COLLOQUIUM MATHEMATICUM

VOL. XXVIII 1973 FASC. 2

A NOTE ON OPEN MAPPINGS OF IRREDUCIBLE CONTINUA

BY

J. B. FUGATE (LEXINGTON) AND L. MOHLER (BUFFALO)

In [2] it has been shown that local homeomorphisms preserve irreducibility of continua. This result cannot be extended to open mappings since, as has been pointed out by Charatonik [1], the circle can be written as the open (continuous) image of an irreducible continuum of type λ. The object of this note is to show that indeed *any* continuum can be realized as the open, continuous image of an irreducible continuum of type λ.

Definition. A continuum I, irreducible between points a and b, is said to be *of type* λ if there is a monotone, continuous function m from I onto the closed unit interval $[0, 1]$ such that $m(a) = 0$, $m(b) = 1$ and $m^{-1}(t)$ has void interior in I for every $t \in I$.

THEOREM. *Let M be a (metric) continuum. Then there exists an irreducible continuum $I(M)$ of type λ which admits an open, continuous mapping onto M.*

Proof. Let C denote the Cantor set. Let a_1^1 and b_1^1 denote the endpoints of the interval of length $\frac{1}{3}$ whose deletion from $[0, 1]$ begins the construction of C. Let a_2^1, b_2^1 and a_2^2, b_2^2 be the endpoints of the two deleted intervals of length $\frac{1}{9}$. In general, let $a_n^1, b_n^1; a_n^2, b_n^2; \ldots; a_n^{2^{n-1}}, b_n^{2^{n-1}}$ be the endpoints of the 2^{n-1} deleted intervals of length 3^{-n}.

Now let $\{x_1, x_2, \ldots\}$ be a countable dense subset of M. $I(M)$ is formed by a series of identifications in the product space $C \times M$. First, identify the points (a_1^1, x_1) and (b_1^1, x_1). Call this point (in the quotient space) p_1^1. Next, identify (a_2^1, x_2) and (b_2^1, x_2), calling this point p_2^1, and identify (a_2^2, x_2) and (b_2^2, x_2), calling this point p_2^2. In general, at the n-th stage, identify (a_n^1, x_n) and (b_n^1, x_n), (a_n^2, x_n) and (b_n^2, x_n) and $(b_n^2, x_n), \ldots, (a_n^{2^{n-1}}, x_n)$ and $(b_n^{2^{n-1}}, x_n)$ to get, respectively, the points $p_n^1, p_n^2, \ldots, p_n^{2^{n-1}}$ in the quotient space. $I(M)$ is the quotient space resulting from all of the above identifications. Let q be the quotient mapping from $C \times M$ onto $I(M)$.

A straightforward argument will show that $I(M)$ is a continuum and that the set $\{p_n^j : n = 1, 2, \ldots, ; j = 1, 2, \ldots, 2^{n-1}\}$ is dense in $I(M)$. Moreover, each p_n^j is a separating point of $I(M)$ which separates the two sets $q(\{0\} \times M)$ and $q(\{1\} \times M)$. Thus if S is any subcontinuum of $I(M)$

which meets $q(\{0\}\times M)$ and $q(\{1\}\times M)$, S must contain all of the points p_n^j. Since these points are dense in $I(M)$, S must then be all of $I(M)$. Thus $I(M)$ is irreducible between any point of $q(\{0\}\times M)$ and any point of $q(\{1\}\times M)$.

Let f be the mapping of C onto the unit interval which identifies endpoints of the deleted intervals. Now define a mapping m of $I(M)$ onto $[0,1]$ as follows: if $(c,x)\in C\times M$, let $m\big(q(c,x)\big)=f(c)$. m is then continuous and monotone (the inverse image of any $t\in[0,1]$ is either a copy of M or two copies of M identified at a point). And for each $t\in[0,1]$, $m^{-1}(t)$ has void interior in $I(M)$ ($f^{-1}(t)$ has a void interior in C). Thus, $I(M)$ is an irreducible continuum of type λ.

Finally, define $g\colon I(M)\to M$ by $g\big(q(c,x)\big)=x$ for any $(c,x)\in C\times M$. It is straightforward to verify that g is open, continuous and surjective.

REFERENCES

[2] J. J. Charatonik, *Confluent mappings and unicoherence of continua*, Fundamenta Mathematicae 56 (1964), p. 213-220.

[2] J. B. Fugate and L. Mohler, *Quasi-monotone and confluent images of irreducible continua*, this fascicle, p. 221-224.

UNIVERSITY OF KENTUCKY
STATE UNIVERSITY OF NEW YORK AT BUFFALO
MATHEMATICS INSTITUTE, UNIVERSITY OF WARWICK

Reçu par la Rédaction le 30. 8. 1972