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DE STEFAN BERGMAN

Stefan Bergman

1895–1977

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Stefan Bergman (1895 – 1977)

in Memoriam

by M. M. SCHIFFER (Stanford)

Stefan Bergman was born on May 5, 1895 in Częstochowa as son of the merchant Bronisław Bergman and his wife Tekla. He received his primary and high school education in his home town and graduated from the local "Gymnasium" in 1913. These years left him with a great love for Polish and Russian literature, and he could cite its classic poets by heart and with enthusiasm. He then entered the School of Engineering in Wrocław. In 1915, he changed over to the School of Engineering in Vienna (Austria), where he finished as Diplomated Engineer. Even in these years of more practical studies he was strongly attracted to the theoretical aspects of the engineering sciences and to problems of pure and applied mathematics. For that reason he entered in 1921 the Institute for Applied Mathematics which had just been established by Richard von Mises at the University of Berlin (then called Friedrich-Wilhelm Universität, now Humboldt Universität). Von Mises was one of the leading theoreticians in aerodynamics and probability theory, with deep interest in all uses of mathematics in engineering and science. His definition of applied mathematics is very characteristic for him and his group. He demanded that applied mathematics be as precise as pure mathematics but that, moreover, all its methods be feasible and practical. His ideas had an enormous impact on Bergman's scientific outlook and he stayed in close personal and professional connection with his teacher until the death of von Mises in 1951. As a member of the Institute for Applied Mathematics he worked on such down to earth problems as the magnetic field in an electric transformer and the distribution of temperature in the stator of a generator. He studied boundary value problems of elasticity and various other problems of potential theory. To obtain a large number of harmonic functions he applied the Whittacker method for creating harmonic functions by means of integrals over analytic functions. This method aroused in him the curiosity and ingenuity of the abstract mathematician. Restricting himself to algebraic-logarithmic analytic functions as generators in the integral, he brought the entire theory of Abelian integrals and Riemann surface theory into play. He created harmonic functions in three-space which are multivalued and have closed branch lines. Here lies the germ for the general theory of integral operators to which he devoted a large part of his research activities and which is one of his great contributions to analysis.

Another outstanding mathematician at Berlin who had great influence on Bergman's scientific development was Erhard Schmidt who together with Hilbert had pioneered in the early theory of integral equations, orthogonal functions and the beginning functional analysis. Shortly after his arrival at Berlin, Bergman participated in Schmidt's seminar and was charged to give a lecture on development of arbitrary functions with finite square integral in terms of an orthogonal set. As he told me, he misunderstood the task and instead of dealing with real functions over a real interval, he attacked the problem for analytic functions over a complex domain. He found the task hard but attacked it couragesly and carried it through. This was the genesis of his famous theory of orthogonal functions and the kernel function. A first fruit was his thesis which gave him the Doctor's degree in 1922. Interestingly enough another student from Schmidt's seminar by the name of Salomon Bochner was also attracted to the problem of orthogonal systems but developed into a different direction of analysis.

Bergman applied his results on orthogonal analytic functions in fluid dynamics, conformal mapping and potential theory, as was natural for a member of the von Mises group in applied mathematics. On the other hand, he studied the theoretical implications of his work and the significance of the central concept of his theory which is now called the *Bergman kernel.* This is one of his major achievements in pure mathematics and has found many important applications. He soon realized that his methods worked as well for the class of analytic functions of several complex variables and that he could define his kernel function in this case, too. The subject was still quite undeveloped in the Twenties and he may be considered as one of the founders of this branch of research which stands today in the center of attention in analysis. His most impressive achievement in this field is the concept of invariant metric. He considered domains in the space of two complex variables which would be mapped onto each other by means of a pair of analytic functions. He could easily obtain the transformation law for the kernel function under such *pseudo-conformal* mapping and by means of it, and its derivatives introduce a metric which was invariant under the mapping. This is a special case of a class of metrics which is now called a *Kähler metric.* It is an outstanding testimony to his intuition for significant ideas and formalism in analysis that he came so early to his concept.

Next, he got interested in the problem of pseudo-conformal equivalence of domains. In analogy to the concept of canonical domains in the theory of conformal mapping, he introduced the concept of representative domain in the case of several variables. He hoped to obtain a classification of the various types of equivalence classes. He characterized some representative domains by extremum problems and worked until his last days on the problem to characterize them geometrically. He also wanted to use invariant metric and representative domains to obtain distortion theorems under pseudo-conformal mapping and to find useful generalizations of the Schwarz lemma to several complex variables.

Another fruitful idea in this theory was the concept of *distinguished boundary*. He discovered that for a large class of domains an analytic function of several complex variables is already completely determined by its value on a relatively small part of the boundary. This part is now called the *Bergman–Shilov boundary* of such a domain. This phenomenon is not met in the function theory for one complex variable and leads to various important consequences. Bergman gave also an integral formula which is an analogue to the classical Cauchy integral formula. It allows us to express the values of a function at interior points of a domain in terms of its values at the distinguished boundary. Then he connected the theory of the boundaries of a domain with that of the kernel function by classifying boundary points in terms of the asymptotic behavior of the kernel under an approach to that boundary point.

He brought many more ideas to the theory. Since it was clear that one could not prescribe arbitrarily the boundary values even of the real part of an analytic function of several variables, he introduced a wider class, his so-called *extended class* for which the boundary value problem was possible. All theoretical insight of such functions would then contain information on the subclass of proper analytic functions.

