ANNALES
POLONICI MATHEMATICI
LIII.1 (1991)

On isomorphisms between fibrations over T^k associated with a T^k action

by MICHAŁ SADOWSKI (Gdańsk)

Abstract. Let M be a closed manifold with a given action of a k-dimensional torus T^k on it. Let ev_*: $\pi_1(T^k, 1) \to \pi_1(M, x)$ be the homomorphism induced by ev: $T^k \ni t \to tx \in M$. For a fixed monomorphism λ: $\pi_1(T^k) \to \pi_1(T^k)$ let $F(\lambda)$ denote the set of all smooth T^k-equivariant fibrations p: $M \to T^k$ such that $p_* \circ \mathrm{ev}_* = \lambda$. We show that two fibrations $p, q \in F(\lambda)$ such that $\mathrm{im}[p_* - q_*] \subseteq \mathrm{im}\,\lambda$ are equivariantly isomorphic. This implies that there are no more than $(\mathrm{card}(\pi_1(T^k)/\mathrm{im}\,\lambda))^{b_1(M)-k}$ isomorphism classes of fibrations belonging to $F(\lambda)$, where $b_1(M)$ is the first Betti number of M. We also prove the affine version and the holomorphic version of this result.

Introduction. Let M be a closed T^k manifold. In this paper we investigate when two T^k-equivariant (see Definition 1.1) fibrations p, q: $M \to T^k$ are equivariantly isomorphic.

T^k-equivariant fibrations over T^k arise naturally in the following situations. First, a fibration over T^k having finite structure group can be treated as a T^k-equivariant fibration with respect to some T^k action on T^k (see Remark 1.1). Next, any homomorphism λ: $\pi_1(M) \to \pi_1(T^k)$ such that $\lambda \circ \mathrm{ev}_*$: $\pi_1(T^k) \to \pi_1(T^k)$ is a monomorphism (where ev_*: $\pi_1(T^k, 1) \to \pi_1(M, x)$ is induced by ev: $T^k \ni t \to tx \in M$) can be lifted to a T^k-equivariant fibration p: $M \to T^k$ (see e.g. [12, Theorem 1], see also [3, Theorem 4.2], [13, §2]).

We start with the discussion of one of the main results of the paper for $k = 1$. Let p, q: $M \to S^1$ be two equivariantly isomorphic S^1-equivariant fibrations. It is clear that $|p_*(\sigma)| = |q_*(\sigma)|$, where $\sigma \in \pi_1(M)$ is the homotopy class of the orbit of the base point of M and p_*, q_*: $\pi_1(M) \to \pi_1(S^1) \cong \mathbf{Z}$ are the induced homomorphisms. We say that the number $|p_*(\sigma)|$ is the *order* of p. It is an important invariant of p (see Remark 1.3). As the order of an S^1-equivariant fibration can be arbitrarily large it is only reasonable to ask how many nonisomorphic fibrations of a given order m there can be. We shall prove that two S^1-equivariant fibrations p, q: $M \to S^1$ such that $|p_*(\sigma)| = |q_*(\sigma)| = m$ and $\mathrm{im}[p_* - q_*$: $\pi_1(M) \to \pi_1(S^1) \approx \mathbf{Z}] \subseteq m\mathbf{Z}$ are equivariantly isomorphic. In particular, there are no more than $|m|^{b_1(M)-1}$ nonisomorphic fibrations of a given order m (see Corollary 1.1). A more general result can be stated as follows.

THEOREM 1.1. *Let p, q: $M \to T^k$ be two fibrations associated with a smooth T^k action on a closed manifold M. If $p_* \circ \mathrm{ev}_* = q_* \circ \mathrm{ev}_*$ and $\mathrm{im}[p_* - q_*] \subseteq \mathrm{im}\, p_* \circ \mathrm{ev}_*$, then there is a T^k-equivariant diffeomorphism f: $M \to M$ such that $p \circ f = q$.*

The holomorphic and affine variants of Theorem 1.1 are discussed in Section 2. Proposition 2.2 allows one to estimate the number of inequivalent Calabi's reductions of the same order of a given flat manifold M (cf. Remark 2.2).

If M is a closed homologically injective S^1 manifold then Theorem 1.1 implies that appropriate automorphisms of $\pi_1(M)$ can be lifted to diffeomorphisms so that Diff(M) has infinitely many connected components (see Corollary 1.2). Note that the problem when a given automorphism of the fundamental group can be lifted to a diffeomorphism was solved for some special (mainly aspherical) manifolds only (as hyperbolic manifolds [10], [11], infranilmanifolds [8], some 3-manifolds [5], [15]). The holomorphic variant of Corollary 1.2 can be derived from Theorem 2.1.

