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DISTRIBUTION OF THE MAXIMUM ARTERIAL BLOOD-PRESSURE IN THE HUMAN POPULATION(1)

1. Introduction. It is known that many medical scientists have different opinions about the limits of the maximum arterial blood-pressure in a population of healthy people. As a rough rule it is often accepted that the normal maximum blood-pressure should be a 100 plus the number of the years of the person. So, for example, the maximum blood-pressure of a person 50 years old should amount to 150 mm Hg.

On the other hand, the blood-pressure measurements carried out on a great number ot people show that this value should be considerably smaller: in above instance amounting to about 140 mm Hg.

P. D. Oldham, a British physician, in his note [13] writes that so far there are age and sex as only considered reasons for the level of blood-pressure, but there are still many other factors to be taken into account. The task which is put forward before contemporary medicine is, as Mr. Oldham stresses, to come to the rule for making a distinction between the normal pressure and the abnormal one, i. e. the high or low blood pressure, and thus to change many prevailed till now unsatisfactory and uncontrolled rules one of which I quoted at the beginning.

I was enabled through the kindness of the physicians of the 2-nd Clinic of Internal Diseases of the Medical Academy in Wrocław, Poland to gather necessary informations. They render available to me the results of 10 000 measurements of the arterial bloodpressure gathered during the period of six years on the population inhabiting Wrocław county(2).

(1) This paper originated from discussion on the Wrocław seminar on applied mathematics led by the late Professor Julian Perkal. The author is truly indebted to Professor Perkal for his help in getting the experimental material, for many valuable hints, and for critical discussion during the research and preparation of the paper.

(2) The measurements were carried out on the patients of the Thyroid Gland Laboratory. A great part of them, if not all, were ill people and the observed blood--pressure distribution was probably somewhat different from that of total population. This is the reason for the necessity of a thorough criticism before any practical application of numerical results presented in the paper. The available material did not permit to obtain the reliable practical criteria but it was sufficient to work out the efficient statistical method now ready to be applied as soon as the appropriate data are collected.

I have decided to limit my study to the measurements taken on men from 30 to 40 years old. That way I got a sample containing 2639 measurements of the arterial blood-pressure.

2. Statistical analysis of the data. The distribution of frequencies of the maximum arterial blood-pressure is given in Table 1.

Although the parameters of asymmetry are rather small ($\gamma_1 = 0{,}056$ and $\gamma_2 = 0{,}013$) we suppose that the observed distribution of frequencies is composed of three distributions,

$$f = c_1 f_1 + c_2 f_2 + c_3 f_3 \qquad (c_1 + c_2 + c_3 = 1), \tag{2.1}$$

where f_1, f_2 and f_3 are densities of probabilities of the low, normal and high maximum arterial blood-pressure([3]). On the ground of the central limit theorem we assume that the density f_2 is a density of Gauss-Laplace distribution. We cannot extend the same supposition to the distributions of the low and high pressure, because we have no biological reasons for that: the low and high pressures, being pathological symptoms, are caused by one, two or several factors present in organism, and therefore they do not satisfy the conditions for the central limit theorem.

TABLE 1

Maximum blood-pressure	Frequencies		
	Abs. freq.	Rel. freq.	Cum. freq.
80	1	0,0004	0,0004
85	2	0,0008	0,0012
90	8	0,0030	0,0042
95	5	0,0019	0,0061
100	34	0,0129	0,0190
105	31	0,0117	0,0307
110	165	0,0625	0,0932
115	98	0,0371	0,1303
120	668	0,2531	0,3834
125	268	0,1015	0,4849
130	738	0,2796	0,7645
135	201	0,0761	0,8406
140	314	0,1189	0,9595
145	24	0,0091	0,9686
150	45	0,0171	0,9857
155	8	0,0030	0,9887
160	20	0,0075	0,9962
165	1	0,0004	0,9966
170	5	0,0019	0,9985
175	1	0,0004	0,9989
180	2	0,0008	0,9997
	2639		

([3]) It is known that a unimodal symmetrical distribution often does not characterize a homogeneity (see [1] and [9]).

Since the appearance of the empiric distribution does not suggest the assumption of the mixture of three distributions (Fig. 1), we confirme it using the normal paper. It is easily seen (Fig. 2) that distribution of the maximum arterial blood-pressure consists of the three distributions.

