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AN EXAMPLE OF A 6-DIMENSIONAL FLAT ALMOST HERMITIAN MANIFOLD

BY

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1. Let M be a C^∞ almost Hermitian manifold, that is, the tangent bundle has an almost complex structure J and a Riemannian metric g such that $g(JX, JY) = g(X, Y)$ for all $X, Y \in \mathscr{X}(M)$, $\mathscr{X}(M)$ denoting the Lie algebra of C^∞ vector fields on M.

Curvature identities are a key to understanding the geometry of almost Hermitian manifolds (see [3] and [4]). In particular, an almost Hermitian manifold such that the Riemann curvature tensor satisfies the Kähler identity

$$R(X, Y, Z, W) = R(X, Y, JZ, JW) \quad \text{for all } X, Y, Z, W \in \mathscr{X}(M)$$

is called an *F-space* [8] or a *para-Kähler manifold* [7]. For this class of manifolds it has been proved [8] that they are Kähler manifolds if the holomorphic sectional curvature μ is pointwise constant and non-zero. The authors have proved in [10] that the case $\mu = 0$ is exceptional by constructing a 4-dimensional flat almost Hermitian manifold which is not a Kähler manifold. This provides also an example of a non-Kähler F-space with vanishing Bochner curvature tensor (see [11] and [12]).

The main purpose of this paper is to give a 6-dimensional non-Kähler flat F-space. This is done in the following way. We start with a study of the tangent bundle of a Riemannian manifold with Riemann connection $\mathring{\nabla}$. Using a metric connection ∇ with torsion, the associated horizontal and vertical lift and the corresponding Sasaki metric, one can prove that the tangent bundle has an almost Hermitian structure with respect to this metric. With this almost complex structure, $T(M)$ is an F-space if and only if M is flat with respect to ∇ and $\mathring{\nabla}$, and $T(M)$ is not a Kähler manifold if and only if $\nabla \neq \mathring{\nabla}$.

So we have only to prove that there exist flat 3-dimensional Riemannian manifolds which have also a flat metric connection with torsion. Such an example has been given in [10] by considering connected Lie groups of dimension 3.

2. First of all we give some well-known results and formulas for the geometry of the tangent bundle $T(M)$ of a C^∞ n-dimensional Riemannian manifold (M, g) with linear connection ∇. For details we refer to [13], [9], and [1].

Let (x^i), $i = 1, 2, \ldots, n$, be a system of local coordinates on M and let (x^i, y^i) be the associated system on $T(M)$. Then, the *horizontal lift* X^{H} and the *vertical lift* X^{V} of a vector field $X \in \mathfrak{X}(M)$ are defined by

$$X^{\mathrm{H}} = X^i \frac{\partial}{\partial x^i} - \Gamma_{hs}{}^i X^h y^s \frac{\partial}{\partial y^i} \quad \text{and} \quad X^{\mathrm{V}} = X^i \frac{\partial}{\partial y^i},$$

where X^i are the components of X with respect to the basis $\partial/\partial x^i$, and $\Gamma_{hs}{}^i$ are the components of ∇. Then, on $T(M)$ there exists a well-defined *almost complex structure* $\tilde{J}$ given by

$$\tilde{J}(X^{\mathrm{H}}) = X^{\mathrm{V}} \quad \text{and} \quad \tilde{J}(X^{\mathrm{V}}) = -X^{\mathrm{H}} \quad \text{for all } X \in \mathfrak{X}(M).$$

