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A CONTINUUM WITHOUT THE FIXED POINT PROPERTY WHICH IS QUASI-HOMEOMORPHIC WITH AN AR-SET

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Below we construct a 2-dimensional locally connected continuum admitting an auto-homeomorphism without the fixed point property and which is quasi-homeomorphic with an AR-set. This is a solution of a question asked in [2].

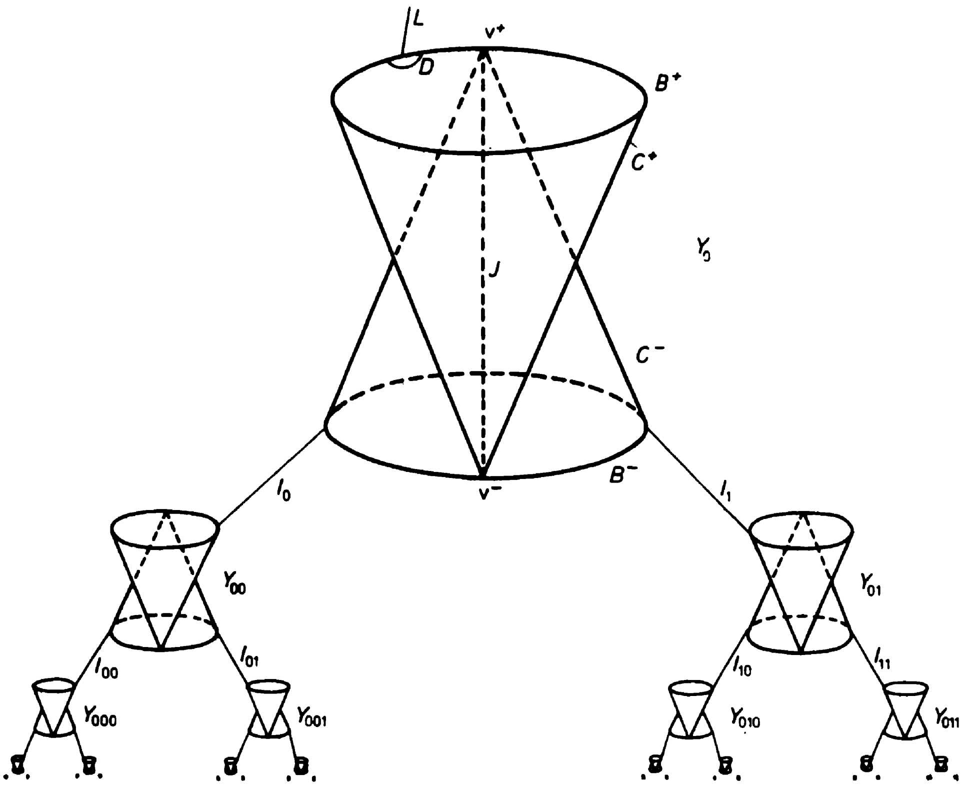
Let B^- and B^+ be two circles in the Euclidean 3-space E^3 given by the relations $x^2+y^2=4$ and $z=\pm\frac{1}{2}$. Take two cones C^- and C^+ whose bases respectively are B^- and B^+ and whose vertices are the points $v^+=(2, 0, \frac{1}{2})$ and $v^-=(-2, 0, -\frac{1}{2})$, respectively. Cones C^- and C^+ have only the line segment J, joining points v^+ and v^-, in common. Let $Y_{\varepsilon_0\varepsilon_1\varepsilon_2\ldots\varepsilon_n}$ be the homeomorphic copies of $C^-\cup C^+$ for each $n=0, 1, 2, \ldots$ where $\varepsilon_0=0$ and $\varepsilon_n\in\{0, 1\}$ for $n>1$, placed in E^3 as in Figure. Adding the Cantor set F and intervals $I_{\varepsilon_1\varepsilon_2\ldots\varepsilon_n}$ for $n=1, 2, \ldots$ and $\varepsilon_n\in\{0, 1\}$ we can obtain a continuum Y pictured in Figure.

The dendritic structure of Y obviously implies that the continuum Y is an AR-set. Observe that

(1) for each $n=1, 2, \ldots$ Y contains a continuum Y_n which is homeomorphic with Y and such that diam $Y_n<1/n$ and the set $\overline{Y\setminus Y_n}\cap Y_n$ is degenerate.

To obtain a continuum X, first we construct Bing's continuum X_0 (see [1] for the details) from the set $C^-\cup C^+$ by the rotation of the intersection of this set with any horizontal plane $z=z_0$, where $-\frac{1}{2}<z_0<\frac{1}{2}$ around the z-axis, by the angle tan πz_0. Now, we replace the part Y_0 in Y by X_0 and so we obtain the continuum X.

If we have done the construction of X enough symmetrically, the rotation around the axis of the symmetry of $X\setminus X_0$ by the angle π and a suitable homeomorphism on X_0 (which is lying in X) (compare [1] and [2]) give us a homeomorphism of X onto itself without a fixed point.



Clearly, for each $\varepsilon > 0$ there is an ε-mapping from X onto Y (we shrink strips close enough to B^+ and B^-). To find an ε-mapping from Y onto X, we first take a small closed disc D in Y_0 (as in Figure) and an ε-mapping f: $Y \to Y'$ where Y' is a one-point union of an arc L and Y such that $L \cap Y = f(\mathrm{bd}\, D)$ is degenerate and $f \mid \overline{Y \setminus D}$ is a homeomorphism. Next, we take a mapping g: $Y' \to X$ such that $g \mid \overline{Y' \setminus L}$ is a homeomorphism from $\overline{Y' \setminus L}$ onto Y_n (compare (1)) with $g(L \cap Y) = \overline{X \setminus Y_n} \cap Y_n$ and sufficiently large n; g maps the arc L onto $\overline{X \setminus Y_n}$ which is locally connected. The composition gf is an ε-mapping from Y onto X. Details are similar to those from [1]. Hence X and Y are quasi-homeomorphic.

REFERENCES

[1] R. H. Bing, *The elusive fixed point property*, The American Mathematical Monthly 76 (1969), p. 119–132.

[2] H. Patkowska, *An example of 2-dimensional quasi-homeomorphic sets*, Colloquium Mathematicum, this fascicle, p. 71–78.

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