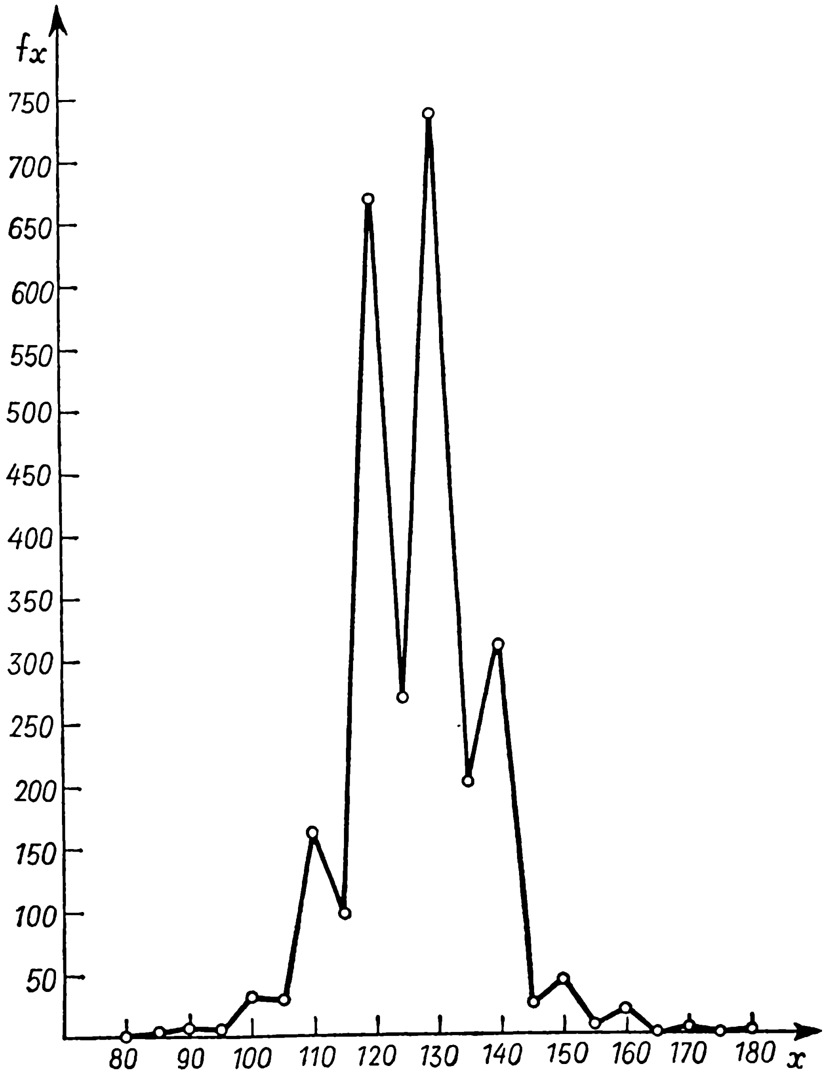

Fig. 1

So far the problem of distinguishing the central part from the mixture of the three hypothetical distributions has no solution. Pearson [16] gave an algorithm for the separation of two mixed normal distributions, but, even in this case, analytical difficulties are such that the algorithm, which demands computation of several higher moments, has no practical meaning. As we cannot accept that low and high pressures have a normal distribution, it is useless to generalize Pearson's method for which the assumption of normally distributed components is substantial.

