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FREQUENCY OF OSCILLATIONS OF A SYNCHRONOUS MOTOR

1. Introduction. In this paper we consider the differential equation

$$\ddot{\theta}+\sin\theta-\sin\theta_0 = 0, \tag{1.1}$$

in which θ_0 is a given constant from the open interval $(0, \pi/2)$, and $\theta = \theta(t)$ is the unknown function. Equation (1.1) occurs in the theory of a synchronous motor as a special case of the more general equation

$$\ddot{\theta}+f(\theta)\dot{\theta}+g(\theta) = D, \tag{1.2}$$

for the power angle $\theta(t)$ as a function of time. Here, $g(\theta)$ and $f(\theta)$ are some known periodic functions with period 2π, and represent the electrodynamical torque and the damping coefficient of the motor, respectively. The quantity D may be considered as the external moment (cf. [5], 5.101; [7], 6.3 and also the references in those works).

In recent years some essential results in the qualitative theory of equation (1.2) have been obtained. Those results explain some effects during the motion of a synchronous motor, noticed empirically (see [2], p. 16-21; [6], p. 190-191; [14], p. 66-80; [1], VII, § 2, § 3). A quantitative analysis of the equation (1.2) is rather less developed ([5], 5. 102; [8], VIII, 8.30-8.43; [9], § 4).

The point $\theta = \theta_0$ is a point of stable equilibrium of equation (1.1). If the deflection $\delta = \theta(t)-\theta_0$ from the equilibrium-point is sufficiently small, equation (1.1) may be approximated by

$$\ddot{\delta}+(\cos\theta_0)\delta = 0.$$

Then, for the frequency ω of the free oscillations of the power angle we have the well-known formula

$$\omega^2 = \cos\theta_0. \tag{1.3}$$

However, that formula cannot be used for large amplitudes of oscillations defined by

$$a = \tfrac{1}{2}(\max\theta-\min\theta). \tag{1.4}$$

In that case, one considers higher terms in the power expansion of $\sin\theta = \sin(\theta_0+\delta)$ in order to get the approximate equation

$$\ddot{\delta}+(\delta-\tfrac{1}{6}\delta^3)\cos\theta_0-\tfrac{1}{2}\delta^2\sin\theta_0 = 0$$

within the third degree of accuracy. The study of the dependence between the amplitude, the frequency of oscillations and the parameter θ_0 involves elliptic functions. In this way a formula more accurate than (1.3) could be obtained. Here we are going to follow another direction of investigation.

Suppose that the solution $\theta = \theta(t)$ of equation (1.1) describes the vibrations about the equilibrium-point θ_0. Denote

(1.5) $$\theta_1 = \min\theta(t), \quad \theta_2 = \max\theta(t).$$

Then, the period of the oscillations T is given by

(1.6) $$T = \sqrt{2}\int_{\theta_1}^{\theta_2}[(\cos\theta-\cos\theta_1)+(\theta-\theta_1)\sin\theta_0]^{-1/2}d\theta,$$

(cf. [4], § 42). Finally, let $\omega = 2\pi/T$ be the corresponding circular frequency of the oscillations. In this paper some approximate formulae for ω as well as the estimates for the error of approximation will be given. The methods are based on papers [10], [11], [12].

2. Range of oscillations. Denote by $g(\theta)$ and $G(\theta)$ the control-function and the potential energy, respectively. In the case of equation (1.1) we have

(2.1) $$g(\theta) = \sin\theta-\sin\theta_0$$

and

(2.2) $$G(\theta) = \int_{\theta_0}^{\theta}g(\theta)d\theta = -\cos\theta+\cos\theta_0-(\theta-\theta_0)\sin\theta_0.$$

From Fig. 1 and also [4], § 42 it can be seen that the quantity θ_1 defined by (1.5) satisfies the conditions

(2.3) $$-\pi-\theta_0 < \theta_1 < \theta_0$$

and

(2.4) $$G(\theta_1) < G(\pi-\theta_0)$$

or, by (2.2),

(2.4') $$\cos\theta_1+\theta_1\sin\theta_0 > (\pi-\theta_0)\sin\theta_0.$$

On the other hand, if θ_1 satisfies (2.3) and (2.4') there exists a periodic solution $\theta = \theta(t)$ of the initial-value problem

(2.5) $$\ddot{\theta}+\sin\theta-\sin\theta_0 = 0, \qquad \theta(0) = \theta_1, \quad \dot{\theta}(0) = 0,$$

describing the oscillations about the equilibrium-point θ_0. Of course, θ_1 is the left turning point of those oscillations.

