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# *SETS OF UNIFORM CONVERGENCE*

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Let $G$ be a compact abelian group and let its dual group $\Gamma$ be an infinite discrete group. We denote by $C(G)$ the space of continuous functions on $G$, and by $A(G)$ the space of continuous functions with absolutely convergent Fourier series. For a subset $E \subset \Gamma$, we let

$$C_E(G) = \{f \in C(G) \colon \hat{f}(\gamma) = 0,\ \gamma \notin E\}.$$

Let $F_N$ be finite subsets of the group $\Gamma$ such that $F_N \subset F_{N+1}$ and $\bigcup_1^\infty F_N = \Gamma$; if

$$f(x) \sim \sum_{\gamma \in \Gamma} \hat{f}(\gamma)\gamma(x)$$

we write

$$S_N(f, x) = \sum_{\gamma \in F_N} \hat{f}(\gamma)\gamma(x).$$

We recall the following definition:

Definition 1. $E \subset \Gamma$ is called a *Sidon set* if $C_E \subset A$; that is, if every function in $C_E$ has an absolutely convergent Fourier series.

The purpose of this note is to study subsets of $\Gamma$ which satisfy a property which is weaker than the property of being a Sidon set. That is we want to study sets defined by the following

Definition 2. A subset $E \subset \Gamma$ is called a *set of uniform convergence*, or a *UC set*, if every function in $C_E$ has a uniformly convergent Fourier series.

It is clear that every Sidon set is a UC set.

In section 1, we give a condition which is equivalent to the definition of a UC set and we exhibit an example of a UC set which is not a Sidon set in an infinite discrete group.

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In section 2, we study UC subsets of $\boldsymbol{Z}$, the dual group of the group $\boldsymbol{T} = \boldsymbol{R}/2\pi\boldsymbol{Z}$. In this case we construct a class of UC sets which are not Sidon sets. This construction is based on that which is given in [4]. We prove that then a UC set cannot contain arbitrarily long arithmetic progressions. Finally we discuss some examples and counter-examples.

In section 3, we discuss the previous arguments in the group $\hat{\boldsymbol{D}}$, the dual group of the Cantor group.

In section 4, we discuss some open problems, in particular the union problem for UC sets.

**1.** Let $G$ be a compact group and let its dual group $\Gamma$ be an infinite discrete abelian group; let $F_N$ be finite subsets of $\Gamma$ such that

$$F_N \subset F_{N+1} \quad \text{and} \quad \bigcup_1^\infty F_N = \Gamma.$$

We have the following characterization of UC sets:

THEOREM 1. *$E \subset \Gamma$ is a UC set if and only if there is a constant $C$ and, for every positive integer $N$, a measure $\mu_N \in M(G)$ with the following properties:*

(i) $\|\mu_N\| \leqslant C$ *for every* $N \in \boldsymbol{N}$;

(ii) $\hat{\mu}_N(\gamma) = 1$ *for* $\gamma \in E \cap F_N$;

(iii) $\hat{\mu}_N(\gamma) = 0$ *for* $\gamma \in E \cap (\Gamma \setminus F_N)$.

Proof. If $E$ is a UC set, then, by the uniform boundedness principle, there exists a constant $C$ such that, for each $N$, the mapping

$$f \to \varphi_N(f) = \sum_{\gamma \in F_N} \hat{f}(\gamma) \quad \text{for } f \in C_E$$

is a bounded linear functional on $C_E$ of norm at most $C$. The Hahn-Banach theorem allows us to extend this functional to $C(G)$, with preservation of norm. Hence there is a measure $\mu_N$ such that

$$\|\mu_N\| = \|\varphi_N\| \leqslant C \quad \text{and} \quad \varphi_N(f) = \int_G f(x)\, d\mu_N(x) \text{ for } f \in C_E.$$

Let $\gamma(x)$ be an element in $E$; then

$$\varphi_N(\gamma) = \int_G \gamma(x)\, d\mu_N(x) = \overline{\hat{\mu}_N(\gamma)}.$$

But, from the definition of $\varphi_N$,

$$\varphi_N(\gamma) = \begin{cases} 1, & \gamma \in F_N, \\ 0, & \gamma \in \Gamma \setminus F_N, \end{cases}$$

so that the assertion is proved.