A concrete example is studied in Section 5. We show that there are two nonisomorphic fibrations p, q: $K \times S^1 \to S^1$ of order 2 on the product of the Klein bottle K by S^1. This example is simpler and more natural than the earlier known ones (cf. [14]).

The following notation will be used. The letter I will denote the canonical identification of the fundamental group with the corresponding deck group. If M is a manifold, then $\tilde{M}$ will denote the universal covering space of M, and Γ the deck group of M. If G is a group, $g_1, \ldots, g_k \in G$, then $\langle g_1, \ldots, g_k \rangle$ will denote the subgroup of G generated by $g_1, \ldots, g_k$, and $Z(G)$ the center of G. By $H_1(\Gamma)$ we will denote $\Gamma/[\Gamma, \Gamma] \approx H_1(M)$. The symbol $FH_1(M)$ (or $FH_1(\Gamma)$) stands for $H_1(M)$ divided by its torsion subgroup and π: $H_1(M) \to FH_1(M)$ is the canonical projection. Note that $FH_1(M)$ is isomorphic to $Z^{b_1(M)}$, where $b_1(M)$ is the first Betti number of M.

1. On isomorphisms between fibrations associated with a smooth torus action. The aim of this section is to prove Theorem 1.1.

DEFINITION 1.1. Let p: $M \to T^k$ be a fibration on a smooth T^k manifold M. The fibration p is *T^k-equivariant* if $p(tx)p(x)^{-1}$ depends on $t \in T^k$ only. A T^k-equivariant fibration p: $M \to T^k$ is *associated with the T^k action* if $p_* \circ \mathrm{ev}_*$ is a monomorphism.

Remark 1.1. (a) The fibration p will be T^k-equivariant in the usual meaning if we take the T^k action $T^k \times T^k \ni (t, u) \to p(tx_0)u \in T^k$ (where $x_0 \in p^{-1}(1)$) on T^k.

(b) Let q: $M \to T^k$ be a smooth fibration. Then *there is a T^k action on M such that q is associated with this action iff the structure group of q can be reduced to a finite group* (see e.g. [12, § 2]; see also the proof of Theorem 1B in [16]).

Under the canonical identification I of $\pi_1(M, x)$ with the deck group Γ the homotopy class of the orbit $\sigma \in \pi_1(M, x)$ of an S^1 action φ_t: $M \to M$, $t \in [0, 1]$, corresponds to the deck transformation $\tilde{\varphi}_1 \in \Gamma$. Recall that an element $\gamma \in \Gamma$ is identified with $\gamma' \in \pi_1(M, x)$, where γ' is represented by the projection of any curve d: $[0, 1] \to \tilde{M}$ such that $d(0) = \tilde{x}$, $d(1) = \gamma(\tilde{x})$. Here $\tilde{x}$ is a point above x. For every $t \in \boldsymbol{R}$ *the group* Γ *commutes with* $\tilde{\varphi}_t$, because $\tilde{\varphi}_0 = \mathrm{id}$ commutes with the discrete group Γ. In particular, $\tilde{\varphi}_1 \in Z(\Gamma)$ and $\sigma \in Z(\pi_1(M))$.

Let f: $X \to Y$ be a continuous map, let $\tilde{X}$, $\tilde{Y}$ denote the universal covering spaces of X and Y, let Γ_1 denote the deck group of X, and let $\tilde{f}$: $\tilde{X} \to \tilde{Y}$ be a map covering f. For any $x \in \tilde{X}$, $\gamma \in \Gamma_1$ we have

$$\tilde{f}(\gamma x) = If_*(I^{-1}(\gamma))(\tilde{f}(x)). \tag{1}$$

Proof of Theorem 1.1. Let φ_t: $M \to M$, $t \in T^k$, be the T^k action on M and let $\tilde{\varphi}_t$: $\tilde{M} \to \tilde{M}$, $t \in \boldsymbol{R}^k$, be the $\boldsymbol{R}^k$ action covering the T^k action. Identify T^k with $\boldsymbol{R}^k/B$, where B is a lattice in $\boldsymbol{R}^k$, and denote $I(\mathrm{im}\, p_* \circ \mathrm{ev}_*) \subseteq B$ by A. Let $\tilde{p}$, $\tilde{q}$: $\tilde{M} \to \boldsymbol{R}^k$ be the fibrations covering p and q respectively. We have $p_* \circ \mathrm{ev}_* = q_* \circ \mathrm{ev}_*$. Using this and the T^k-equivariance of p and q it is easy to verify that