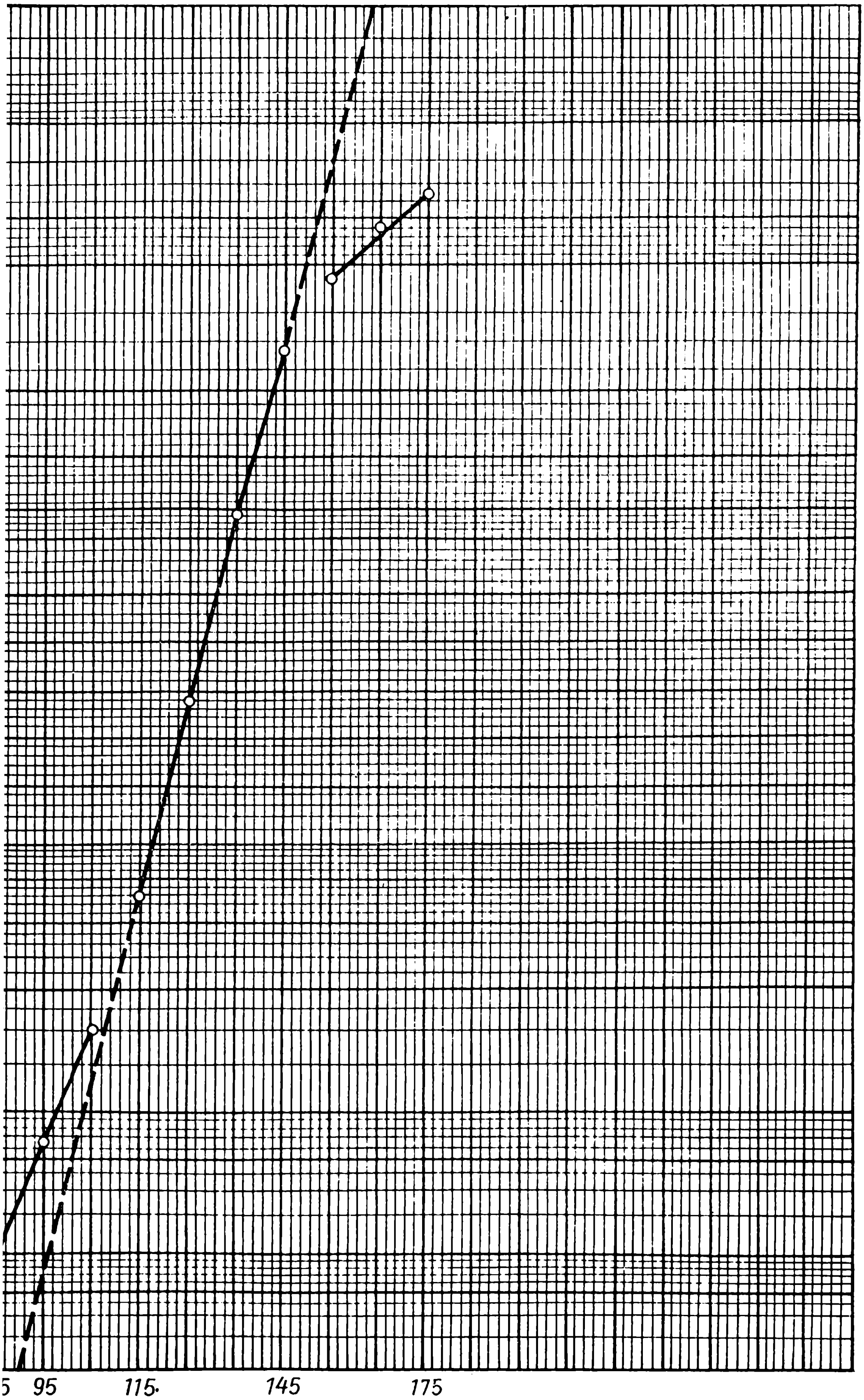

Fig. 2

The method applied in this paper requires first a performing of the following programme: *from the empirical distribution form a both sides truncated sample and, assuming it be a double truncated sample of the Gauss population, estimate its parameters: the mean and the variance*(4).

Having these estimations we will be able to construct the Gauss curve and to compare it with the empirical one. The differences of the ordinates of the empirical and Gauss distribution will allow us to form distributions of the low and high blood-pressure. After all this is done, it will remain only to make a conclusion it is to work out details of the discrimination rule, and to estimate its consequences. The conclusion, which we give on the patient, can be subjected to two kinds of errors: with a probability α we declare an ill man as a healthy one, with a probability β we declare a healthy man as an ill one.

3. Estimation of the parameters of the normal distribution by the double truncated sample. A density of the probability distribution of the random variable corresponding to the maximum arterial blood-pressure with truncated normal distribution may be represented in the form

$$(3.1)\qquad f_2(x) = \begin{cases} \dfrac{(\sigma\sqrt{2\pi})^{-1}\exp\{-(x-\mu)^2/2\sigma^2\}}{P(a \leqslant x \leqslant b)} & \text{for} \quad a \leqslant x \leqslant b, \\ 0 & \text{for} \quad x < a \text{ and } x > b. \end{cases}$$

If we perform a translation $x = y+a$ and express the limits of the truncated distribution in standardized units,

$$(3.2)\qquad \xi_1 = \frac{a-\mu}{\sigma}, \qquad \xi_2 = \frac{b-\mu}{\sigma} = \xi_1 + \frac{d}{\sigma},$$

where $d = b-a$, then from equation (3.1) we get

$$g(y) = \begin{cases} \dfrac{(\sigma\sqrt{2\pi})^{-1}\exp\{-\frac{1}{2}((y+a-\mu)/\sigma)^2\}}{P(\xi_1 \leqslant (x-\mu)/\sigma \leqslant \xi_2)} & \text{for} \quad 0 \leqslant y \leqslant d, \\ 0 & \text{for} \quad y < 0 \text{ and } y > d \end{cases}$$

