ANNALES
POLONICI MATHEMATICI
LIV.1 (1991)

Classical boundary value problems for integrable temperatures in a C^1 domain

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Abstract. We study a Neumann problem for the heat equation in a cylindrical domain with C^1-base and data in h_c^1, a subspace of L^1. We derive our results, considering the action of an adjoint operator on B_TMOC, a predual of h_c^1, and using known properties of this last space.

Introduction. The classical Neumann and Dirichlet problems for the heat equation in the cylindrical domain $D \times (0, T)$ have been studied by E. Fabes and N. Rivière ([5]) using the method of layer potentials. They considered bounded domains of class C^1 and L^p data on the boundary $\partial D \times (0, T)$ for $1 < p < \infty$. For $p = 1$ the problems are still open. As for the Laplacian we must introduce a suitable subspace of L^1 on which the corresponding results can be proved.

In [3] E. Fabes and C. Kenig studied the Neumann problem for the Laplacian with data in h^1, a subspace of L^1 whose dual is BMO. They use the fact that the solvability of the Neumann problem with boundary data in h^1 is closely related to solvability of the Dirichlet problem with boundary data in BMO.

In order to study the Neumann problem for the heat operator with suitable h^1 data, we shall consider the action of an adjoint operator on a subspace B_TMOC of caloric-BMO. The definition of B_TMOC is analogous to that of B_0MOC ([7]).

As usual we must construct h_c^1 (caloric-h^1), a predual of B_TMOC, making use of an atomic representation.

Applying the results obtained on B_0MOC ([7], [8]) we solve the Neumann problem with data in h_c^1.

We would like to thank Professors E. Fabes and U. Neri for many helpful conversations.

§ 1. Definitions and preliminaries. A bounded domain $D \subset \mathbf{R}^n$ is called a C^1 *domain* if for each point $Q \in \partial D$ there exists a ball $B = B(r, Q)$ with center Q and

1980 *Mathematics Subject Classification*: Primary 35K05.
Both authors supported in part by C.N.R. and University of Maryland grants.

radius r and a coordinate system of $\mathbf{R}^n$ with Q as the origin such that with respect to these new coordinates

$$B \cap D = B \cap \{(x', x_n):\ x' \in \mathbf{R}^{n-1}, x_n > \Phi(x')\},$$
$$B \cap \partial D = B \cap \{(x', \Phi(x')):\ x' \in \mathbf{R}^{n-1}\},$$

where $\Phi \in C_0^1(\mathbf{R}^{n-1})$, $\Phi(0) = 0 = (\partial \Phi/\partial x_i)(0)$, $i = 1, \ldots, n$.

We will assume the radius of the ball B can be chosen independently of $Q \in \partial D$, and we call it r_0; it is clearly a geometric constant depending only on the structure of D.

If D is a bounded C^1 domain we will let N_Q denote the unit inner normal to ∂D at Q. We write

$$D_+ = D \times \mathbf{R}^+, \qquad D_T = D \times (0, T),$$
$$\partial D_+ = \partial D \times \mathbf{R}^+, \qquad \partial D_T = \partial D \times (0, T), \qquad \text{for } 0 < T < \infty.$$

$X, Y, \ldots$ are points in D (or $\mathbf{R}^n$), while $P, Q, \ldots$ are points of ∂D. The letters t and s are used for the time variable in $\mathbf{R}^+$. We set

$$\Gamma(X, t) = (\pi t)^{-n/2} \exp(-|X|^2/4t),$$

the fundamental solution of the heat equation, and

$$K(X, t) = \langle \nabla_X \Gamma(X, t), N_Q \rangle = c_n \frac{\langle X - Q, N_Q \rangle}{t^{n/2+1}} \exp\{-|X-Q|^2/4t\},$$

the kernel of the double layer potential.

We recall the definition of the space $\mathrm{BMOC}(\partial D_T)$ ([7]): $f \in \mathrm{BMOC}(\partial D_T)$ if

$$\|f\|_* = \sup_{\Delta} \{|\Delta|^{-1} \int_{\Delta} |f - f_\Delta| \, dQ \, ds\} < \infty \tag{1.1}$$

where $\Delta = \Delta_r(P, t) = \{(Q, s) \in \partial D_T:\ |P - Q| < r, |s - t| < r^2\}$, $f_\Delta = |\Delta|^{-1} \int_\Delta f$. With the identification $f_1 \sim f_2$ if $f_1 - f_2 = \text{constant}$, BMOC is a complete normed space with norm (1.1).

We say that $f \in \mathrm{B_0MOC}$ ([7]) if (1.1) is valid and

$$B_0(f) = \sup_{\Delta^0} \left| |\Delta^0|^{-1} \int_{\Delta^0} f \right| < \infty \tag{1.2}$$

where $\Delta^0 = \Delta^0(P_0) = \{(P, t) \in \partial D_T:\ |P - P_0| < r, 0 < t < r^2\}$. $\mathrm{B_0MOC}$ is a complete normed space with norm

$$\|f\|_{0,*} = B_0(f) + \|f\|_*. \tag{1.3}$$

For our purpose we introduce the space $\mathrm{B}_T\mathrm{MOC}$, which is analogous to $\mathrm{B_0MOC}$, except that the condition (1.2) at $t = 0$ is replaced by a condition at $t = T$: $f \in \mathrm{B}_T\mathrm{MOC}$ if (1.1) is valid and

$$B_T(f) = \sup_{\Delta^T} \left| |\Delta^T|^{-1} \int_{\Delta^T} f \right| < \infty \tag{1.4}$$

where $\Delta^T = \Delta_r^T(P_0) = \{(P, t) \in \partial D_T \colon |P-P_0| < r,\ T-r^2 < t < T\}$. $\mathrm{B}_T\mathrm{MOC}$ is a complete normed space with norm

$$\|f\|_{T,*} = \|f\|_* + B_T(f). \tag{1.5}$$

As done for $\mathrm{B}_0\mathrm{MOC}$ ([7]), it is possible to show that (1.5) is equivalent to the norm

$$\|f\|_{p,T,*} = C_{p,T}(f) + \|f\|_{*,p} \tag{1.6}$$

where

$$C_{p,T}(f) = \sup_{\Delta^T}\{|\Delta^T|^{-1} \int_{\Delta^T} |f|^p\}^{1/p}, \qquad \|f\|_{*,p} = \sup_{\Delta}\{|\Delta|^{-1} \int_{\Delta} |f-f_\Delta|^p\}^{1/p}.$$

We now consider the so-called adjoint of the heat operator, $L^* = \Delta_X + D_t$. Choosing a Banach space $E = E(\partial D_T)$ of functions defined on ∂D_T, we can associate to this operator the Dirichlet problem

$$\begin{cases} L^*u = 0 & \text{in } D_T, \\ u(X, T) = 0, \\ u(X, t)|_{\partial D_T} = f, \end{cases} \tag{1.7}$$

where f is in E.