He tried through many years to visualize the space of four dimensions by using movie pictures in which the time played the role of the fourth space variable and cooperated with film producers to create some teaching movies in four-dimensional geometry.

In 1930 Bergman became a "Privat Dozent" at the University of Berlin. His thesis which he had to submit for his official "Habilitation" dealt with the theory of boundary behavior of the kernel function. He was appointed simultaneously to the Institute of Mathematics and the Institute for Applied Mathematics at the Berlin University. This was at that time a rare distinction. I was then a very young student present at his inaugural lecture and very impressed by his sponsors von Mises and Schmidt who presided in full academic dress on this occasion. The topic of the lecture was about the theory of a wing of an airplane.

During his brief position in Berlin he did mostly research, but he lectured also on his field to advanced graduate students and tried to attract them to the topics of his interest. However, in 1933 the Nazi take-over of Germany cut short his, like many others, scientific career at the University. He left

Germany and found refuge for some years in Russia. In 1934 he became Professor at the University of Tomsk in Sibiria and in 1936 he moved to Tbilisi, Georgia, where he stayed until 1937. The success of his stay in the Soviet Union is best appreciated if one observes that some of his students became leading mathematicians in their own right, such as Vekua, Fuks, Kufarev, etc. In 1937 his position became precarious, because of increase of Stalinism. He left for Paris and worked there under most difficult conditions. He spent most of his time at the Institut Henri Poincaré, where he continued with irrepressible energy and wrote a two volume monograph on the kernel function and its application in complex analysis. It appeared in the series "Memorial des Sciences Mathématiques" and is still today useful and appreciated. His lot was somewhat alleviated by the support and advice of the great French analyst Hadamard. Through his help he was able to immigrate to the United States. He left France in 1939 just before the outbreak of the second World War. He worked from 1939 to 1941 at the M. I. T. in Cambridge, Massachusetts, and at the Yeshiva College in New York. He ended up in 1941 at Brown University, Providence, Rhode Island, which was at that time a real haven for scientist refugees from Europe. He gave there various courses on the kernel function, conformal mapping and fluid dynamics. Among his earliest students were L. Bers and A. Gelbart who worked out his lecture notes.

In 1945 he joined his old teacher and revered friend von Mises at the Harvard Graduate School of Engineering. There he worked intensively on various research projects until 1951 when von Mises died. The conditions of war research had renewed his interest in problems of aerodynamics. He published very useful research reports under the auspices of the National Advisory Council for Aeronautics (now NASA) and the Office of Naval Research. He drew attention to the classical method of Chaplygin to deal with the non-linear equations of a compressible fluid flow in the plane. He developed the method of the hodograph plane in which the flow equations are linearized but at the price of very complicated partial differential equations and rather difficult correspondence between the velocity plane and the physical plane. So Bergman extended, on the one hand, the concept of orthogonal development to the space of solutions of partial differential equations. On the other hand, he generalized his method of integral operators to transform analytic functions into the desired solutions of the partial differential equations. Finally, he used his knowledge of functions of several complex variables to obtain analytic continuation of the solutions across singular lines of the equation.

After I had shown the relation between the harmonic Green's function of a plane domain and its kernel function it was natural to extend the method of orthogonal solutions to problems of partial differential equations and obtain representations for their fundamental solutions. We worked in close cooperation at Harvard from 1946 to 1950 and I remember those days with nostalgia.

In 1952 Bergman became again my colleague when we joined the Mathematics Department at Stanford University. He enjoyed the pleasant California living. He had married in 1950 and with his wife Adele spent there the most secure and peaceful time of his life. He continued until the end in enthusiastic and intensive research and published numerous papers covering all fields of his previous interest.

One cannot end a biographical essay on Stefan Bergman without mentioning the unselfish devotion which he gave to students and young colleagues and the zeal with which he tried to stimulate and attract their interest in mathematical research. He helped them with advice, recommendations and quite often with money when they needed it. A list of mathematicians who have been affected by him would be much too long to be put down here. Instead, I should like to reminisce and tell about my own experience with him. When I saw him in Paris in 1938, he was in a bad and almost hopeless situation. But he forgot about it when he began to talk about mathematics. He tried to convince me that the future of analysis lay in the field of several complex variables and urged me to work and publish in that direction. He helped me to edit my first Notes and spent hours to show me how to prepare the manuscript for the printer, to bring me to the right printing office and so on. The many hours and efforts which he spent for my benefit are paralleled by similar efforts for many other beginners in our Science. It was a great loss to the Stanford Mathematics Department and the mathematical community at large when he died on June 6, 1977. He is survived by his wife Adele.

Bergman's list of publications is most impressive also by its length. I shall therefore only mention his books from which the interested reader can construct easily a more complete bibliography.

List of books by Stefan Bergman

[1] *Sur les fonctions orthogonales de plusieurs variables complexes avec les applications à la théorie des fonctions analytiques*, Mémor. Sci. Math. Paris 106 (1947).

[2] *Sur les fonctions-noyaux d'un domaine et ses applications dans la théorie des transformations pseudo-conformes*, Mémor. Sci. Math. Paris 108 (1948).

[3] *The kernel function and conformal mapping*, Second Edition, Providence (1970).

[4] (with M. Schiffer) *Kernel functions and elliptic differential equations in mathematical physics*, New York (1953).

[5] *Integral operators in the theory of linear partial differential equations*, New York (1969).