On the other hand, if we suppose that a measure satisfying (i), (ii) and (iii) exists, then we have

$$\|S_N f\|_\infty = \|f * \mu_N\|_\infty \leqslant C\|f\|_\infty \quad \text{for every } f \in C_E,$$

so that $E$ is a UC set.

Obviously, every subset of a UC set is a UC set.

Now we exhibit an example of a subset of $\Gamma$ which is a UC set but not a Sidon set.

We say that a finite set $D \subset \Gamma$ is *dissociated* [6] if it does not contain 1 and if the equality

$$\gamma_1^{\varepsilon_1}\gamma_2^{\varepsilon_2}\ldots\gamma_m^{\varepsilon_m} = 1 \quad \text{with } \gamma_j \in D \text{ and } \varepsilon_j \in \{-2, -1, 0, 1, 2\}$$

implies the equality

$$\gamma_1^{\varepsilon_1} = \gamma_2^{\varepsilon_2} = \ldots = \gamma_m^{\varepsilon_m} = 1.$$

We construct a subset $E \subset \Gamma$ in the following way: let $E_1 = \{\gamma_1\}$ with $\gamma_1 \neq 1$. Let us suppose to have constructed $E_k$ and let $n_k$ be the first index such that

$$E_k^2 = \{\gamma_{i_1}\gamma_{i_2};\ \gamma_{i_1}, \gamma_{i_2} \in E_k,\ 1 \leqslant i_1 < i_2 \leqslant k\} \subset F_{n_k}.$$

Then we choose $\gamma_{k+1}$ so that $E_{k+1} = E_k \cup \{\gamma_{k+1}\}$ is a dissociated set and $\{\gamma_{k+1}E_k\} \cap F_{n_k} = \emptyset$. Then we have the following

THEOREM 2. *If*

$$E = \bigcup_{k=1}^{\infty} E_k = \{\gamma_i\}_{i=1}^{\infty},$$

*then $E^2$ is a UC set and it is not a Sidon set.*

Proof. Let $f$ be a continuous function with Fourier coefficients $\hat{f}(\gamma)$ and such that $\hat{f}(\gamma) = 0$ if $\gamma \notin E^2$. For each $N$, we have $n_k \leqslant N < n_{k+1}$ for some $k$ and, therefore,

$$S_N(f, x) = \sum_{\gamma \in F_N} \hat{f}(\gamma)\gamma(x) = \sum_{\gamma \in F_{n_k}} \hat{f}(\gamma)\gamma(x) + \sum_{\gamma \in F_N \setminus F_{n_k}} \hat{f}(\gamma)\gamma(x) = S_N^{(1)} + S_N^{(2)}.$$

We let

$$P(x) = P_k(x) = \prod_{i=1}^{k}\left(1 + \frac{\gamma_i(x) + \bar{\gamma}_i(x)}{2}\right).$$

Then, since $P(x) \geqslant 0$ and $E$ is dissociated, we have

(i) $\hat{P}(0) = \|P\|_1 = 1$,

(ii) $\hat{P}(\gamma_{i_1}\gamma_{i_2}) = 1/4,\ 1 \leqslant i_1 < i_2 \leqslant k.$

But if $\gamma_{i_1}\gamma_{i_2} \in F_{n_k}$, then $i_1$ and $i_2$ are not greater than $k$ and, in view of (ii),

$$\frac{1}{4} S_N^{(1)}(f, x) = \frac{1}{4} \sum_{\gamma \in F_{n_k}} \hat{f}(\gamma)\gamma(x) = P * f(x).$$

Therefore $\|S_N^{(1)}\|_\infty \leqslant 4\|f\|_\infty$.

On the other hand, if $\hat{f}(\gamma) \neq 0$ and $\gamma \in F_N \setminus F_{n_k}$, then $\gamma = \gamma_{k+1}\gamma_i$, $1 \leqslant i \leqslant k$.