If we look for a solution of (1.7) in the form of a double layer potential, we will write it

$$u(X, s) = \int_s^T \int_{\partial D} \frac{\langle X-Q, N_Q\rangle}{(t-s)^{n/2+1}} \exp\{-|X-Q|^2/4(t-s)\}\, g(Q, t)\, dQ\, dt$$

where g is an unknown function in E.

The trace on ∂D_T of $u(X, s)$ gives rise to the operator

$$\Phi(g)(P, s) = \int_s^T \int_{\partial D} \frac{\langle P-Q, N_Q\rangle}{(t-s)^{n/2+1}} \exp\{-|P-Q|^2/4(t-s)\}\, g(Q, t)\, dQ\, dt. \tag{1.8}$$

When $E = L^p(\partial D_T)$, Φ is a singular integral operator. Using, with the obvious modifications, the techniques and results of [5] for the operator

$$J(g)(P, t) = \int_0^t \int_{\partial D} \frac{\langle P-Q, N_Q\rangle}{(t-s)^{n/2+1}} \exp\{-|P-Q|^2/4(t-s)\}\, g(Q, s)\, dQ\, ds \tag{1.9}$$

one can show that Φ is bounded linear operator on $L^p(\partial D_T)$, $1 < p < \infty$. Moreover, if $(a, b) \subset (0, T)$ then

$$\|\Phi(\chi_{(a,b)} f)\|_{L^p(\partial D \times (a,b))} \leqslant \omega(b-a) \|f\|_{L^p(\partial D \times (a,b))} \tag{1.10}$$

where $\omega(\delta) \to 0$ as $\delta \to 0^+$

The solvability of (1.7), with the method of the double layer potential, is related, as for the analogous problem for the heat operator, to the invertibility of the operator $c_n I + \Phi$ where $c_n > 0$ and I is the identity operator.

This can be obtained directly as in [5] using property (1.10), or simply observing that Φ is the adjoint of J', where

$$(1.11)\quad J'(g)(P,t) = -\int_0^t\int_{\partial D} \frac{\langle P-Q, N_P\rangle}{(t-s)^{n/2+1}} \exp\{-|P-Q|^2/4(t-s)\}\, g(Q,s)\, dQ\, ds,$$

and using the results of [5] for this operator.

So the problem (1.7), for $E = L^p(\partial D_T)$, $1 < p < \infty$, has a unique solution if the convergence of $u(X, s)$ to f is understood in the non-tangential sense. When $E = \mathrm{B}_T\mathrm{MOC}$ the continuity of Φ in E and the invertibility of $c_n I + \Phi$ in E can be obtained, with only slight modifications, in the same manner as for the operator J in $\mathrm{B}_0\mathrm{MOC}$ (see [7], [8]).

§ 2. The space $h_c^{1,q}(\partial D_T)$. We begin by giving the definition of a "caloric" atom. We say that a function $a = a(P, t)$ is a $(1, q)$ *c-atom*, $1 < q < \infty$, if

(i) the support of a is contained in $\Delta \subset \partial D_T$, with $\Delta = \Delta_r(P_0, t_0)$,
(ii) $\left(\int_\Delta |a(P, t)|^q\right)^{1/q} \leqslant |\Delta|^{-1+1/q}$,
(iii) $\left|\int_\Delta a(P, t)\right| \leqslant r/(T-t_0+r^2)^{1/2}$

When $q = \infty$, (ii) becomes (ii′) $|a(P, t)| \leqslant |\Delta|^{-1}$

The definition of c-atom is a generalization of the classical definition of atom, see ([1]); in fact the condition $\int_{\partial D_T} a = 0$ is replaced by the more general condition (iii).

We define $h_c^{1,q}$, $1 < q < \infty$, to be the space of functions admitting an atomic decomposition

$$f = \sum_{j=1}^{\infty} \lambda_j a_j$$

where the a_j are $(1, q)$ c-atoms, $\sum_{j=1}^{\infty} |\lambda_j| < \infty$ and the convergence is in the sense of $L^1(\partial D_T)$. The infimum of the numbers $\sum_{j=1}^{\infty} |\lambda_j|$ taken over all such representations will be denoted by $\|f\|_{1,q}$; it turns out to be a norm on $h_c^{1,q}(\partial D_T)$.

THEOREM 2.1. *$\mathrm{B}_T\mathrm{MOC}_{q'}(\partial D_T)$ is the dual space of $h_c^{1,q}$, with $1/q + 1/q' = 1$, i.e. for each $l \in \mathrm{B}_T\mathrm{MOC}$ and for all $f = \sum_{j=1}^{\infty} \lambda_j a_j \in h_c^{1,q}$, the mapping $f \to \langle l, f\rangle = \lim_{n\to\infty} \sum_{j=1}^{n} \lambda_j \int_{\partial D_T} l a_j$ is a well defined linear functional on $h_c^{1,q}$. Moreover, each continuous functional has this form.*

Proof. Fix $l \in \mathrm{B}_T\mathrm{MOC}$ and let a be a $(1, q)$ c-atom. Then $|\langle l, a\rangle| = \left|\int_\Delta la\right| \leqslant \left|\int_\Delta (l-l_\Delta)a\right| + \left|l_\Delta \int_\Delta a\right| = I' + I''$ By Hölder's inequality we have

$$I' \leqslant \left(\int_\Delta |a|^q\right)^{1/q}\left(\int_\Delta |l-l_\Delta|^{q'}\right)^{1/q'} \leqslant \left(|\Delta|^{-1}\int_\Delta |l-l_\Delta|^{q'}\right)^{1/q'} \leqslant \|l\|_{*,q'}.$$