(4) The problem of the estimation of parameters of a general population by means of truncated sample has been dealt with by Pearson and Lee [14] and Pearson [15], Fisher [10], Hald [11], Stevens [19] and Cochran [2]. Cohen [3], [4] generalized all the cases of the truncated sample, using results of the above-mentioned forerunners; he estimated the mean and the variance by using the method of maximum likelihood. Des Raj in [7] and in [8] has come to the same results using the method of the moments. The author's method is based on the results contained in these papers.

or

$$(3.3)\qquad g(y) = \begin{cases} \dfrac{(\sigma\sqrt{2\pi})^{-1}\exp\{-\frac{1}{2}(\xi_1+y/\sigma)^2\}}{\Phi(\xi_2)-\Phi(\xi_1)} & \text{for} \quad 0 \leqslant y \leqslant d, \\ 0 & \text{for} \quad y<0 \text{ and } y>d, \end{cases}$$

where

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} \exp\left\{-\frac{u^2}{2}\right\} du.$$

Let us suppose now that our empirical distribution between a and b represents a simple sample with n elements $x_1, x_2, \ldots, x_n$. Having performed the translation $x = y+a$, we get the values $y_1, y_2, \ldots, y_n$. The likelihood function for this sample is a function of the parameters ξ_1 and σ. It can be expressed in the form

$$(3.4)\qquad L = \left[\frac{1/\sigma\sqrt{2\pi}}{\Phi(\xi_2)-\Phi(\xi_1)}\right]^n \exp\left\{-\frac{1}{2}\sum_{j=1}^{n}\left(\xi_1+\frac{y_j}{\sigma}\right)^2\right\}.$$

The system of equations

$$(3.5)\qquad \frac{\partial \log L}{\partial \xi_1} = 0 \quad \text{and} \quad \frac{\partial \log L}{\partial \sigma} = 0$$

gives the maximum likelihood estimates $\hat{\xi}_1$ and $\hat{\sigma}$. The first equation leads to

$$\hat{\sigma}\left[\frac{\varphi(\hat{\xi}_2)-\varphi(\hat{\xi}_1)}{\Phi(\hat{\xi}_2)-\Phi(\hat{\xi}_1)}+\hat{\xi}_1\right]+\frac{1}{n}\sum_{j=1}^{n} y_j = 0.$$

If by

$$\nu_1 = \frac{1}{n}\sum_{j=1}^{n} y_j$$

we denote the first moment of the truncated and translated empirical distribution of the maximum arterial blood-pressure, we get the first equation in the form

$$(3.6)\qquad \hat{\sigma}(Z_2-Z_1+\hat{\xi}_1)+\nu_1 = 0,$$

where

$$(3.7)\qquad Z_i = \frac{\varphi(\hat{\xi}_i)}{\Phi(\hat{\xi}_2)-\Phi(\hat{\xi}_1)} \quad (i = 1, 2).$$

The second equation in a similar way may be reduced to the form

$$(3.8)\qquad dZ_2-\hat{\sigma}+\hat{\xi}_1\nu_1+\frac{1}{\hat{\sigma}}\nu_2 = 0,$$

where ν_2 is the second moment of the sample.

Equations (3.6) and (3.8) will give the maximum likelihood estimates $\hat{\xi}_1$ and $\hat{\sigma}$ of the parameters ξ_1 and σ. In view of (3.2) the maximum likelihood estimate $\hat{\mu}$ of the parameter μ may be obtained from the equation

$$\hat{\mu} = a - \hat{\sigma}\hat{\xi}_1. \tag{3.9}$$

The solution of equations (3.6) and (3.8) needs, however, the application either of the graphic methods of nomogramic character or of an algebraic method of the iterative character. We shall apply here an iterative procedure which gives approximate values with necessary exactness [18]. For this reason let us transform equations (3.6) and (3.8) to a more convenient form. If we express $\hat{\sigma}$, according to (3.2), by

$$\hat{\sigma} = \frac{d}{\hat{\xi}_2 - \hat{\xi}_1} \tag{3.10}$$

and put this into equations (3.6) and (3.8), we get

$$Z_1 - Z_2 - \hat{\xi}_1 = (\hat{\xi}_2 - \hat{\xi}_1)\frac{\nu_1}{d} \tag{3.11}$$

and

$$d^2 Z_2(\hat{\xi}_2 - \hat{\xi}_1) - d^2 + \hat{\xi}_1 \nu_1 d(\hat{\xi}_2 - \hat{\xi}_1) + \nu_2(\hat{\xi}_2 - \hat{\xi}_1)^2 = 0. \tag{3.12}$$

