to 1.13; thus the above-mentioned deformation is rather not too great.

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