Given the left turning-point θ_1 we find the right turning-point θ_2 by standard methods from the equation $G(\theta_2) = G(\theta_1)$ (cf. [4], § 42). In this way we obtain the transcendental equation

$$\cos\theta_2 - \cos\theta_1 + (\theta_2 - \theta_1)\sin\theta_0 = 0, \tag{2.6}$$

where the solution $\theta_0 < \theta_2 < \pi - \theta_0$ is needed in our case. Obviously, the required solution of equation (2.6) exists provided conditions (2.3) and (2.4′) hold (see Fig. 1).

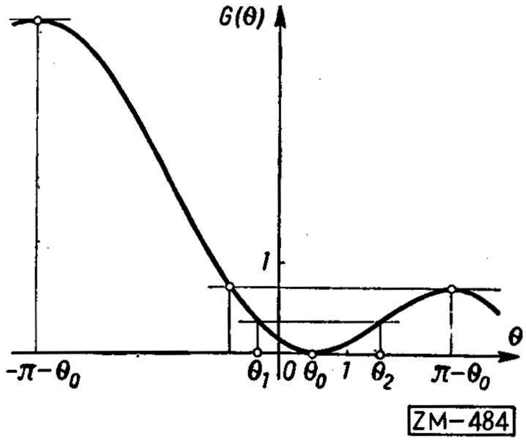

Fig. 1. Case $\theta_0 = 0{,}5$; $\theta_1 = -0{,}3$

Now we give a method of finding θ_2. We first find the dependence between the quantities θ_1, θ_2 and the amplitude

$$a = \frac{\theta_2 - \theta_1}{2}$$

of the oscillations. Accordingly put

$$\theta_1 = q - a, \qquad \theta_2 = q + a. \tag{2.7}$$

By substituting these expressions in (2.6) we obtain the equation

$$\sin q = \frac{a\sin\theta_0}{\sin a}. \tag{2.8}$$

Next we define two sequences $\{a^{(n)}\}$ and $\{q^{(n)}\}$ by the formulae

$$\begin{aligned} &a^{(0)} = +0, \quad q^{(n)} = \arcsin\frac{a^{(n)}\sin\theta_0}{\sin a^{(n)}}, \\ &a^{(n+1)} = q^{(n)} - \theta_1, \quad n = 0, 1, 2, \ldots \end{aligned} \tag{2.9}$$

It can be proved that:

(i) *there exist* $\lim_{n\to\infty} q^{(n)}$ *and* $\lim_{n\to\infty} a^{(n)}$,

(ii) $\theta_2 = \lim_{n\to\infty}[q^{(n)}+a^{(n)}]$

if (2.3) *and* (2.4′) *hold.* Here is the plan of the proof:

1. We show that the graphs of the functions $y = a\sin\theta_0/\sin a$ and $y = \sin(a+\theta_1)$ are located as in Fig. 2.