Let

$$Q(x) = Q_k(x) = \gamma_{k+1} \prod_{i=1}^{k} \left(1 + a_i \frac{\gamma_i(x) + \bar{\gamma}_i(x)}{2}\right),$$

where $a_i = 1$ if $\gamma_{k+1}\gamma_i \in F_N$ and $a_i = 0$ if $\gamma_{k+1}\gamma_i \notin F_N$, i.e. $\gamma_{k+1}\gamma_i \in F_{n_{k+1}} \setminus F_N$. Then

(i) $\|Q(x)\|_1 = \hat{Q}(\gamma_{k+1}) = 1$,

(ii) $\hat{Q}(\gamma_{k+1}\gamma_i) = a_i/2$, $1 \leqslant i \leqslant k$.

Therefore,

$$f * Q(x) = \sum_{\gamma \in F_{n_{k+1}}} \hat{f}(\gamma)\hat{Q}(\gamma)\gamma(x) = \frac{1}{2} \sum_{\gamma \in F_N \setminus F_{n_k}} \hat{f}(\gamma)\gamma(x),$$

and so

$$\|S_N^{(2)}\|_\infty = 2\|f * Q\|_\infty \leqslant 2\|f\|_\infty.$$

In conclusion,

$$\|S_N f\|_\infty \leqslant 4\|f\|_\infty + 2\|f\|_\infty = 6\|f\|_\infty.$$

From this inequality it follows that $E^2$ is a UC set. The fact that $E^2$ is not a Sidon set is well known and follows, e.g., from the lemma of [4] or [3]

**2.** Throughout this section $G$ is the group $\boldsymbol{T} = \boldsymbol{R}/2\pi\boldsymbol{Z}$, and then $\Gamma$ is the group $\boldsymbol{Z}$. If

$$F_N = \{0, \pm 1, \pm 2, \ldots, \pm N\} \quad \text{and} \quad f(x) \sim \sum_{-\infty}^{+\infty} \hat{f}(n)\, e^{inx},$$

we have the usual definition

$$S_N(f, x) = \sum_{-N}^{N} \hat{f}(n)\, e^{inx}.$$

THEOREM 3. *Let $n_j$ be a strictly increasing sequence of positive integers with the property that*

$$n_j > n_1 + n_2 + \ldots + n_{j-1}$$

*and let*

$$E^s = \{n_{j_1} + n_{j_2} + \ldots + n_{j_s} \colon 1 \leqslant j_1 < \ldots < j_s\}.$$

*Then $E^s$ is a UC set, but if $s \geqslant 2$, it is not a Sidon set.*

Proof. We will prove by induction that $E^s$ is a UC set. The assertion is true for $s = 1$, since $E = E^1$ is a Sidon set. Let us suppose that $E^{s-1}$ is a UC set and let $f \epsilon E^s$. We will show that

$$\|S_N f\|_\infty \leqslant (2^s + C)\|f\|_\infty,$$

where $C$ is the UC constant relative to $E^{s-1}$. We can assume that $N \neq n_j$ for all $j$. Therefore, $n_k < N < n_{k+1}$ for some $k$. We have

$$S_N(f, x) = \sum_{n<n_k} \hat{f}(n)\, e^{inx} + \sum_{n_k<n\leqslant N} \hat{f}(n) e^{inx} = S_N^{(1)}(f, x) + S_N^{(2)}(f, x).$$

With the same argument as used in theorem 2, we prove that

$$\|S_N^{(1)} f\|_\infty \leqslant 2^s \|f\|_\infty.$$

If $n_k < n \leqslant N < n_{k+1}$ and $\hat{f}(n) \neq 0$, then $n = n_k + n_{j_1} + \ldots + n_{j_{s-1}}$ for some $1 \leqslant j_1 < \ldots < j_{s-1} \leqslant k-1$. Thus

$$S_N^{(2)}(f, x) = \sum_{n_{j_1}+\ldots+n_{j_{s-1}} \leqslant N-n_k} \hat{f}(n_k + n_{j_1} + \ldots + n_{j_{s-1}}) \exp[i(n_k + n_{j_1} + \ldots + n_{j_{s-1}})x].$$