In the same manner

$$I'' \leqslant \frac{r}{(T-t_0+r^2)^{1/2}} \big||\Delta|^{-1} \int_\Delta l\big| \leqslant \frac{r}{(T-t_0+r^2)^{1/2}} |\Delta|^{n/(n+1)} \{|\Delta|^{-1} (\int_\Delta |l|^{n+1})^{1/(n+1)}\}$$
$$\leqslant c(|\Delta_{\bar r}^T|^{-1} \int_{\Delta_{\bar r}^T} |l^{n+1}|)^{1/(n+1)} \leqslant c\|l\|_{q',T,*}$$

where $\bar r = (T-t_0+r^2)^{1/2}$, and c depends only on the dimension of the space. More generally, for any $f = \sum_{j=1}^{\infty} \lambda_j a_j \in h_c^{1,q}$ we have $\langle l, f\rangle = \lim_{n\to\infty} \sum_{j=1}^{n} \lambda_j \int_{\Delta_j} l a_j$. By the previous argument we have $|\langle l, a_j\rangle| \leqslant c\|l\|_{q',T,*}$ for all j. Therefore

$$|\langle l, f\rangle| \leqslant c\{\lim_{n\to\infty} \sum_{j=1}^{n} |\lambda_j|\} \|l\|_{q',T,*} \leqslant c\|f\|_{1,q'} \|l\|_{q',T,*}$$

and so $\mathrm{B}_T\mathrm{MOC}_{q'} \subsetneq h_c^{1,q}$.

To prove the reverse inclusion we first observe that any function $f \in L^{q'}(\Delta)$ such that $\|f\|_{L^{q'}} = 1$ and $\mathrm{supp}(f) \subset \Delta$ can be normalized to be a $(1, q')$ c-atom. In fact, letting

$$g_1 = (|\Delta|^{1/q'}/|\Delta_{\bar r}^T|) f \tag{2.1}$$

we have $\mathrm{supp}(g_1) \subset \Delta$ and

$$(\int_\Delta |g_1|^{q'})^{1/q'} \leqslant |\Delta|^{1/q'}/|\Delta_{\bar r}^T| \leqslant |\Delta|^{-1+1/q'}$$

In addition

$$\big|\int_\Delta g_1\big| \leqslant \frac{|\Delta|^{1/q'}}{|\Delta_{\bar r}^T|} \int_\Delta |f| \leqslant \frac{|\Delta|^{1/q'}}{|\Delta_{\bar r}^T|} |\Delta|^{1/q} \leqslant c\frac{r}{(T-t_0+r^2)^{1/2}},$$

so (iii) holds.

Furthermore, besides the caloric g_1 we can associate to f another caloric atom

$$g_2 = (f-f_\Delta)/(2|\Delta|^{1/q}) \tag{2.2}$$

having integral zero on ∂D_T.

Suppose now L is a linear functional on $h_c^{1,q'}(\partial D_T)$. Given any Δ in ∂D_T and any $f \in L_0^{q'}(\Delta)$, where $L_0^{q'}(\Delta)$ is the class of all $f \in L^{q'}(\Delta)$ such that $\mathrm{supp}(f) \subset \Delta$, f belongs to $h_c^{1,q'}(\Delta)$. Assuming, as we may, that $\|f\|_{L^{q'}(\Delta)} = 1$ and writing $f = g_1|\Delta_{\bar r}^T||\Delta|^{-1/q'}$, since g_1 is a $(1, q')$ c-atom, it follows that

$$\|f\|_{h_c^{1,q'}} \leqslant |\Delta_{\bar r}^T|/|\Delta|^{1/q'}$$

Therefore, for any $f \in L_0^{q'}(\Delta)$

$$|L(f)| \leqslant \|L\| \frac{|\Delta_{\bar r}^T|}{|\Delta|} \|f\|_{L^{q'}(\Delta)},$$

i.e., L is a bounded linear functional on $L_0^{q'}(\Delta)$. By the Hahn–Banach theorem L extends continuously to all $f \in L^{q'}(\Delta)$. Thus, by the Riesz representation theorem, there exists $l \in L^q(\Delta)$ such that $L(f) = \int_\Delta fl$ for all $f \in L^{q'}(\Delta)$. In particular, for $\Delta = \partial D_T$ it follows that $l \in L^q(\partial D_T)$.

Recall that, given $f \in L^{q'}(\Delta_r^T)$ with $\operatorname{supp}(f) \subset \Delta_r^T$ and norm 1, then g_1, defined by (2.1), is a $(1, q')$ c-atom. Hence

$$|L(g_1)| = \Big||\Delta_r^T|^{-1/q} \int_{\Delta_r^T} fl\Big| = \big|L(|\Delta_r^T|^{-1/q} f)\big| \leq \|L\|. \tag{2.3}$$

Moreover, because g_2 is defined by (2.2), for all $f \in L^{q'}(\Delta)$

$$\Big|\int_\Delta f(l - l_\Delta)\Big| = \Big|\int_\Delta fl - \int_\Delta fl_\Delta\Big| = \Big|\int_\Delta fl - \int_\Delta f\big(|\Delta|^{-1}\int_\Delta l\big)\Big|$$
$$= \Big|\int_\Delta fl - \int_\Delta lf_\Delta\Big| = \Big|\int_\Delta l(f - f_\Delta)\Big|.$$

Hence

$$|\Delta|^{-1/q} \int_\Delta l|f - f_\Delta| = 2\int_\Delta l|f - f_\Delta| 2^{-1} |\Delta|^{-1/q} = 2|L(g_2)| \leq 2\|L\|.$$

Consequently

$$\Big(|\Delta|^{-1} \int_\Delta |l - l_\Delta|^q\Big)^{1/q} \leq 2\|L\|. \tag{2.4}$$

Combining (2.3) and (2.4) it follows that $l \in B_T MOC$.

The space $h_c^{1,q}(\partial D_T)$ is easily shown to be complete, and if $1 < q_1 < q_2 < \infty$ then

$$h_c^{1,\infty} \subset h_c^{1,q_2} \subset h_c^{1,q_1}.$$

The spaces $h_c^{1,q}$ defined above are not a particular case of the spaces $H^{1,q}$ associated to a space of homogeneous type, introduced by R. Coifman and G. Weiss in [1].

In fact, in our space ∂D_T, there is no distance for which the anisotropic surface disc Δ is a sphere. In [9] we studied the geometric structure of ∂D_T in relation to the surface disc Δ, and we established some different results. In particular, we have the following

THEOREM 2.2. *For any* $1 < q < \infty$, $h^{1,q} = h^{1,\infty}$ *and the two norms are equivalent.*

This theorem enables us to define the space h_c^1 as one of the spaces $h_c^{1,q}$, $1 < q < \infty$, and to use the most convenient one in the proofs of the theorems. In the following we will use the space $h_c^{1,2}$ and clearly the same results are valid for any $h_c^{1,q}$.