$$\tilde{p}(\tilde{\varphi}_t(x)) - \tilde{p}(x) = h(t) = \tilde{q}(\tilde{\varphi}_t(x)) - \tilde{q}(x) \tag{2}$$

for $t \in \boldsymbol{R}^k$, $x \in \tilde{M}$, and for some linear map h: $\boldsymbol{R}^k \to \boldsymbol{R}^k$. It is also easy to see that $h(B) = A$. Since $p_* \circ \mathrm{ev}_*$ is a monomorphism, h is an isomorphism and $h^{-1}(A) = B$. Consider now $\tilde{\beta}$: $\tilde{M} \ni x \to h^{-1}(\tilde{q}(x) - \tilde{p}(x)) \in \boldsymbol{R}^k$. Applying (1) and the assumption that $\mathrm{im}[p_* - q_*] \subseteq \mathrm{im}\, p_* \circ \mathrm{ev}_*$ we have

$$\tilde{\beta}(\gamma x) - \tilde{\beta}(x) = h^{-1}\big(Iq_*(I^{-1}(\gamma)) - Ip_*(I^{-1}(\gamma))\big) \in h^{-1}(A) \subseteq B$$

for every $\gamma \in \Gamma$. It follows that there is a map β: $M \to T^k$ covered by $\tilde{\beta}$. By (2), $\tilde{\beta} \circ \tilde{\varphi}_t = \tilde{\beta}$ for $t \in \boldsymbol{R}^k$ so that

$$\beta \circ \varphi_t = \beta \quad \text{for } t \in T^k. \tag{3}$$

Consider f, g: $M \to M$ given by

$$f(x) = \varphi_{\beta(x)}(x), \quad g(x) = \varphi_{-\beta(x)}(x). \tag{4}$$

Then $f \circ g = g \circ f = \mathrm{id}_M$ so that f is a T^k-equivariant diffeomorphism. By (2), $\tilde{p}(\tilde{\varphi}_{\tilde{\beta}(x)}(x)) = \tilde{p}(x) + h(\tilde{\beta}(x)) = \tilde{q}(x)$ and accordingly $p \circ f = q$. This finishes the proof of Theorem 1.1.

COROLLARY 1.1. *Let* ψ: $\boldsymbol{Z}^k \to \boldsymbol{Z}^k$ *be a given monomorphism. There are no more than* $(\mathrm{card}(\boldsymbol{Z}^k/\mathrm{im}\,\psi))^{b_1(M)-k}$ *isomorphism classes of fibrations* p: $M \to T^k$ *satisfying* $p_* \circ \mathrm{ev}_* = \psi$ *that are associated with a given* T^k *action on* M.

Proof. We give the proof for $k = 1$ only leaving the verification of the general case to the reader. Every homomorphism α: $\pi_1(M) \to \pi_1(S^1)$ factors through $FH_1(M) \approx \boldsymbol{Z}^{b_1(M)}$ so that there is a canonical isomorphism $\mathrm{HOM}(\pi_1(M), \boldsymbol{Z}) \approx \mathrm{HOM}(\boldsymbol{Z}^{b_1(M)}, \boldsymbol{Z})$. Let $\psi(\sigma) = m$ and let $E(m)$ denote the set of all epimorphisms λ: $FH_1(M) \to \boldsymbol{Z}$ such that $\lambda(\sigma) = m$. Let $s = b_1(M) - 1$ and let $FH_1(M) = \boldsymbol{Z} \times \boldsymbol{Z}^s$ be a fixed direct sum decomposition such that the image of σ in $FH_1(M)$ is contained in $\mathbf{Z}$. Consider the equivalence relation $\sim$ on $E(m)$ defined by the requirement that $\lambda_1 \sim \lambda_2$ if and only if $\operatorname{im}[\lambda_1 - \lambda_2] \subseteq m\mathbf{Z}$. By Theorem 1.1 we have no more isomorphism classes of our fibrations than the elements of $E(m)/\sim \subseteq \mathbf{Z}^s/m\mathbf{Z}^s$. This completes the proof of Corollary 1.1.