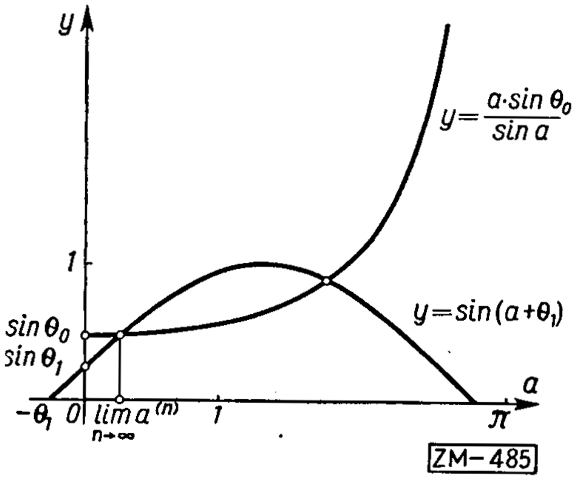

Fig. 2. Case $\theta_0 = 0{,}5$; $\theta_1 = 0{,}25$

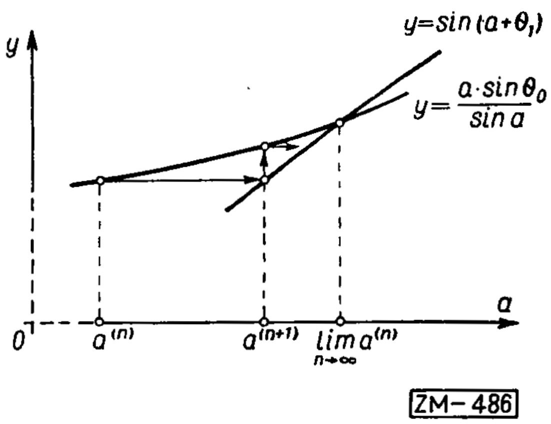

Fig. 3

2. We verify that

$$a^{(n)} < a^{(n+1)} < a, \qquad q^{(n)} < q^{(n+1)} < q$$

(see Fig. 3), from which (i) follows. Then the quantity $\bar{\theta}_2 = \lim_{n\to\infty}[q^{(n)}+ + a^{(n)}]$ satisfies equation (2.6) and the inequality $\theta_0 < \bar{\theta}_2 < \pi-\theta_0$. Since there exists only one root of (2.6) in the interval $(\theta_0, \pi-\theta_0)$, we obtain (ii). The details of the proof are omitted. Relation (ii) gives a simple rule to find θ_2 with any required numerical accuracy.

3. Auxilary control-functions. In the initial value problem (2.5) the control-function $g(\theta) = \sin\theta-\sin\theta_0$ is non-odd with regard to the point of equilibrium θ_0. The problem may be replaced by two similar problems with odd control-functions

$$g_1(\delta) = -\operatorname{sgn}\delta\cdot g(\theta_0-|\delta|), \tag{3.1}$$
$$g_2(\delta) = \operatorname{sgn}\delta\cdot g(\theta_0+|\delta|).$$

Then $g(\theta_0+\delta) = g_1(\delta)$ for $\delta \leqslant 0$ and $g(\theta_0+\delta) = g_2(\delta)$ for $\delta \geqslant 0$ and instead of the initial problem (2.5) we can consider the following initial value problems:

$$\ddot{\delta}+g_1(\delta) = 0, \tag{3.2}$$
$$\delta(0) = \theta_1-\theta_0, \quad \dot{\delta}(0) = 0$$

and

$$\ddot{\delta}+g_2(\delta) = 0, \tag{3.3}$$
$$\delta(0) = \theta_2-\theta_0, \quad \dot{\delta}(0) = 0.$$

Put

$$a_1 = \theta_0-\theta_1, \quad a_2 = \theta_2-\theta_0. \tag{3.4}$$

In view of the inequalities $-\pi-\theta_0 < \theta_1 < \theta_0$ and $\theta_0 < \theta_2 < \pi-\theta_0$, by (3.1), we get

$$\delta\cdot g_i(\delta) > 0 \quad \text{for} \quad 0 < |\delta| \leqslant a_i.$$

Thus the initial-value problems (3.2) and (3.3) have periodic solutions describing the oscillations about the point $\delta = 0$ with the amplitudes a_1 and a_2, respectively (see [4], § 42).

Denote by T_1 and T_2 the periods of those oscillations. It is easy to prove that

$$T = \tfrac{1}{2}(T_1+T_2), \tag{3.5}$$

where T is given by formula (1.6).

Put

$$\omega = \frac{2\pi}{T}, \quad \omega_1 = \frac{2\pi}{T_1}, \quad \omega_2 = \frac{2\pi}{T_2} \tag{3.6}$$

and suppose that

$$\omega_1 \approx \omega_1', \quad \omega_2 \approx \omega_2', \tag{3.7}$$

where ω_1' and ω_2' are any approximations of ω_1 and ω_2. Let ω' be defined by the equation

$$\frac{1}{\omega'} = \frac{1}{2}\left(\frac{1}{\omega_1'}+\frac{1}{\omega_2'}\right). \tag{3.8}$$