§3. The Neumann problem with data in $h_c^1(\partial D_T)$. Given $g \in h_c^1(\partial D_T)$, we consider the Neumann problem for the heat equation

$$(3.1)\quad \begin{cases} Lu(X,t) = \Delta_X u - D_t u = 0 & \text{for all } (X,t)\in\partial D_T,\\ \lim\limits_{t\to 0^+} u(X,t) = 0 & \text{uniformly on compact subsets of } D,\\ \partial_{N_P} u(X,t)\to g(P,z). \end{cases}$$

The last statement is that $\partial_{N_P}u(X,t)$ approaches $g(P,z)$ in non-tangential sense, i.e.

$$\lim_{\substack{(X,t)\to(P,z)\\ (X,t)\in\Gamma(P,z)}} \langle \nabla_X u(X,t), N_P\rangle = g(P,z) \quad \text{a.e.}$$

where

$$\Gamma(P,z) = \{(X,t)\colon |X-P|+|t-z|^{1/2} < (1+\beta)\operatorname{dist}(X,\partial D)\}\cap D_T,$$

β a constant giving the opening of the cone Γ.

We look for a solution of (3.1) in the form of a single layer potential

$$u(X,t) = -2\int_0^t \int_{\partial D} \frac{\exp\{-|X-Q|^2/4(t-s)\}}{(t-s)^{n/2}} f(Q,s)\,dQ\,ds$$

where f is an unknown function in $h_c^1(\partial D_T)$.

The first step is to study the behavior on the boundary of

$$\langle \nabla_X u(X,t), N_P\rangle = \int_0^t \int_{\partial D} \frac{\langle X-Q, N_P\rangle}{(t-s)^{n/2+1}} \exp\{-|X-Q|^2/4(t-s)\} f(Q,s)\,dQ\,ds$$

whose trace on ∂D_T is the singular integral operator

$$(3.2)\quad J'(f) = \lim_{\varepsilon\to 0^+} \int_0^{t-\varepsilon} \int_{\partial D} \frac{\langle P-Q, N_P\rangle}{(t-s)^{n/2+1}} \exp\{-|P-Q|^2/4(t-s)\} f(Q,s)\,dQ\,ds$$

$$= \lim_{\varepsilon\to 0^+} \int_0^{t-\varepsilon} \int_{\partial D} K'(P-Q, t-s) f(Q,s)\,dQ\,ds.$$

As already remarked, J' is the adjoint of $-\Phi$ and is continuous from $L^p(\partial D_T)$ to $L^p(\partial D_T)$, $1 < p < \infty$.

Now we will show that J' is a continuous operator from $h_c^1(\partial D_T)$ to $h_c^1(\partial D_T)$. To do this, we need some lemmas.

LEMMA 3.1. *If $a = a(P,t)$ is a $(1,2)$ c-atom with support in $\Delta = \Delta_r(P_0, t_0)$, then*

$$(3.3)\quad \Big|\int_{\partial D_T} J'(a)\Big| \leqslant c\frac{r}{(T-t_0+r^2)^{1/2}}, \quad c \text{ independent of } a.$$

Proof. We consider first the case $r^2 < T-t_0$. Recalling that $\sup_\varepsilon |J'_\varepsilon(a)| \in L^2$ (see [5]), we can write

$$\int_{\partial D_T} J'(a) = \int_{\partial D_T} \lim_{\varepsilon\to 0^+} J'_\varepsilon(a) = \lim_{\varepsilon\to 0^+} \int_{\partial D_T} J'_\varepsilon(a)$$

$$= \lim_{\varepsilon\to 0^+} \int_0^T \int_{\partial D} \{ \int_0^{t-\varepsilon} \int_{\partial D} K'(P-Q,\, t-s) a(Q,s)\, dQ\, ds\}\, dP\, dt$$

$$= \lim_{\varepsilon\to 0^+} I_\varepsilon.$$

Now performing the change of variable $z = |P-Q|^2/(t-s)$, we have

$$I_\varepsilon = \int_\Delta a(Q,s)\left\{ \int_{\partial D} \frac{\langle P-Q,\, N_P\rangle}{|P-Q|^n} \Big(\int_{|P-Q|^2/(T-s)}^{|P-Q|^2/\varepsilon} z^{n/2-1} e^{-4z}\, dz\Big) dP \right\} dQ\, ds$$

$$= \int_\Delta a(Q,s)\{ \int_{\partial D} \ldots \Big(\int_0^{|P-Q|^2/\varepsilon} \ldots dz\Big) dP\}\, dQ\, ds$$

$$- \int_\Delta a(Q,s)\{ \int_{\partial D} \ldots \Big(\int_0^{|P-Q|^2/(T-s)} \ldots dz\Big) dP\}\, dQ\, ds = I'_\varepsilon - I''$$

Using the results in [7], we have

$$\lim_{\varepsilon\to 0^+} |I'_\varepsilon| = (\omega_n/2)\Gamma(n/2) \Big| \int_\Delta a \Big| \leqslant c \frac{r}{(T-t_0+r^2)^{1/2}}.$$

Moreover,

$$I'' = \int_\Delta a(Q,s)\{ \int_{\partial D} \ldots \Big(\int_0^{|P-Q|^2/(T-s)} \ldots dz - \int_0^{|P-Q|^2/(T-t_0)} \ldots dz\Big) dP\}\, dQ\, ds$$

$$+ \int_\Delta a(Q,s)\{ \int_{\partial D} \ldots \Big(\int_0^{|P-Q|^2/(T-t_0)} \ldots dz\Big) dP\}\, dQ\, ds = I''_1 + I''_2.$$

If $s < t_0$, observing that $(T-s)(T-t_0)^{-1} < 2$, we have

$$|I''_1| \leqslant \int_\Delta |a(Q,s)| \left\{ \int_{\partial D} \frac{|P-Q|}{T-s} \frac{1}{(T-t_0)^{n/2-1}} \frac{r^2}{T-t_0} e^{-4(|P-Q|^2/(T-s))}\, dP \right\} dQ\, ds$$

$$\leqslant c \frac{r^2}{T-t_0} \int_\Delta |a(Q,s)| \left\{ \frac{1}{(T-s)^{n/2-1}} \int_{\partial D} \frac{|P-Q|}{(T-s)^{1/2}} e^{-4(|P-Q|^2/(T-s))}\, dP \right\} dQ\, ds.$$

Since $\dim(\partial D) = n-1$, the term $\{\quad\}$ is uniformly bounded in $T-s$ and P, so that

$$|I''_1| \leqslant c \frac{r^2}{T-t_0} \leqslant c \frac{r}{(T-t_0+r^2)^{1/2}}.$$

Reasoning in the same manner, for $s > t_0$ we obtain $|I''_1| \leqslant cr/(T-t_0+r^2)^{1/2}$.

