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Dependence of meromorphic mappings

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Abstract. This note deals with a general aspect connected with dependence of meromorphic mappings of reduced complex spaces. We state results on complex m-bases and give applications concerning extension properties of fields of meromorphic functions.

1. Notions and notations, preliminary propositions

All complex spaces considered in this note are assumed reduced and of finite dimension.

1.1. Let X, Y be complex spaces. A *meromorphic mapping* from X into Y is a correspondence f, i.e. a set valued mapping, given by a graph $G_f \subset X \times Y$ with the following properties:

(1) G_f is an analytic set in $X \times Y$,

(2) the projection map $\check{f}\colon G_f \to X$ is a proper modification map(1); we write $f\colon X \rightharpoonup Y$. One has the diagram

$$\begin{array}{ccc} & G_f & \longrightarrow X \times Y \\ \check{f} \swarrow & & \searrow \hat{f} \\ X & \xrightarrow[f]{} & Y \end{array}$$

where $\hat{f}$ denotes the projection map from G_f into Y. If $x \in X$, then $f(x) = \hat{f}(\check{f}^{-1}(x))$; $f(x)$ is always a non-empty compact analytic set in Y. For any subset $A \subset X$ one has $f(A) = \bigcup_{x \in A} f(x)$.

Remarks. (a) A holomorphic map $f\colon X \to Y$ is particularly a meromorphic mapping (we do not distinguish between a set consisting of one element and the element), in this case $\check{f}$ is biholomorphic.

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(1) This notion of meromorphic mapping is due to R. Remmert [9], [11]. For properties of meromorphic mappings see also [14], [15], [21].

(b) If the foregoing condition (2) is dropped, f is an *a-holomorphic correspondence* from X to Y(²); we then use the notation $f\colon X \multimap Y$. One has again $f(x) = \hat{f}(\check{f}^{-1}(x))$ for $x \in X$ and $f(A) = \bigcup_{x\in A} f(x)$ for $A \subset X$. The restriction $f|A\colon A \multimap Y$ of f is defined by setting $G_{f|A}\colon = G_f \cap (A \times Y)$. If (2) is replaced by the weaker condition that the projection map $\check{f}$ is merely a proper map, $f\colon X \multimap Y$ is said to be *c-a-holomorphic.* Hence every meromorphic mapping is a *c-a*-holomorphic correspondence.

A point $x \in X$ is called a *singularity* of the meromorphic mapping $f\colon X \rightharpoonup Y$ if there is no open neighbourhood U of x such that $f|U$ is a holomorphic map; x is a *point of indeterminacy* of f if $\operatorname{card} f(x) > 1$. The set S_f of singularities of f is a nowhere dense analytic set in X.

EXAMPLE. Define $f\colon C^2 \rightharpoonup P_1$ by $(z_1, z_2) \mapsto z_1 : z_2$ for $(z_1 z_2) \neq (0, 0)$ and $(0, 0) \mapsto P_1$. Then $G_f = \{(z_1, z_2, u_1 : u_2) \in C^2 \times P_1 \colon z_1 u_2 - z_2 u_1 = 0\}$, $S_f = \{(0, 0)\}$; $(0, 0)$ is a point of indeterminacy of f.

A *meromorphic function* φ on the complex space X is a meromorphic mapping $\varphi\colon X \rightharpoonup P_1$ such that $\varphi(X) \neq \emptyset$ and $\varphi(U)$, for every open subset U of X, does not reduce to the point $\infty = (1:0)$ of P_1. The meromorphic functions on a non-empty complex space X form a ring $K(X)$ (the rational operations in $K(X)$ being defined adequately); if, moreover, X is irreducible, then $K(X)$ is a field.

1.2. Let $f\colon X \multimap Y$ be an a-holomorphic correspondence.

f is called *reducible* resp. *irreducible* if G_f is reducible resp. irreducible. If f is reducible and $G_f = \bigcup_j G^{(j)}$ is the decomposition of G_f into irreducible components, the a-holomorphic correspondences f_j given by $G_{f_j} := G^{(j)}$ are called the *irreducible components* of f. If f is c-a-holomorphic, then so are the f_j.