Remark 1.2. Let $p, q\colon M \to S^1$ be two S^1-equivariant fibrations and let $\sigma \in \pi_1(M)$ be the homotopy class of the orbit. Assume that $p_*(\sigma) = q_*(\sigma) = m$, $\operatorname{im}[p_* - q_*] \subseteq m\mathbf{Z}$ and $p(x_0) = q(x_0)$. Let β and f be as above. Then

$$f_*(\gamma) = \sigma^{\beta_*(\gamma)}\gamma \quad \text{for every } \gamma \in \pi_1(M). \tag{5}$$

This equality can be verified as follows. Consider the diffeomorphism $\tilde{f}\colon \tilde{M} \to \tilde{M}$ given by $\tilde{f}(z) = \tilde{\varphi}_{\tilde{\beta}(z)}(z)$. For the notational convenience the same letter will denote an element of the fundamental group and the corresponding deck transformation. Let $\gamma \in \pi_1(M)$. By (1), $\tilde{\beta}(\gamma(z)) = \tilde{\beta}(z) + \beta_*(\gamma)$. As $\tilde{\varphi}_t$ commutes with Γ it follows that

$$\tilde{f}(\gamma z) = (\tilde{\varphi}_{\beta_*(\gamma)}\tilde{\varphi}_{\tilde{\beta}(z)}\gamma)(z) = (\tilde{\varphi}_1^{\beta_*(\gamma)}\gamma\,\tilde{\varphi}_{\tilde{\beta}(z)})(z) = (\tilde{\varphi}_1^{\beta_*(\gamma)}\gamma)(\tilde{f}(z)).$$

But $\tilde{\varphi}_1 \in \Gamma$ is canonically identified with σ (see above). By (1) again, $\tilde{f}(\gamma z) = f_*(\gamma)(\tilde{f}(z))$. Hence $f_*(\gamma) = \sigma^{\beta_*(\gamma)}\gamma$, which is our claim.

Remark 1.3. Let $p\colon M \to S^1$ be an S^1-equivariant fibration of order r. One can check (for details we refer to [12]) that *an r-fold covering space of M is equivariantly diffeomorphic to $F \times S^1$, where F is a fiber of p.*

Let M be a homologically injective S^1 manifold. This means that $\mathrm{ev}_*\colon H_1(T) \to H_1(M)$ is a monomorphism. If $b_1(M) > 1$, then the argument given in the proof of Corollary 1.1 shows that we have two different homomorphisms $\lambda, \mu\colon \pi_1(M, x_0) \to \mathbf{Z}$ such that $m = \lambda(\sigma) = \mu(\sigma) \neq 0$ and $\operatorname{im}[\lambda - \mu] \subseteq m\mathbf{Z}$, where σ is as above. By [12, Theorem 1.1], there are S^1-equivariant fibrations $p, q\colon M \to S^1$ such that $p_* = \lambda$, $q_* = \mu$ and $p(x_0) = q(x_0)$. According to Theorem 1.1 there is a diffeomorphism $f\colon M \to M$ such that $p \circ f = q$. We have $f_*(\gamma) = \sigma^{\beta_*(\gamma)}\gamma$ for $\gamma \in \pi_1(M)$ (see Remark 1.2) so that from the nontriviality of β_* it follows that the homomorphism $k \to f_*^k \in \operatorname{Aut}(\pi_1(M, x_0))$ is a monomorphism. This shows the following.

COROLLARY 1.2. *Let M be a closed manifold that admits a homologically injective S^1 action and let $b_1(M) > 1$. Then the group of all diffeomorphisms of M has infinitely many connected components.*

2. The affine case and the holomorphic case. In this section the same notation as in Section 1 will be used. The first result we want to prove is the holomorphic variant of Theorem 1.1.

THEOREM 2.1. *Let $p, q\colon M \to T^{2n}$ be two T^{2n}-equivariant holomorphic fibrations associated with a homologically injective holomorphic torus action on* *a complex manifold M. Assume that $p_* \circ \mathrm{ev}_* = q_* \circ \mathrm{ev}_*$ and $\mathrm{im}[p_* - q_*] \subseteq \mathrm{im}\, p_* \circ \mathrm{ev}_*$. Then there is a biholomorphic and T^{2n}-equivariant map $f\colon M \to M$ such that $p \circ f = q$.*

Proof. Let h, φ_t, β, $\tilde{\beta}$, h, f be as in the proof of Theorem 1.1. We have $f = \mu \circ \beta'$, where $\beta'\colon M \ni x \to (\beta(x), x) \in T^{2n} \times M$, $\mu\colon T^{2n} \times M \ni (t, x) \to \varphi_t(x) \in M$. It suffices to check that the map $\tilde{\beta} = h^{-1}(\tilde{q} - \tilde{p})$ is holomorphic. This follows from the assumption that p and q are holomorphic and from the equality $h(t) = \tilde{p}(\tilde{\varphi}_t(x_0)) - \tilde{p}(x_0)$, where x_0 is a point of M.