This yields an approximation $\omega \approx \omega'$ with the following estimate for the error:

$$\frac{|\omega-\omega'|}{\omega} \leqslant \frac{\omega_2'}{\omega_1'+\omega_2'}\cdot\frac{|\omega_1-\omega_1'|}{\omega_1}+\frac{\omega_1'}{\omega_1'+\omega_2'}\cdot\frac{|\omega_2-\omega_2'|}{\omega_2}. \tag{3.9}$$

In fact, by (3.5) and (3.6) we have

$$\frac{1}{\omega} = \frac{1}{2}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)$$

and by (3.8) it follows that

$$\frac{\omega-\omega'}{\omega} = \frac{\omega_2'}{\omega_1'+\omega_2'}\cdot\frac{\omega_1-\omega_1'}{\omega_1}+\frac{\omega_1'}{\omega_1'+\omega_2'}\cdot\frac{\omega_2-\omega_2'}{\omega_2},$$

which implies (3.9).

4. The approximations for ω_1 and ω_2. The considerations of the preceding sections permit the use of the well-known approximate formulae

(4.1) $$\omega_i^2 \approx (\omega_i')^2 = \frac{1}{\pi a_i}\int_0^{2\pi} g_i(a_i \sin\alpha)\sin\alpha\, d\alpha$$

obtained by asymptotic methods (cf. [3], § 2). In those formulae the functions g_i are given by (3.1) and a_i are the amplitudes of the oscillations determined by (3.2), (3.3) and (3.4) (see [10]; [11], § 1, § 4).

In view of (2.1) and (3.1) we obtain for $g_i(\delta)$ the following expressions:

(4.2) $$g_i(\delta) = \sin\delta\cos\theta_0 - (-1)^i \operatorname{sgn}\delta\cdot(1-\cos\delta)\sin\theta_0; \qquad i = 1, 2.$$

Substituting (4.2) in (4.1), we get

$$(\omega_i')^2 = (\pi a_i)^{-1}\cos\theta_0 \int_0^{2\pi} \sin(a_i\sin\alpha)\sin\alpha\, d\alpha -$$
$$-(-1)^i(\pi a_i)^{-1}\sin\theta_0 \int_0^{2\pi} [1-\cos(a_i\sin\alpha)]\,|\sin\alpha|\, d\alpha.$$

After calculations with the use of integral tables ([13], 3.524.5; 3.524.3) we get the following expressions:

(4.3) $$(\omega_i')^2 = A_1(a_i)\cos\theta_0 - (-1)^i A_2(a_i)\sin\theta_0,$$

where

(4.3′) $$A_1(a) = \frac{2J_1(a)}{a} = \sum_{n=0}^{\infty}(-1)^n \frac{a^{2n}}{2^{2n} n!(n+1)!},$$
$$A_2(a) = \frac{4}{\pi}\sum_{n=0}^{\infty}(-1)^n \frac{a^{2n+1}}{(2n+1)!!(2n+3)!!}.$$

We thus have the required approximations for ω_1 and ω_2.

In [12], § 3, the estimates of the errors

$$e_i = \frac{\omega_i' - \omega_i}{\omega_i}$$

have been found as

(4.4) $$0 \leqslant e_i \leqslant 0{,}43\varrho_i^{5/2}\omega_i'(\Omega_i^4 - \omega_i'^4),$$

where

(4.5) $$\Omega_i^4 = \frac{1}{\pi a_i^2}\int_0^{2\pi} g_i^2(a_i\sin\alpha)\, d\alpha,$$

and the constants ϱ_i will be given later. (In [12] an other notation was used: $g_i = g$, $\varrho_i \equiv r^{-1}$, $\omega_i' \equiv \omega_{as}$ and $\Omega_i \equiv \omega_{op}$.)

By substituting (4.2) in (4.5) we have

(4.6) $\Omega_i^4 = B_{11}(a_i)\cos^2\theta_0 - (-1)^i 2B_{12}(a_i)\sin\theta_0\cos\theta_0 + B_{22}(a_i)\sin^2\theta_0,$

where we write

$$B_{11}(a) = \frac{1}{\pi a^2}\int_0^{2\pi} \sin^2(a\sin\alpha)\,d\alpha,$$

$$B_{22}(a) = \frac{1}{\pi a^2}\int_0^{2\pi} [1-\cos(a\sin\alpha)]^2\,d\alpha,$$

$$B_{12}(a) = \frac{1}{\pi a^2}\int_0^{2\pi} [1-\cos(a\sin\alpha)]\sin|a\sin\alpha|\,d\alpha.$$