From (3.11) we get

$$\hat{\xi}_1 = Z_1 - Z_2 - (\hat{\xi}_2 - \hat{\xi}_1)\frac{\nu_1}{d}$$

and putting this into (3.12) we come to the square equation in $\hat{\xi}_2 - \hat{\xi}_1$:

$$(\hat{\xi}_2 - \hat{\xi}_1)^2(\nu_2 - \nu_1^2) + (\hat{\xi}_2 - \hat{\xi}_1)(d^2 Z_2 + \nu_1 d Z_1 - \nu_1 d Z_2) - d^2 = 0. \tag{3.13}$$

We are only interested in one root of equation (3.13),

$$\hat{\xi}_2 - \hat{\xi}_1 = \frac{\nu_1 d Z_2 - d^2 Z_2 - \nu_1 d Z_1 + \sqrt{(\nu_1 d Z_2 - d^2 Z_2 - \nu_1 d Z_1)^2 + 4d^2(\nu_2 - \nu_1^2)}}{2(\nu_2 - \nu_1^2),}, \tag{3.14}$$

because it easy to see that the second root is negative and, by definition, $\hat{\xi}_2$ is greater than $\hat{\xi}_1$. From (3.11) and (3.13) we get

$$\hat{\xi}_1 = \frac{Z_1 - Z_2 + \frac{\nu_1}{d}\cdot\hat{\xi}_2}{1 - \nu_1/d} \tag{3.15}$$

and

$$(3.16)\qquad \hat{\xi}_2 = Z_1 - Z_2 + \frac{d-\nu_1}{2d(\nu_2-\nu_1^2)} \times$$

$$\times [\nu_1 dZ_2 - d^2 Z_2 - \nu_1 dZ_1 + \sqrt{(\nu_1 dZ_2 - d^2 Z_2 - \nu_1 dZ_1)^2 + 4d^2(\nu_2 - \nu_1^2)}\,].$$

Equations (3.15) and (3.16) may be solved by an iterative procedure,

$$(3.17)\qquad \hat{\xi}_2^{(i+1)} = Z_1^{(i)} - Z_2^{(i)} + \frac{d-\nu_1}{2d(\nu_2-\nu_1^2)} \times$$

$$\times [\nu_1 dZ_2^{(i)} - d^2 Z_2^{(i)} - \nu_1 dZ_1^{(i)} + \sqrt{(\nu_1 dZ_2^{(i)} - d^2 Z_2^{(i)} - \nu_1 dZ_1^{(i)})^2 + 4d^2(\nu_2 - \nu_1^2)}\,],$$

$$(3.18)\qquad \hat{\xi}_1^{(i+1)} = \frac{Z_1^{(i)} - Z_2^{(i)} - \frac{\nu_1}{d}\,\hat{\xi}_2^{(i+1)}}{1-\nu_1/d},$$

where $\hat{\xi}_j^{(i)}$ $(j = 1, 2)$ is the i-th approximation for $\hat{\xi}_j$, and $Z_j^{(i)}$ is the value of Z_j calculated from (3.7) for $\hat{\xi}_j = \hat{\xi}_j^{(i)}$.

Although there are some other methods, especially graphic ones [18], to obtain the initial values $\hat{\xi}_1^{(0)}$ and $\hat{\xi}_2^{(0)}$, according to my opinion, the normal paper (Fig. 2) gives the values $\mu^{(0)}$ and $\sigma^{(0)}$ near enough to the estimates $\hat{\mu}$ and $\hat{\sigma}$. Using formulas (3.2) we may then calculate the initial values $\hat{\xi}_1^{(0)}$ and $\hat{\xi}_2^{(0)}$.

For the preliminary values $\mu^{(0)}$ and $\sigma^{(0)}$ we take values read on the normal paper:

$$(3.19)\qquad \mu^{(0)} \approx 126 \text{ mm Hg}, \qquad \sigma^{(0)} \approx 10 \text{ mm Hg}.$$

The values of a and b we also get on the normal paper:

$$(3.20)\qquad a = 115 \text{ mm Hg}, \qquad b = 140 \text{ mm Hg},$$

and $d = 140 - 115 = 25$ mm Hg.