From the inductive hypothesis we know that for every $N$ there is a measure $\mu_N \epsilon M(\boldsymbol{T})$ such that $\|\mu_N\| \leqslant C$ and

$$\hat{\mu}_N(n) = \begin{cases} 1 & \text{for } n \epsilon E^{s-1},\ |n| \leqslant N - n_k, \\ 0 & \text{for } n \epsilon E^{s-1},\ |n| > N - n_k. \end{cases}$$

Let $d\nu_N = \exp[in_k x]\, d\mu_N$; then

$$\|\nu_N\| = \|\mu_N\| \leqslant C \quad \text{and} \quad \hat{\nu}_N(n_k + n_{j_1} + \ldots + n_{j_{s-1}}) = \hat{\mu}_N(n_{j_1} + \ldots + n_{j_{s-1}}),$$

so that

$$\hat{\nu}_N(n) = \begin{cases} 1 & \text{if } n = n_k + n_{j_1} + \ldots + n_{j_{s-1}} \leqslant N, \\ 0 & \text{if } n = n_k + n_{j_1} + \ldots + n_{j_{s-1}} > N. \end{cases}$$

It follows that

$$S_N^{(2)}(f, x) = (\nu_N * f)(x),$$

and so

$$\|S_N^{(2)}f\|_\infty \leqslant C\|f\|_\infty .$$

The conclusion is as in theorem 2.

We give now a necessary condition in order that $E \subset \boldsymbol{Z}$ be a UC set.

THEOREM 4. *If $E \subset \boldsymbol{Z}$ is a UC set, then $E$ does not contain arbitrarily long arithmetic progressions.*

**Proof.** Suppose that there is a UC set $E$ such that, for infinitely many positive integers $N_j$,

$$\{a+bn\}_{n=0}^{N_j} \subset E, \quad \text{where } a, b \in \boldsymbol{N},\ a < b .$$

Without loss of generality we can assume that $E$ consists of positive integers. Let $\{f_\nu\}_{\nu\in N}$ be trigonometric polynomials of degree $k_\nu$ and such that

$$\|f_\nu\|_\infty \leqslant 1 \quad \text{and} \quad \|S_\nu f_\nu\|_\infty \to \infty .$$

Assume also that $\hat{f}_\nu(n) = 0$ for $n < 0$. We choose a subsequence $\{N_{j_\nu}\}$ of $\{N_j\}$ in such a way that $N_{j_\nu} > k_\nu$. Let

$$\hat{g}_\nu(a+bm) = \hat{f}_\nu(m) \quad \text{and} \quad \hat{g}_\nu(n) = 0 \text{ if } n \neq a+bm .$$

Then

$$\|g_\nu\|_\infty = \|f_\nu\|_\infty \leqslant 1 \quad \text{and} \quad g_\nu \in C_E .$$

On the other hand,

$$S_{a+b\nu}(g_\nu) = \sum_{-(a+b\nu)}^{a+b\nu} \hat{g}_\nu(n)e^{inx} = \sum_{0}^{\nu} \hat{f}_\nu(m)e^{i(a+bm)x} .$$

Therefore,

$$\|S_{a+b\nu}g_\nu\|_\infty = \|S_\nu f_\nu\|_\infty \to \infty ,$$

but this is a contradiction, if $E$ is a UC set.

**Remark.** It is not true that if $E$ is a Sidon set, then $E+E$ is a UC set. In fact, there exists a Sidon set $E$ such that $E+E$ contains all positive integers. This is shown by Ebenstein [2], who has taken the set $E = \{10^n+n\}\cup\{-10^n\}$.