Further, M being a Riemannian manifold with metric tensor g, $T(M)$ is also Riemannian and this with respect to the *diagonal lift* $\tilde{g}$ (or *Sasaki metric*) which is defined by

$$\tilde{g}(X^{\mathrm{H}}, Y^{\mathrm{H}}) = \tilde{g}(X^{\mathrm{V}}, Y^{\mathrm{V}}) = \big(g(X, Y)\big)^{\mathrm{V}} \quad \text{and} \quad \tilde{g}(X^{\mathrm{V}}, Y^{\mathrm{H}}) = \tilde{g}(X^{\mathrm{H}}, Y^{\mathrm{V}}) = 0$$

for all $X, Y \in \mathfrak{X}(M)$. It is easy to see that $\tilde{g}$ is a Hermitian metric with respect to $\tilde{J}$, i.e. $\tilde{g}(\tilde{J}\tilde{X}, \tilde{J}\tilde{Y}) = \tilde{g}(\tilde{X}, \tilde{Y})$ for all $\tilde{X}, \tilde{Y} \in \mathfrak{X}\big(T(M)\big)$. We may conclude that $T(M)$ is an almost Hermitian manifold.

In the following we choose an adapted basis defined by

$$E_i = \left(\frac{\partial}{\partial x_i}\right)^{\mathrm{H}} = \frac{\partial}{\partial x^i} - \Gamma_{is}{}^h y^s \frac{\partial}{\partial y^h}, \tag{1}$$

$$\tilde{J}E_i = E_{\bar{i}} = \left(\frac{\partial}{\partial x^i}\right)^{\mathrm{V}} = \frac{\partial}{\partial y^i}, \tag{2}$$

where $\bar{i} = n + i$. Then, $\tilde{g}$ and $\tilde{J}$ are given by

$$\tilde{g} = \begin{pmatrix} g & 0 \\ 0 & g \end{pmatrix} \quad \text{and} \quad \tilde{J} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix},$$

I being the unit (n, n)-matrix, and g the matrix for M.

3. Now we suppose that ∇ is a metric connection and we calculate the components of the Riemann connection $\tilde{\nabla}$ and the Riemann curvature tensor $\tilde{R}$ associated with the diagonal lift $\tilde{g}$ on $T(M)$.

We put

$$[E_\alpha, E_\beta] = \Omega_{\alpha\beta}{}^\gamma E_\gamma,$$

where $\{E_\alpha, \alpha, \beta, \gamma = 1, 2, \ldots, 2n\}$ is an adapted basis. Using (1) and (2) we find that *all* $\Omega_{\alpha\beta}{}^{\gamma}$ *are zero except*

$$\Omega_{ij}{}^{\bar{h}} = -R_{ijs}{}^{h}y^{s}, \quad \Omega_{i\bar{j}}{}^{\bar{h}} = \Gamma_{ij}{}^{h}, \quad \Omega_{\bar{i}j}{}^{\bar{h}} = -\Omega_{j\bar{i}}{}^{\bar{h}} = -\Gamma_{ji}{}^{h}, \tag{3}$$

where $R_{ijs}{}^{h}$ denotes the components of the curvature tensor of ∇.

The components of the Riemann connection $\tilde{\nabla}$ with respect to $\tilde{g}$ are given by (see [13])

$$\tilde{\Gamma}_{\gamma\beta}{}^{a} = \tfrac{1}{2}\tilde{g}^{as}(D_{\gamma}\tilde{g}_{s\beta} + D_{\beta}\tilde{g}_{s\gamma} - D_{s}\tilde{g}_{\beta\gamma}) + \tfrac{1}{2}(\Omega^{a}{}_{\beta\gamma} + \Omega^{a}{}_{\gamma\beta} + \Omega_{\gamma\beta}{}^{a}),$$

where

$$D_{\gamma}\tilde{g}_{s\beta} = E_{\gamma}\tilde{g}_{s\beta}, \quad \Omega^{a}{}_{\beta\gamma} = \tilde{g}^{as}\Omega_{s\beta}{}^{\theta}\tilde{g}_{\theta\gamma}, \quad \tilde{g}^{as}\tilde{g}_{s\beta} = \delta^{a}_{\beta}.$$