The values of the moments $\nu_1 = 12{,}7066$ and $\nu_2 = 204,\ 6305$ have been taken from Table 1, and the initial values $\xi_1^{(0)} = -1{,}1$, $\xi_2^{(0)} = 1{,}4$ are determined according to formulas (3.2).

The first approximations for $\hat{\xi}_1$ and $\hat{\xi}_2$ we get from (3.17) and (3.18). Using the statistical table for normal distribution [21] we compute: $Z_1^{(0)} = 0{,}2916$ and $Z_2^{(0)} = 0{,}2003$.

Now, according to (3.17) and (3.18) we have $\hat{\xi}_2^{(1)} = 1{,}2800$ and $\hat{\xi}_1^{(1)} = -1{,}1373$ and first approximations for $\hat{\mu}$ and $\hat{\sigma}$, according to (3.10) and (3.9), are

$$(3.21)\qquad \hat{\sigma}^{(1)} = 10{,}3421 \quad \text{and} \quad \hat{\mu}^{(1)} = 126{,}7588.$$

Since the asymptotic variance-covariance matrix of $\hat{\mu}$ and $\hat{\sigma}$ may be approximated as

$$K = \begin{pmatrix} -\dfrac{\partial^2 \log L}{\partial \hat{\mu}^2} & \dfrac{\partial^2 \log L}{\partial \hat{\mu}\partial \hat{\sigma}} \\ \dfrac{\partial^2 \log L}{\partial \hat{\mu}\partial \hat{\sigma}} & -\dfrac{\partial^2 \log L}{\partial \hat{\sigma}^2} \end{pmatrix}^{-1},$$

where

$$\frac{\partial^2 \log L}{\partial \hat{\mu}^2} = \frac{n}{\sigma^2}(Z_2\hat{\xi}_2 + Z_2^2 - 2Z_1Z_2 - \hat{\xi}_1 Z_1 + Z_1^2 - 1),$$

$$\frac{\partial^2 \log L}{\partial \hat{\sigma}^2} = -\frac{n}{\sigma^2}\left[\frac{3\nu^2}{\sigma^2} - 1 + \frac{\nu_1 + 2\hat{\xi}_1\nu_1 + 2dZ_2}{\sigma}\right] +$$

$$+ \frac{nd}{\sigma^4}[(a+d-\mu)(Z_2\hat{\xi}_2 + Z_2^2) - (a-\mu)Z_1Z_2],$$

$$\frac{\partial^2 \log L}{\partial \hat{\mu}\partial \hat{\sigma}} = \frac{nd}{\sigma^3}(\hat{\xi}_2 Z_2 + Z_2^2 - Z_1Z_2) - \frac{\nu_2}{\sigma},$$

we have

$$K = \begin{pmatrix} 0{,}509 & 0{,}327 \\ 0{,}327 & 0{,}077 \end{pmatrix}$$

and

$$\vartheta(\hat{\mu}) = 0{,}509, \qquad \vartheta(\hat{\sigma}) = 0{,}077, \qquad \cos(\hat{\mu}, \hat{\sigma}) \approx 0{,}327.$$

4. Construction of the Gauss distribution with parameters $\hat{\mu}$ and $\hat{\sigma}$ as distribution of the normal maximum arterial blood-pressure and selection of the low and high maximum arterial blood-pressure. Let us introduce the following symbols:

A_k — the random event that the individual taken at random has the blood-pressure equal to $80+5k$ (in mm Hg) or, more exactly, has a blood-pressure in the interval

$$[80+5k-2{,}5,\ 80+5k+2{,}5 \text{ mm Hg}], \qquad k = 0, 1, 2, \ldots, 20.$$

$P(A_k)$ — the probability of the realization of the event A_k. (We accept that the probabilities $P(A_k)$ are approximatively equal to the empirical relative frequencies.)

$P(A_k/N)$ — the probability of the realization of the event A_k under the condition that the pressure is normal (N). (These probabilities are computed from the statistical table [21].)

$P(A_k/L)$ — the probability of the realization of the event A_k under the condition that the pressure is lower (L).

$P(A_k/H)$ — the probability of the realization of the event A_k under the condition that the pressure is higher (H).