Now we split I''_2 as follows:

$$I_2'' = \int_\Delta a(Q,s)\Big\{ \int_{|P-Q|\leqslant 5r} \ldots \Big(\int_0^{|P-Q|^2/(T-t_0)} \ldots dz\Big) dP\Big\}\, dQ\, ds$$
$$+\int_\Delta a(Q,s)\Big\{ \int_{|P-Q|> 5r} \ldots \Big(\int_0^{|P-Q|^2/(T-t_0)} \ldots dz\Big) dP\Big\}\, dQ\, ds = A+B.$$

We have

$$A \leqslant c\int_\Delta a(Q,s)\left\{ \int_{|P-Q|\leqslant 5r} \frac{|P-Q|}{(T-t_0)^{n/2}}\, dP\right\} dQ\, ds \leqslant c\frac{r^2}{(T-t_0)^{n/2}}\int_\Delta a$$
$$\leqslant c\, r/(T-t_0+r^2)^{1/2}.$$

For estimating B, we split it in two parts:

$$B = \int_\Delta a(Q,s)\Big\{ \int_{|P-Q|> 5r} \ldots \Big(\int_0^{|P-Q|^2/(T-t_0)} \ldots dz - \int_0^{|P-P_0|^2/(T-t_0)} \ldots dz\Big) dP\Big\}\, dQ\, ds$$
$$+\int_\Delta a(Q,s)\Big\{ \int_{|P-Q|> 5r} \ldots \Big(\int_0^{|P-P_0|^2/(T-t_0)} \ldots dz\Big) dP\Big\}\, dQ\, ds = B_1+B_2.$$

We have

$$|B_1| \leqslant c\int_\Delta |a(Q,s)|\left\{ \int_{|P-Q|> 5r} |P-Q|^{1-n}\frac{|P-P_0||Q-P_0|}{T-t_0}\left(\frac{|P-Q|^2}{T-t_0}\right)^{(n-2)/2}\right.$$
$$\left.\times \exp\{-|P-Q|^2/(T-t_0)\}\, dP\right\} dQ\, ds \leqslant c\, r/(T-t_0+r^2)^{1/2},$$

$$B_2 = \int_\Delta a(Q,s)\left\{ \int_{|P-Q|> 5r} \left(\frac{\langle P-Q, N_P\rangle}{|P-Q|^n} - \frac{\langle P-P_0, N_P\rangle}{|P-P_0|^n}\right)\Big(\int_0^{|P-P_0|^2/(T-t_0)} \ldots dz\Big) dP\right\}$$
$$\times dQ\, ds + \int_\Delta a(Q,s)\left\{ \int_{|P-Q|> 5r} \frac{\langle P-P_0, N_P\rangle}{|P-P_0|^n}\Big(\int_0^{|P-P_0|^2/(T-t_0)} \ldots dz\Big) dP\right\} dQ\, ds$$
$$= B_2' + B_2''.$$

We can estimate B_2' and B_2'' reasoning as before and we find that both are $\leqslant cr/(T-t_0+r^2)^{1/2}$.

If $r^2 \geqslant T-t_0$, then $2^{-1/2} \leqslant r/(T-t_0+r^2)^{1/2}$, so it is enough to show that $\left|\int_{\partial D_T} J'(a)\right| < c$, where c is a constant independent of a. This can be obtained by introducing a dyadic decomposition of ∂D_T as in the following Lemma 3.2, using the L^2-continuity of J' and estimates of $K'(P-Q, t-s)$ (see [7]), and reasoning as in the last part of case (a) of Lemma 3.3.

LEMMA 3.2. *If $a = a(P,t)$ is a $(1,2)$ c-atom such that $\int_{\partial D_T} a = 0$, then $J'(a) \in h_c^1(\partial D_T)$ and its norm depends only on the dimension of the space.*

Proof. We introduce some notations: if Δ is a surface disc of center (P_0, t_0) and radius r containing the support of a, we denote by Δ_i the surface disc concentric with Δ of radius $2^i r$, and we set

$$\chi_1 = \chi_{\Delta_1} \quad \text{and} \quad \chi_i = \chi_{\Delta_i - \Delta_{i-1}} \quad \text{for } i \geqslant 2.$$

We distinguish two cases: (a) $T - t_0 > r^2$, (b) $T - t_0 \leqslant r^2$.

For case (a), let p denote the integer such that

$$4^{-p-1}(T-t_0) \leqslant r^2 < 4^{-p}(T-t_0). \tag{3.4}$$

We split $J'(a)$ as follows:

$$J'(a) = \left(\chi_1 J'(a) + \frac{\chi_1}{|\Delta_1|} \int_{\partial D_T - \Delta_1} J'(a)\right) + \sum_{i=2}^{p} \left(\chi_i J'(a) + \frac{\chi_i}{|\Delta_i - \Delta_{i-1}|} \int_{\partial D_T - (\Delta_i - \Delta_{i-1})} J'(a)\right)$$

$$+ \sum_{i>p} \chi_i J'(a) - \frac{\chi_1}{|\Delta_1|} \int_{\partial D_T - \Delta_1} J'(a) - \sum_{i=2}^{p} \frac{\chi_i}{|\Delta_i - \Delta_{i-1}|} \int_{\partial D_T - (\Delta_i - \Delta_{i-1})} J'(a)$$

$$= M_1 + \sum_{i=2}^{p} M_i + \sum_{i>p} M_i - R_1 - \sum_{i=2}^{p} R_i.$$

We will show that, up to a multiplicative constant, $2^i M_i$ are $(1, 2)$ c-atoms. We start with M_1: by continuity of $J'(a)$ in $L^2(\partial D_T)$ we have

$$\|M_1\|_2 \leqslant \|J'(a)\|_2 + |\Delta_1|^{-1/2} \Big| \int_{\partial D_T - \Delta_1} J'(a)\Big| \leqslant c|\Delta|^{-1/2}.$$

Moreover, using Lemma 3.1,

$$\Big| \int_{\partial D_T} M_1 \Big| = \Big| \int_{\partial D_T} J'(a) \Big| \leqslant cr/(T - t_0 + r^2)^{1/2}.$$

So M_1, up to a multiplicative constant, is a $(1, 2)$ c-atom.