Using formulas (3) we find that *all* $\Omega^{a}{}_{\beta\gamma}$ *are zero except*

$$\Omega^{i}{}_{j\bar{k}} = -R^{i}{}_{jsk}y^{s}, \quad \Omega^{\bar{i}}{}_{j\bar{k}} = -\Gamma_{j}{}^{i}{}_{k}, \quad \Omega^{i}{}_{\bar{j}\bar{k}} = \Gamma^{i}{}_{jk},$$

where

$$R^{i}{}_{jsk} = g^{im}R_{mjs}{}^{p}g_{pk}, \quad \Gamma_{j}{}^{i}{}_{k} = g^{im}\Gamma_{jm}{}^{p}g_{pk}, \quad \Gamma^{i}{}_{jk} = g^{im}\Gamma_{mj}{}^{p}g_{pk}.$$

Using the fact that *all* $D_{\gamma}\tilde{g}_{s\beta}$ *are zero except*

$$D_{k}\tilde{g}_{ij} = D_{k}\tilde{g}_{\bar{i}\bar{j}} = \frac{\partial g_{ij}}{\partial x^{k}},$$

for the component of $\tilde{\nabla}$ we get

$$\begin{aligned} &\tilde{\Gamma}_{kj}{}^{i} = \mathring{\Gamma}_{kj}{}^{i}, \quad \tilde{\Gamma}_{k\bar{j}}{}^{i} = -\tfrac{1}{2}R^{i}{}_{ksj}y^{s}, \quad \tilde{\Gamma}_{\bar{k}j}{}^{i} = -\tfrac{1}{2}R^{i}{}_{jsk}y^{s}, \\ &\tilde{\Gamma}_{\bar{k}\bar{j}}{}^{i} = \tilde{\Gamma}_{\bar{k}j}{}^{\bar{i}} = \tilde{\Gamma}_{\bar{k}\bar{j}}{}^{\bar{i}} = 0, \quad \tilde{\Gamma}_{kj}{}^{\bar{i}} = -\tfrac{1}{2}R_{kjs}{}^{i}y^{s}, \quad \tilde{\Gamma}_{k\bar{j}}{}^{\bar{i}} = \Gamma_{kj}{}^{i}. \end{aligned} \tag{4}$$

Now, the components of the tensor $\tilde{R}$ are given by (see [13])

$$\tilde{R}_{\delta\gamma\beta}{}^{a} = D_{\delta}\tilde{\Gamma}_{\gamma\beta}{}^{a} - D_{\gamma}\tilde{\Gamma}_{\delta\beta}{}^{a} + \tilde{\Gamma}_{\delta s}{}^{a}\tilde{\Gamma}_{\gamma\beta}{}^{s} - \tilde{\Gamma}_{\gamma s}{}^{a}\tilde{\Gamma}_{\delta\beta}{}^{s} - \Omega_{\delta\gamma}{}^{s}\tilde{\Gamma}_{s\beta}{}^{a},$$

where $D_{\delta}\tilde{\Gamma}_{\gamma\beta}{}^{a} = E_{\delta}(\tilde{\Gamma}_{\gamma\beta}{}^{a})$. These components may now be calculated by using (1)-(3). With the help of the symmetry properties and the first Bianchi identity for $\tilde{R}$ we may conclude that it is sufficient to calculate $\tilde{R}_{khj}{}^{a}$, $\tilde{R}_{k\bar{h}j}{}^{a}$, $\tilde{R}_{k\overline{hj}}{}^{a}$ and $\tilde{R}_{\overline{khj}}{}^{a}$. Finally, we find