For any point $y \in f(X)$ the set $f^{-1}(y) = \check{f}(\hat{f}^{-1}(x))$ is a non-empty analytic set in X, $f^{-1}(y)$ is called the *fibre* of f over y.

If $f(X) \neq \emptyset$ the (global) *rank* $\operatorname{rk} f$ of f equals, by definition, the maximum of the local codimensions of the fibres of the holomorphic map $\hat{f}\colon G_f \to Y$, hence we have in this case

$$\operatorname{rk} f = \operatorname{rk} \hat{f} = \operatorname*{Max}_{z \in G_f} \left(\dim_z G_f - \dim_z \hat{f}^{-1}(\hat{f}(z))\right);$$

if $f(X) = \emptyset$, we set $\operatorname{rk} f\colon = -1$.

Presume now $f\colon X \multimap Y$ c-a-holomorphic. Then f is said to be *semi-proper* (*quasi-proper*, resp.) if, given a compact set C in Y, there is a compact set C' in X such that C' is met by every fibre $f^{-1}(y)$ (by every irre-

(²) As to properties of holomorphic correspondences cf. [14], [16].

ducible component of every fibre $f^{-1}(y)$, resp.), where $y \in C \cap f(X)$ [3]. If the union of the fibres $f^{-1}(y)$, $y \in C \cap f(X)$ for every compact $C \subset Y$, is compact, then f is *proper*.

1.3. Assume $X \neq \emptyset$. Let $f_\nu\colon X \rightharpoonup Y_\nu$ ($\nu = 1, \dots, n$; $n \geqslant 2$) be meromorphic mappings. Define

$$(f_1, \dots, f_n)\colon X \multimap Y_1 \times \dots \times Y_n$$

by

$$x \mapsto f_1(x) \times \dots \times f_n(x) \qquad (x \in X).$$

$(f_1, \dots, f_n)$ is generally not a meromorphic mapping, yet it is always a *c-a*-holomorphic correspondence. But there is a unique meromorphic mapping

$$[f_1, \dots, f_n]\colon X \rightharpoonup Y_1 \times \dots \times Y_n$$

such that

$$G_{[f_1, \dots, f_n]} \subset G_{(f_1, \dots, f_n)}.$$

$[f_1, \dots, f_n]$ is called the *meromorphic function* of $f_1, \dots, f_n$.

Assume now, moreover, that X is irreducible. Define:

(a) $f_2\colon X \rightharpoonup Y$ is *dependent* on $f_1\colon X \rightharpoonup Y_1$, written $f_2 \operatorname{dep} f_1 :\Leftrightarrow \operatorname{rk}[f_1, f_2] = \operatorname{rk} f_1$.

(b) f_1, f_2 are *related*, written $f_1 \operatorname{rel} f_2$: $\Leftrightarrow f_2 \operatorname{dep} f_1$ and $f_1 \operatorname{dep} f_2$.

$f_2 \operatorname{dep} f_1$ holds if and only if there is a non-empty open set U in X such that $f_1|U$ and $f_2|U$ are holomorphic maps and $f_2|U$ is constant on the connected components of the fibres of $f_1|U$. If $f_1 \operatorname{rel} f_2$, then particularly $\operatorname{rk} f_1 = \operatorname{rk} f_2$.

Define $K_{f_1}(X) := \{\varphi \in K(X)\colon \varphi \operatorname{dep} f_1\}$. Then $K_{f_1}(X)$ is a subfield of $K(X)$.

1.4. Let $f\colon X \rightharpoonup Y$, $g\colon Y \rightharpoonup Z$ be meromorphic mappings, assume X, Y, Z non-empty and irreducible. Define

$$g \circ f\colon X \multimap Z,$$

by

$$x \mapsto g\big(f(x)\big) \qquad (x \in X).$$

$g \circ f$ is a *c-a*-holomorphic correspondence but generally not a meromorphic mapping. But if there is a non-empty open subset U in X such that $g \circ f|U\colon U \multimap Z$ is a map (in the usual sense), then there is a unique meromorphic mapping $h\colon X \rightharpoonup Z$ whose graph G_h is contained in the graph $G_{g \circ f}$. We write $h = g \triangle f$; $g \triangle f$ is called the *meromorphic composition* of f and g.