**3.** In this section we study a UC subset of the dual group $\hat{\boldsymbol{D}}$ of the Cantor group $\boldsymbol{D}$ defined by

$$\boldsymbol{D} = \prod_{j=1}^{\infty} \boldsymbol{Z}(2)_j \quad \text{with } \boldsymbol{Z}(2) = \{0, 1\} .$$

We consider the elements of $\hat{\boldsymbol{D}}$ arranged according to the ordering defined by Paley for the Walsh functions. In other words, we let $r_j(x)$

$= (-1)^{x_j}$, where $x = \{x_j\} \in \boldsymbol{D}$ with $x_j \in \boldsymbol{Z}(2)$, and we put

$$w_0 = 1,$$
$$w_n = r_{j_1} r_{j_2} \dots r_{j_p}, \qquad \text{where } n = 2^{j_1-1} + 2^{j_2-1} + \dots + 2^{j_p-1}.$$

Then, for a function $f$ defined on $\boldsymbol{D}$, we write

$$S_n(f, x) = \sum_{k=0}^{n} \hat{f}(w_k) w_k(x_k).$$

We recall (see [4]) that

$$\sup_n \{\|S_n f\|_\infty : f \in C(\boldsymbol{D}),\ \|f\|_\infty \leqslant 1\} = +\infty.$$

Therefore, not all functions in $C(\boldsymbol{D})$ have a uniformly convergent Fourier series. The following theorem is a natural analogue of theorem 3 for subsets of $\hat{\boldsymbol{D}}$, but in this case the proof is more direct. Furthermore, the class of UC sets defined in theorem 5 which follows is closed under finite unions.

THEOREM 5. *Let $r_j(x)$ be the Rademacher functions. For a positive integer $s$, let $E^s$ be defined by*

$$E^s = \{r_{i_1} r_{i_2} \dots r_{i_s} : i_1 < i_2 < \dots < i_s\}.$$

*If $E = E^{s_1} \cup E^{s_2} \cup \dots \cup E^{s_p}$, then $E$ is a UC set. If one of the $s_i$ is greater than one, then $E$ is not a Sidon set.*

Proof. For each $N$, let $M$ be the largest integer such that $w_M \in E$ and $M \leqslant N$. Let

$$M = 2^{j_1-1} + \dots + 2^{j_h-1} \quad \text{with } j_1 > j_2 > \dots > j_h.$$

We write

$$P_1(x) = \prod_{i=1}^{j_1-1} \big(1 + r_i(x)\big),$$

$$P_q(x) = r_{j_1} r_{j_2} \dots r_{j_{q-1}}(x) \prod_{i=1}^{j_q-1} \big(1 + r_i(x)\big) \qquad \text{for } 2 \leqslant q \leqslant h$$

and, lastly,

$$P_{h+1}(x) = r_{j_1} r_{j_2} \dots r_{j_h}(x).$$

For each $N$, the function $Q_N = P_1 + P_2 + \dots + P_{h+1}$, where $h = h(N)$ depends on $N$, satisfies

$$\|Q_N\|_1 \leqslant (\max_{1 \leqslant i \leqslant p} s_i) + 1,$$

$$\hat{Q}_N(w_k) = \begin{cases} 1 & \text{if } w_k \in E,\ k \leqslant N, \\ 0 & \text{if } w_k \in E,\ k > N. \end{cases}$$

Therefore, $E = \bigcup_{1}^{p} E^{s_i}$ is a UC set by theorem 1. If $s_1 > 1$, then $E^{s_i}$ is known not to be a Sidon set.

We prove now the analogue of theorem 4.

THEOREM 6. *If $E \subset \hat{D}$ is a UC set, then $E$ cannot contain subgroups of arbitrarily large finite cardinality.*

Proof. Let $f_k$ be polynomials of degree $d_k$, that is

$$f_k = \sum_{j=0}^{d_k} \hat{f}_k(w_j) w_j(x),$$

such that

$$\|f_k\|_\infty \leqslant 1 \quad \text{and} \quad \|S_{n_k} f_k\|_\infty \to \infty.$$