Remark 2.1. The question of the existence of a holomorphic fibration that is associated with a holomorphic torus action was considered in [1], [2], [9].

Before we state the next result we need some definitions. By a *parallel flow* on a manifold M we mean a flow generated by a parallel vector field. A T^k action on M is *parallel* if each flow induced by the action of a one-parameter subgroup of T^k is parallel. Every isometric T^k action on a nonpositive curvature manifold is parallel (see e.g. [6, Corollary 4.2]). An argument similar to that given in the proof of Theorem 2.1 shows the following.

THEOREM 2.2. *Let M be a closed manifold, let ∇ be a connection on M, and let ∇_0 be the standard flat connection on T^k. Let $p, q\colon M, \nabla \to T^k, \nabla_0$ be two T^k-equivariant affine fibrations associated with a parallel T^k action on M. If $p_* \circ \mathrm{ev}_* = q_* \circ \mathrm{ev}_*$ and $\mathrm{im}[p_* - q_*] \subseteq \mathrm{im}\, p_* \circ \mathrm{ev}_*$, then there is an affine and T^k-equivariant diffeomorphism f such that $p \circ f = q$.*

Remark 2.2. If M is a closed flat manifold such that $b_1(M) > 0$, then there is an isometric S^1 action on M (see e.g. [4, 7]). Every epimorphism $\lambda\colon \pi_1(M) \to \mathbf{Z}$ can be lifted to an affine S^1-equivariant fibration $p\colon M \to S^1$. The resulting splitting of M is said to be *Calabi's reduction* of M (cf. [17, Theorem 3.6.3], [13]). There are infinitely many Calabi's reductions of M. However, if we fix the order r, then we have no more than $r^{b_1(M)-1}$ of them.

3. An example. In this section we show how the results of the paper can help to classify associated fibrations of a fixed order m on a given S^1 manifold M. We consider the simplest nontrivial example when M is the product of the Klein bottle K by S^1 and $m = 2$.

First we define an appropriate affine S^1 action on M. In order to do that recall that K can be represented as $\mathbf{R}^2/\Gamma_1$, where Γ_1 is the subgroup of the isometry group of $\mathbf{R}^2$ generated by $a(x, y) = (x+1, y)$, $b(x, y) = (-x, y+\frac{1}{2})$. Then the one-parameter group of translations $\tilde{\psi}_t\colon \mathbf{R}^2 \to \mathbf{R}^2$, $\tilde{\psi}_t(x, y) = (x, t+y)$, commutes with Γ_1 and consequently determines an S^1 action $\psi_t\colon K \to K$, $t \in [0, 1]$. We have $\tilde{\psi}_1 = b^2$ and $[a, b] = a^2$.

Consider $M = K \times S^1$ with the S^1 action $\varphi_t = \psi_t \times \mathrm{id}_{S^1}$. It is obvious that $\tilde{\varphi}_1 = (b \times \mathrm{id}_{S^1})^2$. The group $\pi_1(M)$ is generated by $a, b \in \pi_1(K)$ corresponding to

a and b and by the generator u of $\pi_1(S^1)$. If u_0, b_0 are the images of u and b in $FH_1(M)$, then $FH_1(M) = \langle b_0, u_0\rangle$. By Corollary 1.1 we have no more than two nonisomorphic fibrations of order 2 that are associated with the action φ_t.

Take the projections λ_1, λ_2: $FH_1(M) \to \langle b_0\rangle$ induced by the direct sum decompositions $FH_1(M) = \langle b_0\rangle \oplus \langle u_0\rangle$ and $FH_1(M) = \langle b_0\rangle \oplus \langle b_0 + u_0\rangle$. The homomorphisms μ_1, μ_2: $\pi_1(M) \to \langle b_0\rangle \approx \boldsymbol{Z}$ induced by λ_1, λ_2 can be lifted to S^1-equivariant fibrations p, q: $M \to S^1$. Let π: $\pi_1(M) \to FH_1(M)$ be the canonical projection. Then $\pi^{-1}(\langle u_0\rangle)$ is the fundamental group of a fiber of p and $\pi^{-1}(\langle u_0 + b_0\rangle)$ is the fundamental group of a fiber of q.