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[3] The concept of a semiproper map is due to N. Kuhlmann [4], [5], the notion quasi-proper was introduced by A. Andreotti and W. Stoll [1].

"$g \triangle f$ exists" always includes that there is an open set $U \subset X$ as just described.

We have the following propositions:

1.4.1. *Sufficient conditions for the existence of* $g \triangle f$ *are*:

(i) $f(X) \not\subset S_g$,

(ii) $\operatorname{rk} f = \dim Y$,

(iii) f *is surjective* (*i.e.* $f(X) = Y$).

1.4.2. *If* $g \triangle f$ *exists, then* $g \triangle f \operatorname{dep} f$.

Let g particulary be a meromorphic function ψ: $Y \rightharpoonup P_1$, assume Y irreducible. Furthermore, assume (ii) $\operatorname{rk} f = \dim Y$ or (iii) f surjective, then $\psi \triangle f$: $X \rightharpoonup P_1$ exists and is a meromorphic function. Moreover, the assignment

$$f^*: K(Y) \to K(X)$$

given by

$$\psi \mapsto \psi \triangle f$$

is a monomorphism of fields. Hence $f^* K(Y)$ is a subfield of $K_f(X)$, and there arises the question

(*) *When is* $K_f(X)$ *finite algebraic over* $f^* K(Y)$?

This general problem was proposed by A. Andreotti and W. Stoll [1] recently; special cases are classic and have been treated by many authors. We will give here some statements with respect to that question.

1.5. Let $f_m: X \rightharpoonup Y_m$, $f: X \rightharpoonup Y$ be meromorphic mappings, presume X, Y_m, Y non-empty and X irreducible. Define:

(1) f_m *m-majorizes uniquely* f: $\Leftrightarrow$ There is a unique meromorphic mapping a_f: $Y_m \rightharpoonup Y$ such that $f = a_f \triangle f_m$; a_f is called induced by f (regarding f_m).

(2) f_m is *m-maximal* $:\Leftrightarrow f_m$ m-majorizes uniquely every meromorphic mapping which is dependent on f_m.

Proposition 1.5.1. *f_m is m-maximal* $\Rightarrow$ *f_m is surjective.*

Proof. Suppose f_m not surjective. Then choose $y_0 \in Y_m \setminus f_m(X)$, set $Z := Y_m \setminus \{y_0\}$ and define f'_m: $X \rightharpoonup Z$ by restriction of f_m (i.e. $f'_m(x) := f_m(x) \subset Z$). f'_m is dependent on f_m, hence there is the induced meromorphic mapping $a_{f'_m}$: $Y_m \rightharpoonup Z$ with $f'_m = a_{f'_m} \triangle f_m$. Furthermore, one has $I^Z_{Y_m} \triangle f'_m = f_m$ ($I^Z_{Y_m}$: $Z \to Y_m$ the inclusion map), hence $f_m = I^Z_{Y_m} \triangle (a_{f'_m} \triangle f_m) = (I^Z_{Y_m} a_{f'_m}) \triangle f_m$. On the other hand $f_m = I_{Y_m} \triangle f_m$ (I_{Y_m} the identy map of Y_m); since f_m is m-maximal it follows $I_{Y_m} = I^Z_{Y_m} \triangle a_{f'_m}$. But this is a contradiction because I_{Y_m} is surjective and $I^Z_{Y_m}$ is not.

Conclusion. *If* f_m: $X \rightharpoonup Y_m$ *is m-maximal* (*X is presumed irreducible*), *then* Y_m *is irreducible. Therefore* $K(Y_m)$ *is a field.*

The next proposition gives a special contribution to the question above.

PROPOSITION 1.5.2. *f_m is m-maximal $\Rightarrow K_{f_m}(X) = f_m^* K(Y_m)$.*

Proof. We have $K_{f_m}(X) \supset f_m^* K(Y_m)$. On the other hand, if $\varphi \in K_{f_m}(X)$, there is the induced meromorphic mapping a_φ: $Y_m \rightharpoonup P_1$ with $\varphi = a_\varphi \triangle f_m$. Then $a_\varphi \in K(Y_m)$ and $\varphi = f_m^* a_\varphi$, therefore $\varphi \in f_m^* K(Y_m)$. It follows $K_{f_m}(X) \subset f_m^* K(Y_m)$, hence $K_{f_m}(X) = f_m^* K(Y_m)$.