$$\begin{aligned} \tilde{R}_{khj}{}^{i} &= \mathring{R}_{khj}{}^{i} + \tfrac{1}{4}(R^{i}{}_{ksm}R_{hjt}{}^{m} - R^{i}{}_{hsm}R_{kjt}{}^{m} + 2R_{khs}{}^{m}R^{i}{}_{jtm})y^{s}y^{t}, \\ \tilde{R}_{k\bar{h}j}{}^{i} &= -\tfrac{1}{2}(\nabla_{k}R^{i}{}_{jsh} - T_{km}{}^{i}R^{m}{}_{jsh} + T_{kj}{}^{m}R^{i}{}_{msh})y^{s}, \\ \tilde{R}_{k\overline{hj}}{}^{i} &= \tfrac{1}{2}R^{i}{}_{khj} - \tfrac{1}{4}(R^{i}{}_{msh}R^{m}{}_{ktj})y^{s}y^{t}, \\ \tilde{R}_{\overline{khj}}{}^{i} &= \tilde{R}_{k\overline{hj}}{}^{\bar{i}} = \tilde{R}_{\overline{khj}}{}^{\bar{i}} = 0, \\ \tilde{R}_{khj}{}^{\bar{i}} &= -\tfrac{1}{2}(\nabla_{k}R_{hjs}{}^{i} - \nabla_{h}R_{kjs}{}^{i} + T_{kj}{}^{m}R_{hms}{}^{i} - T_{hj}{}^{m}R_{kms}{}^{i} + S_{kh}{}^{m}R_{mjs}{}^{i})y^{s}, \\ \tilde{R}_{k\bar{h}j}{}^{\bar{i}} &= \tfrac{1}{2}R_{kjh}{}^{i} + \tfrac{1}{4}(R_{kms}{}^{i}R^{m}{}_{jth})y^{s}y^{t}, \end{aligned} \tag{5}$$

where

$$T_{kj}{}^{m} = \Gamma_{kj}{}^{m} - \mathring{\Gamma}_{kj}{}^{m}, \quad S_{kh}{}^{m} = \Gamma_{kh}{}^{m} - \Gamma_{hk}{}^{m}.$$

4. With the help of these formulas we prove now some interesting results for the almost Hermitian manifold $T(M)$.

THEOREM 1. *$T(M)$ is an F-space if and only if M is flat with respect to ∇ and $\overset{\circ}{\nabla}$ (and thus $T(M)$ is an F-space if and only if $T(M)$ is flat).*

Proof. If $R = \overset{\circ}{R} = 0$, then it follows from (5) that $\tilde{R} = 0$, and hence $T(M)$ is flat, and so it is an F-space.

Conversely, suppose that $T(M)$ is an F-space. Hence we must have $\tilde{R}_{\tilde{X}\tilde{Y}}(\tilde{J}\tilde{Z}) = \tilde{J}\tilde{R}_{\tilde{X}\tilde{Y}}\tilde{Z}$ for all $\tilde{X}, \tilde{Y}, \tilde{Z} \in \mathfrak{X}(T(M))$. With respect to the adapted basis this is equivalent to

$$\tilde{R}_{\delta\gamma\bar{j}}{}^{i} + \tilde{R}_{\delta\gamma j}{}^{\bar{i}} = 0, \qquad \tilde{R}_{\delta\gamma\bar{j}}{}^{\bar{i}} - \tilde{R}_{\delta\gamma j}{}^{i} = 0.$$

Since $\tilde{R}_{\bar{k}\bar{h}j}{}^{\bar{i}} = 0$ we must have, in particular,

$$\tilde{R}_{kh\bar{j}}{}^{\bar{i}} - \tilde{R}_{khj}{}^{i} = 0, \qquad \tilde{R}_{\bar{k}\bar{h}j}{}^{i} = 0. \tag{6}$$

These conditions must be satisfied on the zero cross-section, i.e. at the points where $y^i = 0$. At these points we have

$$\tilde{R}_{kh\bar{j}}{}^{\bar{i}} = R_{khj}{}^{i}, \qquad \tilde{R}_{khj}{}^{i} = \overset{\circ}{R}_{khj}{}^{i}, \qquad \tilde{R}_{\bar{k}\bar{h}j}{}^{i} = \tfrac{1}{2}(R^{i}{}_{jhk} - R^{i}{}_{jkh})$$

and this implies with (6) that $R = \overset{\circ}{R} = 0$.