As u and a commute it follows that $\pi^{-1}(\langle u_0\rangle) = \langle u, a\rangle \approx \boldsymbol{Z} \oplus \boldsymbol{Z}$ and accordingly the fibers of p are diffeomorphic to T^2. The group $\pi^{-1}(\langle b_0 + u_0\rangle) = \langle bu, a\rangle$ is not abelian. Hence the fibers of p are not even homotopy equivalent to the fibers of q. Note that $(bu)^2 = \tilde{\varphi}_1 u$ commutes with a and consequently the fibers of q are diffeomorphic to the Klein bottle.

Acknowledgements. The author is grateful to Zbyszek Marciniak for his careful reading of the first version of the paper and for useful suggestions which led better and more complete formulation of the stated material. He would also like to express his thanks to Andrzej Szczepański for his interest.

References

[1] E. Calabi, *On Kähler manifolds with vanishing canonical class*, in: Algebraic Geometry and Topology, Princeton Math. Series, Princeton Univ. Press, 1957, 78–89.

[2] J. B. Carrell, *Holomorphically injective complex toral actions*, in: Proc. Second Conference on Compact Transformation Groups, Part 2, Lecture Notes in Math. 299, Springer, 1972, 205–234.

[3] P. E. Conner and F. Raymond, *Injective operations of the toral groups*, Topology 10 (1971), 283–296.

[4] D. Gromoll and J. Wolf, *Some relations between the metric structure and the algebraic structure of the fundamental group in the manifolds of nonpositive curvature*, Bull. Amer. Math. Soc. 77 (1971), 545–552.

[5] K. Johanson, *Homotopy Equivalences of 3-Manifolds with Boundaries*, Lecture Notes in Math. 761, Springer, 1979.

[6] S. Kobayashi, *Transformation Groups in Differential Geometry*, Springer, Berlin 1972.

[7] H. B. Lawson and S. T. Yau, *Compact manifolds of nonpositive curvature*, J. Differential Geometry 7 (1972), 211–228.

[8] K. B. Lee and F. Raymond, *The role of Seifert fiber spaces in transformation groups*, in: Contemp. Math. 36, A.M.S., 1975, 367–426.

[9] Y. Matsushima, *Holomorphic vector fields and the first Chern class of a Hodge manifold*, J. Differential Geometry 3 (1969), 477–480.

[10] G. D. Mostow, *Strong Rigidity of Locally Symmetric Spaces*, Ann. of Math. Stud. 78, Princeton Univ. Press, Princeton, N.J., 1973.

[11] H. Munkholm, *Simplices of maximal volume in hyperbolic space, Gromov's norm and Gromov's proof of Mostow's rigidity theorem (following Thurston)*, in: Proc. Symp. in Algebraic Topology Siegen 1979, Lecture Notes in Math. 788, Springer, 1980, 109–125.

[12] M. Sadowski, *Equivariant splittings associated with smooth toral actions*, in: Proc. Conf. on Algebraic Topology, Poznań 1989, Lecture Notes in Math., to appear.

[13] —, *Closed parallel 1-forms on infranilmanifolds and Calabi reductions*, Colloq. Math. 59 (1990), 269–278.

[14] J. L. Tollefson, *3-Manifolds fibering over S^1 with non-unique connected fiber*, Proc. Amer. Math. Soc. 21 (1969), 79–80.

[15] F. Waldhausen, *On irreducible 3-manifolds which are sufficiently large*, Ann. of Math. 87 (1968), 56–88.

[16] J. Welsh Jr., *Manifolds that admit parallel vector fields*, Illinois J. Math. 30 (1986), 9–18.

[17] J. Wolf, *Spaces of Constant Curvature*, McGraw-Hill, 1967.

INSTYTUT MATEMATYKI, UNIWERSYTET GDAŃSKI
INSTITUTE OF MATHEMATICS, UNIVERSITY OF GDAŃSK
Wita Stwosza 57, 80-952 Gdańsk, Poland

Reçu par la Rédaction le 15.05.1989
Révisé le 8.05.1990