We recall a further definition:

(3) Let f_m be m-maximal, presume f_m rel f. Then the pair (f_m, Y_m) is called a *complex m-base with respect to f*, and Y_m is the *base space* of the complex m-base.

Two complex m-bases (f_m, Y_m) and $(\tilde{f}_m, \tilde{Y}_m)$ with respect to the same meromorphic mapping f: $X \rightharpoonup Y$ are "bimeromorphically equivalent": One has the meromorphic mappings a_f: $Y_m \rightharpoonup Y$, $a_{\tilde{f}_m}$: $Y_m \rightharpoonup \tilde{Y}_m$ induced by $f, \tilde{f}_m$ regarding f_m, furthermore the meromorphing mappings $\tilde{a}_f$: $\tilde{Y}_m \rightharpoonup Y$, $\tilde{a}_{f_m}$: $\tilde{Y}_m \rightharpoonup Y_m$ induced by f, f_m regarding $\tilde{f}_m$. Then $\tilde{a}_{f_m} \triangle a_{\tilde{f}_m} = I_{Y_m}$, $a_{\tilde{f}_m} \triangle \tilde{a}_{f_m} = I_{\tilde{Y}_m}$ and $a_f = \tilde{a}_f \triangle a_{\tilde{f}_m}$, $\tilde{a}_f = a_f \triangle \tilde{a}_{f_m}$.

2. Existence of complex m-bases

Let X, Y be non-empty complex spaces, presume X irreducible. Let f: $X \rightharpoonup Y$ be a meromorphic mapping.

THEOREM I. *A complex m-base (f_m, Y_m) with respect to f exists in each of the following cases:*

(1) *f is nowhere degenerated, i.e. all fibres of f are puredimensional analytic sets of dimension* $\dim X - \mathrm{rk} f$.

(2) *There exists an irreducible analytic set A in X with the following property: The c-a-holomorphic correspondence $f|A$: $A \multimap Y$ has an irreducible component f': $A \multimap Y$ which is proper and whose rank equals* $\mathrm{rk} f$.

(3) *f is quasi-proper.*

In each of these cases one has $\dim Y_m = \mathrm{rk} f$. *In cases* (2), (3) *the meromorphic mapping a_f: $Y_m \rightharpoonup Y$ induced by f is proper.*

As to the proof we remark:

Case (1). One can assume that X is normal and that f is a holomorphic map. Now the corresponding statement holds for *holomorphic* maps of X which are dependent on f (cf. [13]): There is a surjective and nowhere degenerated holomorphic map $\tilde{f}$: $X \to Y$ such that $\tilde{f}$rel$f$ and that every holomorphic map Φ: $X \to Z$ dependent on f is (holomorphically) majorized by $\tilde{f}$, i.e. there is a holomorphic map β_Φ: $\tilde{Y} \to Z$ with $\Phi = \beta_\Phi \circ \tilde{f}$. Then $(\tilde{f}, \tilde{Y})$ is a complex m-base with respect to f: This can be shown by using a result of *N.* Kuhlmann [6] on complex bases of *holomorphic* maps.

Case (2) was proved in [14].

Case (3). The existence of (f_m, Y_m) can be shown similarly as in the proof of case (2) in [14]. One has to use the generalization of Remmert's mapping theorem for semiproper holomorphic maps by N. Kuhlmann [4], [5] (see also [1], [20]). A related result on the existence of complex bases with respect to holomorphic maps was obtained recently by N. Kuhlmann [7].

The proofs yield also the relation $\dim Y_m = \mathrm{rk} f$, furthermore, in cases (2), (3) the properness of a_f.