THEOREM 2. *$T(M)$ is a quasi-Kähler manifold if and only if $\nabla = \overset{\circ}{\nabla}$.*

Proof. A manifold $T(M)$ is quasi-Kähler if and only if (see [2]-[4])

$$\tilde{\nabla}_{\tilde{X}}(\tilde{J})\tilde{Y} + \tilde{\nabla}_{\tilde{J}\tilde{X}}(\tilde{J})\tilde{J}\tilde{Y} = 0.$$

Hence, after substituting E_i for $\tilde{X}$ and E_j for $\tilde{Y}$, and using (4), we obtain

$$\tilde{\Gamma}_{ij}^{\bar{k}} - \tilde{\Gamma}_{\bar{i}\bar{j}}^{\bar{k}} = \Gamma_{ij}^{k} - \overset{\circ}{\Gamma}_{ij}^{k} = 0$$

and, consequently, $\nabla = \overset{\circ}{\nabla}$.

Conversely, if $\nabla = \overset{\circ}{\nabla}$, then $T(M)$ is an almost Kähler manifold, and hence also quasi-Kähler (see [2] and [3]).

Theorem 2 implies also that $T(M)$ *is quasi-Kähler if and only if it is almost Kähler*. We note that an almost Kähler manifold is an almost Hermitian manifold with closed fundamental 2-form (Kähler form).

It is also interesting to state the following known theorem [9]:

THEOREM 3. *$T(M)$ is Hermitian if and only if $\nabla = \overset{\circ}{\nabla}$ and $\overset{\circ}{R} = 0$, i.e. if and only if it is a Kähler manifold.*

From this we have

COROLLARY 1. *$T(M)$ is a nearly Kähler manifold if and only if it is a Kähler manifold, i.e. if and only if $\nabla = \overset{\circ}{\nabla}$ and $\overset{\circ}{R} = 0$.*

Proof. A manifold $T(M)$ is nearly Kähler if and only if $\tilde{\nabla}_{\tilde{X}}(\tilde{J})\tilde{X} = 0$ for all $X \in \mathfrak{X}(T(M))$. Such a manifold is necessarily quasi-Kähler [2], and hence we have $\nabla = \nabla$. This means that $T(M)$ is also almost Kähler [1], and since nearly and almost Kähler means Kähler, we have the required result from Theorem 3.

The converse is trivial, since any Kähler manifold is nearly Kählerian.

We may now state the main theorem of this section:

MAIN THEOREM. *Let M be a Riemannian manifold which has also a metric connection with torsion. If M is flat with respect to this connection and to the Riemannian connection, then the tangent bundle $T(M)$ has a flat almost Hermitian structure which is not a Kähler structure.*

5. We give now an example of a manifold M which satisfies the conditions given in the Main Theorem.

Let G be a connected Lie group and $\mathfrak{G}$ its Lie algebra. We shall identify $\mathfrak{G}$ with the tangent space $T_e(G)$ at the neutral element of G. Further, let $A, B, \ldots$ denote elements of $\mathfrak{G}$, and $\tilde{A}, \tilde{B}, \ldots$ the corresponding left invariants on G.

There is a natural bijection between the set of positive-definite non-degenerate bilinear forms $\varphi\colon \mathfrak{G} \times \mathfrak{G} \to \boldsymbol{R}$ and the set of left invariant Riemannian metrics g on G (see [6], p. 200, and [5], p. 125). This bijection is given by

$$\big(g(\tilde{A}, \tilde{B})\big)_e = \varphi(A, B) \qquad \text{for all } A, B \in \mathfrak{G}.$$

There is also a bijection between the set of bilinear maps $\tau\colon \mathfrak{G} \times \mathfrak{G} \to \mathfrak{G}$ and the set of left invariant linear connections. This is defined by (see [5], p. 92)

$$(\nabla_{\tilde{A}} \tilde{B})_e = \tau(A, B) \qquad \text{for all } A, B \in \mathfrak{G}.$$