According to the Bayes formula we come to the following:

$$P(N/A_k) = \frac{P(A_k/N)P(N)}{P(A_k)}, \tag{4.1}$$

$$P(L/A_k) = \frac{P(A_k/L)P(L)}{P(A_k)}, \tag{4.2}$$

$$P(H/A_k) = \frac{P(A_k/H)P(H)}{P(A_k)}. \tag{4.3}$$

It is obvious that

$$P(N/A_k)+P(L/A_k)+P(H/A_k) = 1. \tag{4.4}$$

If we accept that for $115 < x < 140$ we have the frequencies of the normal distribution exclusively, then we may calculate the probability $P(N)$ from the condition

$$D_k = P(A_k)-P(N).\ P(A_k/N) = 0 \quad (7 < k < 11), \tag{4.5}$$

whence $P(N) = 0{,}8733$.

In Table 2 we have the probabilities $P(A_k)$, $P(N)P(A_k/N)$ and the differencies D_k.

TABLE 2

A_k	$P(A_k)$	$P(N)P(A_k/N)$	D_k
80	0,0004	0,0000076	0,00003
85	0,0012	0,0000573	0,00114
90	0,0039	0,0003877	0,0035
95	0,0062	0,0016	0,0046
100	0,0184	0,0063	0,0121
105	0,0252	0,0190	0,0062
110	0,0754	0,0457	0,0297
115	0,0928	0,0897	0,0029
120	0,1464	0,1347	—
125	0,1492	0,1633	—
130	0,1656	0,1579	—
135	0,1168	0,1220	—
140	0,1000	0,0762	0,0239
145	0,0448	0,0367	0,0081
150	0,0310	0,0136	0,0175
155	0,0073	0,0045	0,0030
160	0,0058	0,0010	0,0048
165	0,0025	0,0002	0,0024
170	0,0019	0,00003	0,0019
175	0,0004	0,000003	0,0004
180	0,0008	0,0000008	0,000079

By the aid of the differencies D_k from Table 2, we get the probabilities $P(L)$, $P(N)$ and $P(H)$, i.e. the coefficients c_1, c_2 and c_3 in formula (2.1):

$$\begin{aligned} c_1 &= P(L) = 0{,}0601, \\ c_2 &= P(N) = 0{,}8733, \\ c_3 &= P(H) = 0{,}0620. \end{aligned} \tag{4.6}$$

By the aid of formulas (4.1), (4.2) and (4.3) we get Table 3.

TABLE 3

A_k	$100P(N/A_k)$ $^0/_0$	$100P(L/A_k)$ $^0/_0$	$100P(H/A_k)$ $^0/_0$
80	1,89	98,11	—
85	4,76	95,24	—
90	9,94	90,06	—
95	26,51	73,49	—
100	34,40	65,60	—
105	75,54	24,46	—
110	60,57	39,43	—
115	96,83	3,17	—
120	100	—	—
125	100	—	—
130	100	—	—
135	100	—	—
140	76,15	—	23,85
145	81,87	—	18,13
150	43,94	—	56,06
155	61,01		38,99
160	18,06	—	81,94
165	8,38	—	91,62
170	1,61		89,39
175	0,87	—	99,13
180	0,11	—	99,89

Table 3 shows the percentages of healthy people aged from 30 to 40 and ill persons with high and low blood-pressure for every value of the maximum arterial blood-pressure. So, for example, when a man aged from 30 to 40 has maximum arterial blood-pressure 155 mm Hg, then with the probability 0,6101 we declare this man as healthy, i.e. that he has a normal maximum arterial blood-pressure and with the probability 0,3899 that he has a high maximum arterial blood-pressure([5]).

In that way Table 3 shows the results of this study.

([5]) The author wants once more to draw the attention of the reader to the possibility of substantial bias of the numerical results due to the selection of empirical data (see footnote (2), p. 99). Although all results may be different if the whole analysis was repeated on the ground of a representative sample of total population, one may expect that this would effect more the frequencies of low, normal and high blood-pressure, $P(L)$, $P(N)$ and $P(H)$, rather than the critical limits a, b and the parameters μ and σ.

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ROZKŁAD SKURCZOWEGO CIŚNIENIA KRWI W POPULACJI LUDZKIEJ

STRESZCZENIE

Autor opracował statystycznie materiał 2639 lekarskich pomiarów ciśnienia krwi (biorąc pod uwagę tylko ciśnienia skurczowe) u mężczyzn w wieku od 30 do 40 lat. Tablica 1 i rys. 1 przedstawiają rozkład tych pomiarów. Empiryczna dystrybuanta narysowana na papierze normalnym (rys. 2) potwierdza przypuszczenie, że mamy tu do czynienia z mieszaniną trzech rozkładów:

$$f = c_1 f_1 + c_2 f_2 + c_3 f_3 \qquad (c_1 + c_2 + c_3 = 1).$$

Poszczególne składniki można interpretować jako rozkład (f_1) wyników pomiaru przy podciśnieniu, rozkład (f_2) przy ciśnieniu normalnym i rozkład (f_3) przy nadciśnieniu. Na podstawie rys. 2 można ponadto przyjąć, że środkowy składnik f_2 jest gęstością rozkładu normalnego Gaussa, a rozkłady skrajne f_1 i f_3 nie zachodzą na siebie.