Suppose that $E$ contains subgroups of arbitrarily large finite cardinality. Then passing, if necessary, to a subsequence of $\{d_k\}$, we can suppose that $E$ contains subgroups of cardinality $2^{h_k}$, where $2^{h_k - 1} < d_k \leqslant 2^{h_k}$; let $s_1, s_2, \ldots, s_{h_k}$ be a basis for such a subgroup and let us suppose that $s_i$ precedes $s_{i+1}$ in the Paley ordering. Let $a$: $\hat{D} \to \hat{D}$ be a group-isomorphism such that $a(s_i) = r_i$ for $1 \leqslant i \leqslant h_k$. Let $F_k = S_{n_k} f_k$, and write $\hat{F}_k^*(w_j) = \hat{F}_k(a(w_j))$ and $\hat{f}_k^*(w_j) = \hat{f}_k(a(w_j))$. We have then

$$\|F_k^*\|_\infty = \|F_k\|_\infty \to \infty \quad \text{and} \quad \|f_k^*\|_\infty = \|f_k\|_\infty \leqslant 1.$$

But $F_k^* = S_{m_k} f_k^*$, where $m_k$ is such that $a(w_{m_k}) = w_{n_k}$; therefore, we have a sequence $f_k^*$ of continuous $E$-functions, whose norms are bounded by 1 and such that $\|S_{m_k} f_k^*\|_\infty \to \infty$. This is a contradiction because $E$ is a UC set.

We exhibit now an example of a Sidon set $E \subset \hat{D}$ such that

$$E^2 = \{w_h w_k \colon h \neq k;\ w_h, w_k \in E\}$$

is not a UC set, since it will contain subspaces of arbitrarily high cardinality. We define a sequence $G_i$ of subsets of $\hat{D}$ by induction on $i$. Let $W_1$ be a subspace of $\hat{D}$ of cardinality $n_1$ and let us consider $n_1$ Rademacher functions $r_{1,1}, r_{1,2}, \ldots, r_{1,n_1}$ whose index is greater than the index of each element in $W_1$. If

$$W_1 = \{w_{1,1}, \ldots, w_{1,n_1}\},$$

we let then

$$G_1 = \{w_{1,j} r_{1,j} \colon j = 1, 2, \ldots, n_1\}.$$

If $i \geqslant 2$, $W_i$ will be a subspace of $\hat{\boldsymbol{D}}$ with cardinality $n_i > n_{i-1}$ and will be disjoint from the subspace generated by $r_1, r_2, \ldots, r_{m_{i-1}}$, where $m_{i-1}$ is the greatest index of the functions $r_{i-1,1}, \ldots, r_{i-1,n_{i-1}}$ when they are arranged according to the ordering. We consider then $n_i$ Rademacher functions $r_{i,1}, \ldots, r_{i,n_i}$ such that their index is greater than the index of each function in $W_i$, and we let

$$G_i = \{w_{i,j} r_{i,j}\colon j = 1, 2, \ldots, n_i\}, \qquad \text{where } W_i = \{w_{i,1}, \ldots, w_{i,n_i}\}.$$

Finally, we let

$$G = \bigcup_{i=1}^{\infty} G_i \quad \text{and} \quad R = \bigcup_{i=1}^{\infty} \{r_{i,j(i)}\colon 1 \leqslant j(i) \leqslant n_i\}.$$

Both $G$ and $R$ are Sidon sets because they are independent subsets of $\hat{\boldsymbol{D}}$; therefore $E = G \cup R$ is a Sidon set [1], but $E^2$ is not a UC set since $E^2 \supset W_i$ for each $i$.

**4.** It is known that the union of two Sidon sets is a Sidon set [1]. It is natural to ask now whether the union $E_1 \cup E_2$ of two UC sets $E_1$ and $E_2$ is a UC set. (**P 944**)

In the general case, we are unable to give an answer to this question even if the UC sets are those defined in theorem 3 with respect to the same sequence $\{n_k\}$. For UC subsets of $\hat{\boldsymbol{D}}$ we have only the partial answer given in theorem 5.

Another natural open question concerning UC sets is the following: does every set which is not a Sidon set contain a UC set which is still not a Sidon set? (**P 945**)

Finally, it is natural to ask whether the UC constants found in theorems 2 and 4 can be improved. More, in general, the relation between a Sidon constant and a UC constant of a finite set is still unclear. (**P 946**)

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