Let g be the left invariant metric on G corresponding to φ. Then one can prove that the Riemannian connection $\mathring{\nabla}$ associated with g is a left invariant connection. Precisely, $\mathring{\nabla}$ is the linear connection associated with the bilinear map τ which is entirely defined by the conditions

$$\varphi\big(\tau(A, B), C\big) + \varphi\big(B, \tau(A, C)\big) = 0, \tag{7}$$

$$\tau(A, B) - \tau(B, A) = [A, B] \tag{8}$$

for all $A, B, C \in \mathfrak{G}$. In fact (see [6], p. 201), τ is given by

$$2\varphi\big(\tau(A, B), C\big) = \varphi([A, B], C) + \varphi([C, A], B) + \varphi(A, [C, B]).$$

The Riemann curvature tensor $\mathring{R}$ of $\mathring{\nabla}$ operates on left invariant vector fields as follows:

$$(\mathring{R}_{\tilde{A}\tilde{B}} \tilde{C})_e = \tau\big(A, \tau(B, C)\big) - \tau\big(B, \tau(A, C)\big) - \tau([A, B], C).$$

Thus, $\mathring{R} = 0$ if and only if

$$\tau(A, \tau(B, C)) - \tau(\tau(A, B), C) = \tau(B, \tau(A, C)) - \tau(\tau(B, A), C) \quad \text{for all } A, B, C \in \mathfrak{G}. \tag{9}$$

We may conclude (see [10])

THEOREM 4. *Let G be a connected Lie group and $\mathfrak{G}$ its Lie algebra. If $\mathfrak{G}$ has a positive-definite non-degenerate symmetric bilinear form φ and a bilinear map τ which satisfy* (7)-(9), *then G is a flat Riemannian manifold.*

If this Lie algebra is not commutative, then G has also a metric connection which is flat and which differs from the Riemann connection. Indeed, it is sufficient to consider the classical $(-)$-connection of Cartan-Schouten (see [5], p. 94). It is denoted by $\bar{\nabla}$ and is associated with the null map τ, i.e.

$$(\bar{\nabla}_{\tilde{A}} \tilde{B})_e = 0 \qquad \text{for all } A, B \in \mathfrak{G}.$$

It is easy to see that $\bar{\nabla} g = 0$, that is, this connection is metric. On the other hand, the torsion is given by

$$\big(\bar{T}(\tilde{A}, \tilde{B})\big)_e = -[A, B] \qquad \text{for all } A, B \in \mathfrak{G}.$$

Consider now the Lie algebra of dimension 3 over $\boldsymbol{R}$ defined by

$[\cdot]$	A_1	A_2	A_3
A_1	0	0	A_2
A_2	0	0	$-A_1$
A_3	$-A_2$	A_1	0

We define a positive-definite symmetric bilinear form $\varphi\colon \mathfrak{G} \times \mathfrak{G} \to \boldsymbol{R}$ by

$$\varphi(A_i, A_j) = \delta_{ij},$$

and a bilinear map $\tau\colon \mathfrak{G} \times \mathfrak{G} \to \mathfrak{G}$ by

$$\tau(A_i, A_j) = 0$$

except

$$\tau(A_3, A_1) = -A_2 \qquad \text{and} \qquad \tau(A_3, A_2) = A_1.$$

Then, conditions (7)-(9) are verified [10] and this algebra, clearly, is not commutative.

We note that this Lie algebra may be represented as a subalgebra of $\mathrm{gl}(3, \boldsymbol{R})$ by putting

$$A_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Hence the Lie algebra is formed by the matrices

$$\begin{pmatrix} 0 & x_3 & x_1 \\ -x_3 & 0 & x_2 \\ 0 & 0 & 0 \end{pmatrix}, \quad x_i \in \boldsymbol{R},\ i = 1, 2, 3,$$

and the corresponding Lie group is formed by

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ -a_{12} & a_{11} & a_{23} \\ 0 & 0 & 1 \end{pmatrix}, \quad \text{where } a_{11}^2 + a_{12}^2 = 1.$$

It is a connected subgroup of $\mathrm{Gl}(3, \boldsymbol{R})$.

This provides thus an example of the required class of manifolds.

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