These integrals, by well-known formulae (cf. [13], 3.528,3; 3.421.1), are equal to:

$$B_{11}(a) = \frac{1}{a^2}[1-J_0(2a)],$$

(4.7) $$B_{22}(a) = \frac{1}{a^2}[3-4J_0(a)+J_0(2a)],$$

$$B_{12}(a) = \frac{4}{\pi}\sum_{n=0}^{\infty}(-1)^n\frac{4^{n+1}-1}{[(2n+3)!!]^2}a^{2n+1}.$$

Then, by (4.6) and (4.7) we calculate Ω_i.

Now, the constants ϱ_i in (4.4) are (see [12], § 2)

(4.8) $$\varrho_i = \frac{1}{2}\sup_{(0,a_i)}\frac{a_i^2-x^2}{G_i(a_i)-G_i(x)}, \qquad i = 1, 2,$$

where

$$G_i(x) = \int_0^x g_i(x)\,dx.$$

From the expression for $g_2(x)$, by (4.2), it can be verified that $g_2(x)$ represents a soft control-function. In this case the equality

$$\sup_{(0,a_2)}\frac{a_2^2-x^2}{G_2(a_2)-G_2(x)} = \lim_{x\to a_2}\frac{a_2^2-x^2}{G_2(a_2)-G_2(x)}$$

holds (see [11], § 4). Hence

(4.9) $$\varrho_2 = \frac{a_2}{\sin\theta_2-\sin\theta_0}.$$

The determination of ϱ_1 may be more involved. If $a_1 \leqslant \theta_0$, the function $g_1(x)$ represents a hard control-function in the interval $\langle -a_1, a_1 \rangle$. Therefore

$$\sup_{(0,a_1)} \frac{a_1^2 - x^2}{G_1(a_1) - G_1(x)} = \lim_{x\to 0} \frac{a_1^2 - x^2}{G_1(a_1) - G_1(x)}$$

(see [11], § 4) and by (4.8) and (4.2) we have

$$\varrho_1 = \frac{a_1^2}{2(\cos\theta_0 - \cos\theta_1 + a_1 \sin\theta_0)}. \tag{4.10}$$

If $a_1 > \theta_0$, the function $(a_1^2 - x^2)/[G_1(a_1) - G_1(x)]$ may attain its maximal value at an interior point of the interval $\langle -a_1, 0 \rangle$. Then it is necessary to solve a transcendental equation. In this case, however, instead of determining ϱ_1 we can use the inequality

$$\varrho_1 \leqslant \frac{a_1}{2\sin\frac{1}{2}a_1 \min[\cos\theta_0, \cos(\theta_0 - \frac{1}{2}a_1)]}, \tag{4.11}$$

which follows simply from the estimate

$$\sup_{(0,a_1)} \frac{a_1^2 - x^2}{G_1(a_1) - G_1(x)} \leqslant \sup_{(0,a_1)} \frac{2x}{g_1(x)}.$$

In this way we have prepared all the formulae needed for finding the approximate values of ω_1 and ω_2, and for estimating the errors.

5. Numerical example. As an illustration of the method given in the preceding sections let us assume in the initial-value problem (2.5) the following numerical data:

$$\theta_0 = 0{,}5, \quad \theta_1 = 0{,}25. \tag{5.1}$$

First we check that conditions (2.3) and (2.4′) hold. Thus the initial-value problem (2.5) has a periodic solution.

In order to estimate the corresponding cyclic frequency ω we calculate the right turning-point of the oscillations, using the iteration-process (2.9) for example. Substituting the numerical values (5.1) we find

$$a^{(5)} = 0{,}256\,024, \quad q^{(5)} = 0{,}506\,024.$$

The next iteration leaves the above six decimals unchanged. Hence, by 2(ii), we may assume