In case (1) a_f is generally not proper, but this holds again if a particular condition is satisfied:

Assume that the irreducible complex space X has countable topology and dimension n, let $f: X \rightharpoonup Y$ be nowhere degenerated and of rank r. Denote by $A_r(Y)$ the normal complex space of r-dimensional analytic prime germs of Y, let $\pi: A_r(Y) \to Y$ be the projection map. One has a lifting $'f: X \rightharpoonup A_r(Y)$ of f such that $f = \pi \circ 'f$; $'f(X)$ is an open subset of $A_r(Y)$. Every fibre $'f^{-1}('y)$, $'y \in 'f(X)$, is a pure $(n-r)$-dimensional analytic set in X and each of its irreducible components is equipped with a positive integral order with respect to $'y$. Moreover, $'f^{-1}('y)$ can be analytically triangulated ([3], [8]). Hence $'f^{-1}('y)$ is the support of a (finite or infinite) oriented simplicial $2(n-r)$-cycle $\xi('y)$. Denote by $\mathfrak{H}_k(X)$ the k-th homology group of the complex of locally finite oriented singular chains in X, then ξ determines an element $c('y)$ of $\mathfrak{H}_{2(n-r)}(X)$. $c('y)$ does not depend on the triangulation of $'f^{-1}('y)$, moreover, if $'y_1, 'y_2 \in 'f(X)$, then $c('y_1) = c('y_2)$. Hence the element $c('y) \in \mathfrak{H}_{2(n-r)}(X)$ depends only on f, we write $c('y) =: c(f)$.

Now we have

THEOREM Ia. *Assume X irreducible and of countable topology, let $f: X \rightharpoonup Y$ be nowhere degenerated. Presume $c(f) \neq 0$. Then the base space Y_m of any complex m-base (f_m, Y_m) with respect to f is compact.*

This implies particularly that the induced meromorphic mapping a_f is proper in this case.

We sketch the proof:

One can start with some simplifying assumptions: Y, Y_m are normal complex spaces of dimension $\mathrm{rk} f = r$; f, f_m, a_f are holomorphic maps with f surjective, f_m nowhere degenerated and a_f discrete. Then $A_r(Y)$ can be identified with Y and $'f$ with f. Suppose now Y_m not compact. Choose a point $y_0 \in Y$, then $a_f^{-1}(y_0) = \{y^{(\nu)}\}$ is a non-empty countable set without a point of accumulation in Y_m. Each $y^{(\nu)}$ can be connected with the ideal boundary of Y_m by a piecewise real-analytic simple arc with support W_ν in Y_m such that the W_ν are disjoint and that every compact set in Y_m is met by utmost finitely many W_ν. The set $\tilde{W} :=$

$\bigcup_\nu f_m^{-1}(W_\nu) \subset X$ can be triangulated. It follows that $\tilde{W}$ is the support of an oriented simplicial $\big(2(n-r)+1\big)$ — chain whose boundary is a chain (with support $f^{-1}(y_0)$) representing $c(y_0) = c(f)$. But this means $c(f) = 0$ which is in contradiction with the hypothesis.

3. Applications

X, Y denote non-empty irreducible complex spaces in this section.

With respect to problem (*) p. 110 of Andreotti–Stoll Theorem I and Ia imply

THEOREM II. *Let $f\colon X \rightharpoonup Y$ be a meromorphic mapping such that* $\operatorname{rk} f = \dim Y$. *Assume that the hypothesis of case* (2) *or case* (3) *in Theorem* I *or the hypothesis of Theorem* Ia *is fulfilled. Then* $[K_f(X)\colon f^*K(Y)] < \infty$.

Proof. Let (f_m, Y_m) be a complex m-base with respect to f. The induced meromorphic mapping $a_f\colon Y_m \rightharpoonup Y$ is proper, furthermore one has $\operatorname{rk} a_f = \operatorname{rk} f$ and $\dim Y_m = \operatorname{rk} f = \dim Y$. Hence (Y_m, a_f, Y) is a proper meromorphic covering of Y with a finite number s of sheets. Then every meromorphic function $\eta \in K(Y_m)$ satisfies an equation

$$\eta^s + (\psi_1 \triangle a_f)\cdot \eta^{s-1} + \ldots + (\psi_s \triangle a_f) = 0,$$

where $\psi_\sigma \in K(Y)$ $(\sigma = 1, \ldots, s)$. This implies $[K(Y_m)\colon a_f^* K(Y)] \leqslant s$. On the other hand, $f_m^*\colon K(Y_m) \to K(X)$ is injective and one has $f_m^* K(Y_m) = K_{f_m}(X) = K_f(X)$, $f_m^*\big(a_f^* K(Y)\big) = (a_f \triangle f_m)^* K(Y) = f^* K(Y)$. Therefore $[K_f(X)\colon f^*K(Y)] \leqslant s$.