Dla estymacji parametrów rozkładu f_2 autor obcina z obu stron rozkład empiryczny (ograniczając materiał do przedziału, w którym wpływy rozkładów skrajnych można pominąć) i z obciętej próbki szacuje parametry uciętego rozkładu normalnego (3.1) przy pomocy postępowania iteracyjnego (3.17) i (3.18). Jako wartości krytyczne na podstawie rys. 2 przyjęto wielkości (3.20). Również z wykresu odczytano wyjściowe wartości parametrów (3.19). Po jednym kroku iteracyjnym uzyskano estymacje (3.21). Wzory (4.6) podają oceny współczynników c_1, c_2 i c_3, czyli prawdopodobieństwa, że przypadkowo wybrany osobnik ma podciśnienie (L), ciśnienie normalne (N) lub nadciśnienie (H). Tablica 3 podaje prawdopodobieństwa tych samych zdarzeń w zależności od wyniku pomiaru ciśnienia (A_k).

Autor zaleca dużą ostrożność przy interpretacji uzyskanych wyników liczbowych. Materiał empiryczny wzięto z zapisów Przychodni Badania Tarczycy przy II Klinice Wewnętrznej Akademii Medycznej we Wrocławiu. Pacjenci tej przychodni byli przeważnie ludźmi chorymi i obserwowany rozkład pomiarów ciśnienia może znacznie odbiegać od nieznanego rozkładu w pełnej populacji. Dlatego główny nacisk położono w pracy na opis metody statystycznej, która po zebraniu reprezentatywnego materiału może być zastosowana do dyskryminacji pomiarów ciśnienia.

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РАСПРЕДЕЛЕНИЕ МАКСИМАЛЬНОГО КРОВЯНОГО ДАВЛЕНИЯ В ПОПУЛЯЦИИ ЛЮДЕЙ

РЕЗЮМЕ

Автор обработал статистически материал 2639 врачебных измерений кровяного давления (принимая во внимание только максимальное давление) у мужчин в возрасте от 30 до 40 лет. Таблица 1 и рис. 1 представляют распределение этих измерений. Эмпирическая функция распределеления, начерчена на нормальной

бумаге (рис. 2) подтверждает предположение, что имеем здесь дело со смесью трёх распределений

$$f = c_1 f_1 + c_2 f_2 + c_3 f_3 \quad (c_1 + c_2 + c_3 = 1).$$

Отдельные слагаемые можно интерпретировать как распределение (f_1) результатов измерений при пониженном давлении, распределение (f_2) при нормальном давлении и распределение (f_3) при гипертонии. На основании рис. 2 можно, кроме того, принять, что слагаемое f_2 является плотностью нормального распределения Гаусса, а распределения f_1 и f_3 не заходят на себя.

Для оценки параметров распределения f_2 автор срезает с обеих сторон эмпирическое распределение (ограничивая материал до промежутка, в котором влиянием распределений f_1 и f_2 можно пренебречь) и по урезанной пробе оценивает параметры урезанного нормального распределения (3.1) пользуясь методом (3.17) и (3.18). В качестве критических значений принято на основании рис. 2 величины (3.20). Также из графика найдены исходные значения параметров (3.19). После одного итерационного шага получена оценка (3.21). Формул (4.6) дают оценки коэффициентов c_1, c_2 и c_3, являющиеся вероятностями того, что случайно взятый мужчина имеет пониженное давление (L), нормальное давление ($И$) либо повышенное давление (H). Таблица 3 дает вероятности от результатов измерения давления (A_k).

Автор рекомендует большую осторожность при истолковании полученных численных результатов. Эмпирический материал взят из записок Клиники исследования щитовидной железы при II Клинике внутренних заболеваний Медицинского института во Вроцлаве. Пациенты этой клиники являются преимущественно больными и наблюдаемое распределение может значительно отличаться от неизвестного распределения в полной популяции. Поэтому главный упор в работе был положен на описание статистического метода, который после собрания выборочного материала может быть применен к дискриминации измерений кровяного давления.