$$\theta_2 = q^{(5)} + a^{(5)} = 0{,}762\,048 \tag{5.2}$$

with accuracy 10^{-6}. Now, from (3.4) we get

$$a_1 = 0{,}250\,000, \quad a_2 = 0{,}262\,048 \tag{5.3}$$

and consequently, by (4.3′), we calculate

$$A_1(a_1) = 0{,}9922080, \qquad A_2(a_1) = 0{,}1056620,$$

$$A_1(a_2) = 0{,}9914413, \qquad A_2(a_2) = 0{,}1107085.$$

Using (4.3) we obtain

$$\omega_1'^2 = 0{,}9214015, \quad \omega_2'^2 = 0{,}8169947$$

or

(5.4) $$\omega_1' = 0{,}9598966, \quad \omega_2' = 0{,}9038776.$$

Finally formula (3.8) gives

(5.5) $$\omega' = 0{,}931045,$$

which is an approximation of the frequency ω of oscillations. In order to estimate the error of that approximation we calculate by (4.7) and (5.3) the numerical values

$$B_{11}(a_1) = 0{,}9844830, \quad B_{11}(a_2) = 0{,}9829637,$$

$$B_{12}(a_1) = 0{,}1047841, \quad B_{12}(a_2) = 0{,}1096985,$$

$$B_{22}(a_1) = 0{,}0116174, \quad B_{22}(a_2) = 0{,}0127527.$$

Then formula (4.6) yields

$$\Omega_1^4 = 0{,}8490439, \quad \Omega_2^4 = 0{,}6676539.$$

Now, by (5.4), the estimation formulae (4.4) give

(5.6) $$0 \leqslant e_1 \leqslant 0{,}0000261 \cdot \varrho_1^{5/2}, \quad 0 \leqslant e_2 \leqslant 0{,}0000675 \cdot \varrho_2^{5/2}.$$

The numerical values of ϱ_i are calculated by substituting in formulae (4.9) and (4.10) the numerical data (5.1), (5.2) and (5.3). We get

$$\varrho_1 = 1{,}0954750, \quad \varrho_2 = 1{,}2426466.$$

Hence

$$0 \leqslant e_1 \leqslant 0{,}0000328, \quad 0 \leqslant e_2 \leqslant 0{,}0001161.$$

Finally, by (5.4), formula (3.9) gives

(5.7) $$0 \leqslant \frac{\omega' - \omega}{\omega} \leqslant 0{,}000076$$

and

(5.8) $$\omega = 0{,}931045 - \vartheta \cdot 0{,}000071,$$

where $0 \leqslant \vartheta \leqslant 1$.

More accurate computations performed by the Department of Numerical Methods at Wrocław University show the quality of the estimate given by (5.8). By the use of the electronic computer Elliott 803 the values of the integral (1.6) have been tabulated and those computations have shown that for $\theta_0 = 0{,}5$, $a = 0{,}256024$,

$$\omega = 0{,}931018 15.$$

Consequently

$$\omega' - \omega = 0{,}000027,$$

which shows that the actual error is 2,7 times smaller than its estimate given by (5.8).

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O CZĘSTOŚCI DRGAŃ GENERATORA SYNCHRONICZNEGO

STRESZCZENIE

W pracy bada się równanie

$$\ddot{\theta} + \sin\theta - \sin\theta_0 = 0, \tag{1.1}$$

gdzie θ_0 jest stałą należącą do przedziału otwartego $(0, \pi/2)$, $\theta = \theta(t)$ jest funkcją poszukiwaną. Równanie (1.1) występuje w teorii generatora synchronicznego. Praca zawiera wzory dla przybliżonego obliczania okresu drgań określonych przez równanie (1.1), a także wzory pozwalające oszacować dokładność przybliżeń. Na zakończenie podany jest przykład numeryczny.

А. РЫБАРСКИ и Е. СТШЕЛЕЦКИ (Вроцлав)

О ЧАСТОТЕ КОЛЕБАНИЙ СИНХРОННОГО ГЕНЕРАТОРА

РЕЗЮМЕ

В работе изучается уравнение

$$\ddot{\theta} + \sin\theta - \sin\theta_0 = 0, \tag{1.1}$$

где θ_0 является постоянной, принадлежащей к открытому интервалу $(0, \pi/2)$ а $\theta = \theta(t)$ искомой функцией. Уравнение это встречается в теории синхронного генератора. Приводятся формулы для приближенного вычисления частоты колебаний определяемых уравнением (1.1) а также формулы, позволяющие оценить точность получаемых приближений. Работа заканчивается численным примером.