Remark. The statement that $K_f(X)$ is finite algebraic over $f^*K(Y)$ if f is quasi-proper with $\operatorname{rk} f = \dim Y$, was first proved, with a different method, by A. Andreotti and W. Stoll in [1] (it does not mean a restriction that f was assumed there holomorphic); another proof is due to N. Kuhlmann [7]. As to the case where f satisfies condition (2) in Theorem I, see [14].

The field of meromorphic functions on a non-empty compact and irreducible complex space of dimension n is, by a theorem of Chow–Thimm [2], [19] (see also [10]), isomorphic to an algebraic function field over $\boldsymbol{C}$ whose transcendence degree is utmost n. Applying this to the case where the base space Y_m of a complex m-base (f_m, Y_m) with respect to $f\colon X \rightharpoonup Y$ is compact, we obtain because of $K(Y_m) \cong K_f(X)$ the

COROLLARY OF THEOREM II. *If the hypothesis of Theorem* Ia *is fulfilled, then $K_f(X)$ is isomorphic to a finite algebraic extension of a field $\boldsymbol{C}(z_1, \ldots, z_r)$ of rational functions in r variables with $r \leqslant \operatorname{rk} f$.*

Finally, we consider special situations in which Theorem II yields well-known statements.

1) Let X be compact. Choose meromorphic functions $\varphi_\varkappa \in K(X)$ ($\varkappa = 1, \ldots, k$; $k \geqslant 1$), set $Y := \boldsymbol{P}_1^k$ and $f := [\varphi_1, \ldots, \varphi_k]$ if $k \geqslant 2$ or $f := \varphi_1$ if $k = 1$, assume $\mathrm{rk} f = k$ ($\varphi_1, \ldots, \varphi_k$ are then called *independent*; one has $k \leqslant \dim X$). f is proper, hence f fulfils the hypothesis of case (2) and of case (3) in Theorem I; furthermore f is surjective. Then $[K_f(X): f^* K(\boldsymbol{P}_1^k)] < \infty$ by Theorem II. But $K(\boldsymbol{P}_1^k) = \boldsymbol{C}(z_1, \ldots, z_k)$. Hence *$K_f(X)$ is isomorphic to an algebraic function field over $\boldsymbol{C}$ with transcendence degree k.* This is a theorem due to W. Thimm [19]; if the system $\varphi_1, \ldots, \varphi_k$ is chosen as a maximal system of independent meromorphic functions on X, then $K_f(X) = K(X)$, and one gets the theorem of Chow–Thimm.

2) X is not assumed compact. Choose again a system $\varphi_1, \ldots, \varphi_k$ ($k \geqslant 1$) of meromorphic functions on X, set as above $Y := \boldsymbol{P}_1^k$, $f := [\varphi_1, \ldots, \varphi_k]$ if $k \geqslant 2$ or $f := \varphi_1$ if $k = 1$, assume $\mathrm{rk} f = k$. Furthermore, assume that there is a point $x_0 \in X$ such that $\dim f(x_0) = k$ (x_0 is a point of indeterminacy of f of "maximal degree"). Then $f(x_0) = \boldsymbol{P}_1^k$, hence f is surjective; moreover, f fulfils the hypothesis of case (2) in Theorem I with $A := \{x_0\}$. Then $[K_f(X): f^* K(\boldsymbol{P}_1^k)] = [K_f(X): f^* \boldsymbol{C}(z_1, \ldots, z_k)] < \infty$ by Theorem II. Hence *$K_f(X)$ is isomorphic to an algebraic function field over $\boldsymbol{C}$ with transcendence degree k.* This statement is a theorem again due to W. Thimm [17], [18]. A proof of this theorem was also given by R. Remmert [12]; compare also [14], [15].

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