For $2 \leqslant i \leqslant p$ we have

$$\|M_i\|^2 \leqslant c\Big\{ \int_{\Delta_i - \Delta_{i-1}} J'(a)^2 + |\Delta_i|^{-1}\Big(\int_{\partial D_T - (\Delta_i - \Delta_{i-1})} J'(a)\Big)^2 \Big\} = c\{I_1 + I_2\}.$$

Recalling that $\int_\Delta a = 0$, if we use the mean value theorem and the estimates on K' we obtain, following the techniques in ([7], p. 303),

$$I_1 = \int_{\Delta_i - \Delta_{i-1}} \Big(\int_\Delta (K'(P-Q, t-s) - K'(P-P_0, t-t_0)) a(Q, s)\, dQ\, ds\Big)^2 dP\, dt$$

$$\leqslant c2^{-2i} |\Delta_i|^{-1},$$

while, from Lemma 3.1 and Hölder's inequality,

$$I_2 \leqslant c\Big\{ |\Delta_i|^{-1} \Big(\int_{\partial D_T} J'(a)\Big)^2 + |\Delta_i|^{-1} \Big(\int_{\Delta_i - \Delta_{i-1}} J'(a)\Big)^2 \Big\}$$

$$\leqslant c|\Delta_i|^{-1} \frac{r^2}{(T - t_0 + r^2)} + c\int_{\Delta_i} J'(a)^2 \leqslant c|\Delta_i|^{-1} \left(\frac{r^2}{(T - t_0 + r^2)} + 2^{-2i} \right)$$

$$\leqslant c2^{-2i} |\Delta_i|^{-1}$$

We also have

$$\Big|\int_{\partial D_T} M_i\Big| = \Big|\int_{\partial D_T} J'(a)\Big| \leqslant c\frac{r}{(T-t_0+r^2)^{1/2}} \leqslant c2^{-i}\frac{2^i r}{(T-t_0+r^2)^{1/2}}$$

$$\leqslant c2^{-i}\frac{2^i r}{(T-t_0+(2^i r)^2)^{1/2}}.$$

So $2^i M_i$, up to a multiplicative constant, is a (1, 2) c-atom.

For $i > p$, we proceed in the same manner in order to show that $2^i M_i$ satisfies property (ii). For (iii) we recall that

$$2^i r/(T-t_0+(2^i r)^2)^{1/2} > 2^{-1/2}.$$

So

$$\Big|\int_{\partial D_T} M_i\Big| = \Big|\int_{\Delta_i-\Delta_{i-1}} J'(a)\Big|$$

$$\leqslant \Big|\int_{\Delta_i-\Delta_{i-1}} \Big(\int_{\Delta}(K'(P-Q, t-s)-K'(P-P_0, t-t_0))a(Q, s)\,dQ\,ds\Big)dP\,dt\Big|$$

$$\leqslant c2^{-i}.$$

Next we can write

$$R_1+\sum_{i=2}^{p} R_i = \Big(\frac{\chi_1}{|\Delta_1|}\int_{\partial D_T-\Delta_1} J'(a)+\frac{\chi_2}{|\Delta_2-\Delta_1|}\int_{\Delta_1} J'(a)\Big)$$

$$+\Big(\frac{\chi_2}{|\Delta_2-\Delta_1|}\int_{\partial D_T-\Delta_2} J'(a)+\frac{\chi_3}{|\Delta_3-\Delta_2|}\int_{\Delta_2} J'(a)\Big)+\ldots+\frac{\chi_p}{|\Delta_p-\Delta_{p-1}|}\int_{\partial D_T-\Delta_p} J'(a)$$

$$= T_1+\ldots+T_p.$$

We show that $2^i T_i$, $1 \leqslant i \leqslant p$, up to a multiplicative constant, is a (1, 2) c-atom, with support in Δ_{i+1}, except for T_p whose support is in Δ_p. Clearly, $2^i T_i$ satisfies property (iii). For (ii) we write for $1 \leqslant i \leqslant p$,

$$T_i = \frac{\chi_i}{|\Delta_i-\Delta_{i-1}|}\int_{\partial D_T-\Delta_i} J'(a)+\frac{\chi_{i+1}}{|\Delta_{i+1}-\Delta_i|}\int_{\partial D_T} J'(a)$$

$$-\frac{\chi_{i+1}}{|\Delta_{i+1}-\Delta_i|}\int_{\partial D_T-\Delta_i} J'(a) = I_1+I_2-I_3.$$

Using Lemma 3.1 and (3.4) we have

$$\int_{\Delta_{i+1}} I_2^2 \leqslant c|\Delta_{i+1}|^{-1}\Big(\int_{\partial D_T} J'(a)\Big)^2 \leqslant c2^{-2(i+1)}|\Delta_{i+1}|^{-1},$$

$$\int_{\Delta_{i+1}} I_3^2 \leqslant c|\Delta_{i+1}|^{-1}\sum_{q>i+1}\Big(\int_{\Delta_q-\Delta_{q-1}} J'(a)\Big)^2 \leqslant c2^{-2i}|\Delta_{i+1}|^{-1}$$

The same estimate is valid for I_1.

For case (b): $T-t_0 \leqslant r^2$, since $r/(T-t_0+r^2)^{1/2} > 2^{-1/2}$ we write

$$J'(a) = \chi_1 J'(a) + \sum_{i \geqslant 2} \chi_i J'(a) = \sum_{i \geqslant 1} M_i$$

and reasoning as before it is not hard to show that $2^i M_i$ is, up to a multiplicative constant, a $(1, 2)$ c-atom.

LEMMA 3.3. *If $a = a(P, t)$ is a $(1, 2)$ c-atom with support in $\Delta = \Delta_r(P, t_0)$, then $J'(a) \in h_c^1(\partial D_T)$ and its norm depends only on n.*

Proof. Write

$$a = \left(a - \frac{\chi_\Delta}{|\Delta|}\int_\Delta a\right) + \frac{\chi_\Delta}{|\Delta|}\int_\Delta a.$$

Since the function in brackets is an atom with mean value zero, by the previous lemma, we are reduced to the case in which the atom has the form

$$a = \frac{\chi_\Delta}{|\Delta|}\frac{1}{(T-t_0+r^2)^{1/2}}.$$

As usual, we distinguish two cases: (a) $T-t_0 > r^2$, (b) $T-t_0 \leqslant r^2$.

Case (a). Using the notation of Lemma 3.2, we write

$$J'(a) = \chi_1 a + \sum_{i \geqslant 2} \chi_i a = \sum_{i \geqslant 1} M_i.$$

If $i \leqslant p$, p the integer of Lemma 3.2, we have

$$\int_{\Delta_{i+1}} M_i = \frac{1}{|\Delta|}\frac{r}{(T-t_0+r^2)^{1/2}} \int_{\Delta_{i+1}-\Delta_i} \Big(\int_\Delta K'(P-Q, t-s)\, dQ\, ds\Big)\, dP\, dt$$
$$\leqslant c2^{-i}\frac{2^i r}{(T-t_0+r^2)^{1/2}} \leqslant c2^{-i}\frac{2^i r}{(T-t_0+(2^i r)^2)^{1/2}},$$

while

$$\int_{\Delta_{i+1}} M_i^2 = \frac{1}{|\Delta|^2}\frac{r^2}{T-t_0+r^2} \int_{\Delta_{i+1}-\Delta_i} \Big(\int_\Delta K'(P-Q, t-s)\, dQ\, ds\Big)^2 dP\, dt$$
$$\leqslant c|\Delta_{i+1}|^{-1}\frac{r^2}{T-t_0+r^2} \leqslant c2^{-2i}|\Delta_{i+1}|^{-1}.$$

If $i > p$, since $2^{2i}r^2 + t_0 > T$ and $T > t > s$ we have

$$\int_{\Delta_{i+1}} M_i^2 = \frac{1}{|\Delta|^2} \int_{\Delta_{i+1}-\Delta_i} \Big(\int_\Delta K'(P-Q, t-s)\, dQ\, ds\Big)^2 dP\, dt \leqslant \frac{r^{n-1}2^{i(n-1)}}{2^{2i(n+1)}r^{2(n+1)}}|T-t_0|$$
$$\leqslant \frac{c}{2^{i(n+1)}r^{n-1}}\frac{|T-t_0|}{2^{2i}r^2} \leqslant c\frac{2^{2p-2i}}{2^{i(n-1)}r^{n-1}}\frac{1}{2^{2i}r^2}$$
$$\leqslant c\frac{2^{2p-2i}}{2^{i(n-1)}r^{n-1}}\frac{1}{|T-t_0|} \leqslant c2^{2(p-i)}|\Delta_{i+1}|^{-1}$$

Case (b) is shown by the same reasoning as in the last part of the previous case.

COROLLARY 3.1. *The operator J' is continuous from $h_c^1(\partial D_T)$ to $h_c^1(\partial D_T)$.*

Moreover, since $h_c^1(\partial D_T)$ is the predual of $\mathrm{B}_T\mathrm{MOC}$, the invertibility of $c_n I + J'$ follows from the invertibility of $c_n I + \Phi$ on $\mathrm{B}_T\mathrm{MOC}$ (see § 1).

§ 4. Boundary behavior of single layer potential. In the next theorem we study the behavior of the single layer potential near the boundary. Consider the parabolic cone

$$\Gamma(P, t) = \{(X, z)\colon |X-P|+|z-t|^{1/2} < (1+\beta)\operatorname{dist}(X, \partial D)\} \cap D_T$$

where $\beta > 0$ gives the opening of Γ For u caloric in D_T

$$N(u)(P, t) = \sup_{(X,s)\in\Gamma(P,t)} |u(X, s)|$$

is the tangential maximal function of u and for an atom $a(P, t)$, $u_a(X, t)$ is the single layer potential of a, i.e.

$$u_a(X, z) = -2\int_0^t \int_{\partial D} \frac{\exp\{-|X-Q|^2/4(z-s)\}}{(z-s)^{n/2}} a(Q, s)\, dQ\, ds.$$

Since, given an atom $a = a(P, t)$ with support in $\Delta = \Delta_r(P_0, t_0)$, we can write

$$a = \left(a - \frac{\chi_\Delta}{|\Delta|}\int_\Delta a\right) + \frac{\chi_\Delta}{|\Delta|}\int_\Delta a,$$

the study of u_a near the boundary reduces to the study of two cases:

(a) a is an atom with mean value zero,

(b) a is an atom of the form $a = c(\chi_\Delta/|\Delta|)r/(T-t_0+r^2)^{1/2}$

Then we have the two lemmas.

LEMMA 4.1. *If a is an atom of the form* (b), *then $N(\nabla_X u_a) \in L^1(\partial D_T)$ and $\|N(\nabla_X u_a)\|_{L^1} \leqslant c$, where $c = c_\beta$ is independent of a (β gives the opening of Γ).*

Proof. As shown in [5], $N(\nabla_X u_a)$ is in $L^2(\partial D_T)$ and

$$\|N(\nabla_X u_a)\|_{L^2(\partial D_T)} \leqslant c\|a\|_{L^2(\partial D_T)}.$$

If $\Delta_l = \Delta_l(P_0, t_0)$ has the same meaning as in Lemma 3.2 we write

$$\int_{\partial D_T} N(\nabla_X u_a) = \int_{\Delta_{3r}} N(\ldots) + \int_{\partial D_T - \Delta_{3r}} N(\ldots) = I_1 + I_2.$$

We have, from the previous observation,

$$|I_1| \leqslant c r^{(n+1)/2}\|a\|_{L^2} \leqslant c.$$

In order to estimate I_2 we distinguish two cases:

(ω) $\quad T-t_0 > r^2$, $\quad$ (ψ) $\quad T-t_0 \leqslant r^2$

Case (ω). If p is the integer introduced in Lemma 3.2, we split I_2 as follows:

$$I_2 = \sum_{i>3}^{p} \int_{\Delta_i - \Delta_{i-1}} N(\ldots) + \sum_{i>p} \int_{\Delta_i - \Delta_{i-1}} N(\ldots) = \sum_{i>3}^{p} L_i + \sum_{i>p} L_i.$$

For $(P, t) \in \Delta_i - \Delta_{i-1}$, consider the cone $\Gamma(P, t)$. If $(X, z) \in \Gamma(P, t)$ and (X, z) is such that $|X-P| + |z-t|^{1/2} \leqslant 2^{i-2}r$, we have:

$$\begin{aligned} |P-Q| \geqslant 2^{i-1}r \quad &\text{implies} \quad |X-Q| = |X-P+P-Q| \\ &\geqslant \big||P-Q| - |X-P|\big| \geqslant c2^{i-1}r, \\ |z-s| \geqslant 2^{2i-2}r^2 \quad &\text{implies} \quad |z-s| = |z-t+t-s| \\ &\geqslant \big||t-s| - |z-t|\big| \geqslant c2^{2i-2}r^2. \end{aligned}$$

From this, it follows easily, using estimates in ([7], p. 303), that

$$|\nabla_X u_a| \leqslant c_\beta \frac{1}{2^{(i-1)(n+1)}} \frac{1}{r^{n+1}} \frac{r}{(T-t_0+r^2)^{1/2}}.$$

On the other hand, for $(X, z) \in \Gamma(P, t)$ with $|X-P| + |z-t|^{1/2} \geqslant 2^{i-2}r$ we have $|X-Q| \geqslant \operatorname{dist}(X, \partial D) \geqslant c_\beta(|X-P| + |z-t|^{1/2}) \geqslant c_\beta 2^{i-2}r$ and then

$$|\nabla_X u_a| \leqslant c_\beta \frac{1}{2^{(i-1)(n+1)}} \frac{1}{r^{n+1}} \frac{r}{(T-t_0+r^2)^{1/2}}.$$

So for $i \leqslant p$, recalling the observations of Lemma 3.2 on p,

$$|L_i| \leqslant c_\beta \frac{|\Delta_i - \Delta_{i-1}|}{r^{n+1} 2^{(i-1)(n+1)}} \frac{r}{(T-t_0+r^2)^{1/2}} \leqslant c_\beta 2^{-i}$$

and for $i > p$ (see once again Lemma 3.2)

$$|L_i| \leqslant c_\beta \frac{2^{(i-1)(n-1)} r^{n-1} (T-t_0)}{r^{n+1} 2^{(i-1)(n+1)}} \leqslant c_\beta 2^{2(i-p)}.$$

Collecting these results we have $I_2 \leqslant C_\beta$.

LEMMA 4.2. *If a is an atom of the form (ψ) then $N(\nabla_X u_a) \in L^1(\partial D_T)$ and $\|N(\ldots)\|_{L^1} \leqslant c$ where $c = c_\beta$ is independent of a (β gives the opening of Γ).*

Proof. Splitting as in the previous lemma we have

$$\int_{\partial D_T} N(\nabla_X u_a) = \int_{\Delta_{3r}} N(\ldots) + \int_{\partial D_T - \Delta_{3r}} N(\ldots) = I_1 + I_2.$$

For the same reason as in Lemma 4.1, $|I_1| \leqslant c$, while to estimate I_2, we use the fact that a has mean value zero and we write

$$\nabla_X u_a(X, z) = \int_0^t \int_{\partial D} (\bar{K}(X-Q, z-s) - \bar{K}(X-P_0, z-t_0)) a(Q, s)\, dQ\, ds$$

where

$$\bar{K}(X-Q,\, z-s) = \frac{X-Q}{(z-s)^{(n+2)/2}} \exp(-|X-Q|^2/4(z-s)).$$

Recalling the estimates of $\bar{K}$ (see [7]), introducing once again the decomposition of Lemma 3.2, and reasoning as in the previous lemma we have $|I_2| \leqslant c$.

THEOREM 4.1. *There exists a constant $c > 0$, independent of f, such that for any $f \in h_c^1(\partial D_T)$,*

$$\|N(\nabla_X u_f)\|_{L^1(\partial D_T)} \leqslant c\|f\|_{h_c^1(\partial D_T)}.$$

Moreover, writing $u = u_f$,

$$\langle \nabla_X u(X, t), N_P \rangle \to (c_n I + J') f(P, z)$$

pointwise for almost every $(P, z) \in \partial D_T$ as $(X, t) \to (P, z)$, $(X, t) \in \Gamma(P, z)$, where $c_n = \omega_n H(0)/2$ and J' is the operator defined by (1.11).

Proof. Since $c_n I + J'$ is invertible on $h_c^1(\partial D_T)$, we only have to show the non-tangential convergence of $\partial_{N_P} u_f$ to $(c_n I + J')f$, where $f \in h_c^1$.

We know ([5]) that this convergence holds when $f \in L^2(\partial D_T)$ and then also when f is an atom. Using very simple arguments (see [4], p. 7) one can easily show that this convergence holds for any $f \in h_c^1(\partial D_T)$.

In order to solve the Neumann problem (3.1) it remains to show the uniqueness of the solution. To this end, we show

LEMMA 4.3. *Suppose $Lu = 0$, $N(\nabla_X u) \in L^1(\partial D_T)$ and $u(X, t) \to 0$ as $t \to 0^+$ uniformly on compact subsets of D. If $\partial_{N_P} u = 0$ on ∂D_T, then $u = 0$ on D_T.*

Proof. Using techniques similar to those of Lemma 1.6 of [3], one can show that there exists an integer $q > 1$ such that $N(\nabla_X u) \in L^q$ and moreover $u = 0$ on ∂D_T. Then the fact that $u = 0$ in D_T is a consequence of Theorem 2.2 and Theorem 2.3 of [5].

References

[1] R. Coifman and G. Weiss, *Extensions of Hardy spaces and their use in analysis*, Bull. Amer. Math. Soc. 83 (1977), 569–645.

[2] E. Fabes, M. Jodeit and N. Rivière, *Potential techniques for boundary value problems on C^1-domain*, Acta Math. 141 (1978), 165–186.

[3] E. Fabes and C. Kenig, *On the space H^1 of a C^1-domain*, Ark. Mat. 19 (1981), 1–22.

[4] E. Fabes, C. Kenig and U. Neri, *Carleson measures, H^1 duality and weighted BMO in non-smooth domains*, Indiana Univ. Math. J. 30 (1981), 574–581.

[5] E. Fabes and N. Rivière, *Dirichlet and Neumann problems for the heat equation on C^1-cylinder*, in: Proc. Sympos. Pure Math. 35, Part 2, Amer. Math. Soc., 1979, 179–196.

[6] C. Fefferman and E. Stein, *H^p spaces of several variables*, Acta Math. 129 (1972), 137–193.

[7] A. Grimaldi, U. Neri and F. Ragnedda, *BMO continuity for some heat potentials*, Rend. Sem. Mat. Univ. Padova 72 (1984), 289–305.

[8] —, —, —, *Invertibility of some heat potentials in BMO-norms*, ibid. 75 (1986), 77–90.

[9] A. Grimaldi and F. Ragnedda, *Properties and geometrical structure of the boundary of a C^1-cylindrical domain*, Rend. Sem. Mat. Univ. Cagliari, to appear.

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Reçu par la Rédaction le 02.12.1989