COLLOQUIUM MATHEMATICUM

VOL. XX 1969 FASC. 2

P R O B L È M E S

P 209, R 1. J. H. Conway and Richard K. Guy sent the following solution:

The answer is "yes". Define

$$u_0 = 0,\ u_1 = 1,\ u_{n+1} = 2u_n - u_{n-r} \quad (n \geqslant 1), \tag{1}$$

where r is the nearest integer to $\sqrt{2n}$, and

$$a_i = u_{k+2} - u_{k+2-i} \quad (1 \leqslant i \leqslant k+2). \tag{2}$$

For $k \geqslant 21$, $a_{k+2} \leqslant 2^k$, and it can be shown that the sums of the subsets of $\{a_1, a_2, \ldots, a_{k+2}\}$ are distinct, at least for $k \leqslant 40$. We conjecture, but are so far unable to prove, that this property of distinctness of sums holds for all k. Even if this were not true, the original question can still be answered in the affirmative for $k \geqslant 21$, since, from a set of $k+2$ numbers with the required property we can derive a set of $k+l+2$ such numbers, by multiplying each of the original set by 2^l and adjoining the numbers 2^{j-1} $(1 \leqslant j \leqslant l)$.

V. 1, p. 119.

Letter of April 1, 1968.

P 372, R 1. In the case of f monotone the answer is positive (1).

IX. 2, p. 240.

(1) Gordon G. Johnson, *Concerning commuting functions*, Nieuw Archief voor Wiskunde (3) 16 (1968), p. 19-24.

P 526, R 2. Another counterexample has been found (2).

XIV. p. 355, et XVII, p. 367.

(2) David M. Clark, *Varieties with isomorphic free algebras*, ce fascicule, p. 187.

P 531, R 1. A starting point for a solution has been proposed (3).

XIV, p, 355.

(3) D. A. Higgs, *Matroids and duality,* ce fascicule, p. 215.

P 555, R 1. The answer is negative even if the relation is a closed partial order (4).

XV. 2, p. 220.

(4) E. D. Tymchatyn and L. E. Ward, Jr., *On three problems of Franklin and Wallace concerning partially ordered spaces,* ce fascicule, p. 229-236.

P 556, R 1. If (X, R) is a topological lattice the answer is positive, but in general it is not (4).

XV. 2, p. 220.

P 557, R 1. E. D. Tymchatyn and L. E. Ward have given (4) some sufficient and necessary conditions for a compact quasi-ordered Hausdorff space to contain a closed chain equivalent partial order. A variety of related information is also supplied there.

XV. 2, p. 221.

P 627, R 1. C. Ryll-Nardzewski sent the following solution:

The answer is negative. Modifying slightly the example of Mycielski mentioned there we produce a chain $\mathfrak{A}_0 \prec \mathfrak{A}_1 \prec \mathfrak{A}_2 \prec \ldots$ of countable atomic compact relational structures such that $\bigcup_{n<\omega} \mathfrak{A}_n$ is not atomic compact. Let $+$ denote the ordinal addition of linear orders. Let $\alpha, \beta > 0$ be countable ordinal numbers such that if $R_\alpha = \langle \alpha, \leqslant \rangle$ and $R_\beta = \langle \beta, \leqslant \rangle$, then $R_\alpha \prec R_\alpha + R_\beta$. We put $R_{\beta,0} = \langle 1, \leqslant \rangle$, $R_{\beta,n+1} = R_\beta + R_{\beta,n}$ and $\mathfrak{A}_n = R_\alpha + R_{\beta,n}$. Hence $\mathfrak{A}_{n+1}$ is obtained from $\mathfrak{A}_n$ by inserting a segment of type β right after the initial segment of type α of $\mathfrak{A}_n$. Hence $\bigcup_{n<\omega} \mathfrak{A}_n$ is not well ordered and not atomic compact, while $\mathfrak{A}_n$ are of course countable and atomic compact and the relation $\mathfrak{A}_n \prec \mathfrak{A}_{n+1}$ is easy to check (extending a segment of a linear order $\mathfrak{A}$ by an elementary extension of this segment always produces an elementary extension of $\mathfrak{A}$).

XIX. 1, p. 34.

Letter of May 25, 1968.

P 654, R 1. A positive answer has been given (5).

XX, p. 126.

(5) W. Żelazko, *On m-convex B_0-algebras of type ES*, ce fascicule, p. 299-304.

S. FAJTLOWICZ et E. MARCZEWSKI (WROCŁAW)

P 665-P 667. Formulés dans la communication *On some properties of the family of independent sets in abstract algebras.*

Ce fascicule, p. 190 et 191.

D. A. HIGGS (WATERLOO, ONTARIO, CANADA)

P 668. Formulé dans la communication *Matroids and duality.*

Ce fascicule, p. 220.

A. C. SHERSHIN (TAMPA, FLORIDA)

P 669 and **P 670.** Formulés dans la communication *Algebraic results concerning Green's $\mathscr{H}$-slices.*

Ce fascicule, p. 225.

A. R. BEDNAREK (GAINSVILLE, FLORIDA)

P 671. Formulé dans la communication *A note on the least element map.*

Ce fascicule, p. 228.

L. FILIPCZAK (ŁÓDŹ)

P 672. Formulé dans la communication *Exemple d'une fonction continue privée de dérivée symétrique partout.*

Ce fascicule, p. 253.

S. J. TAYLOR (LONDON)

P 673. A set $E \subset R^n$ is called *polar* for a Markov process $X(t, \omega)$ if $P_x\ \{[t > 0; X(t, \omega) \epsilon E] = \emptyset\}$ is zero for all $x \epsilon R^n$. When $X(t, \omega)$ is a process with independent increments with a symmetric stable distribution of order a, it is known that E is polar of and inly if E has zero Riesz capacity of order $n-a$. Are the polar sets the same for all stable distributions of order a in R^n?

New Scottish Book, Probl. 818, 9. V. 1968

Z. CIESIELSKI (SOPOT)

P 674. Take a partition $0 = t_0 < \ldots < t_n = 1$ and consider the set X_n of all continuous functions on $[0, 1]$ which are linear in every interval (t_{i-1}, t_i). Suppose p and q satisfy the conditions $1 < p < \infty$, $0 < q \leqslant \delta_{i+1}/\delta_i \leqslant 1$, where $\delta_i = t_i - t_{i-1}$, $1 \leqslant i \leqslant n$.

Choose in X the orthonormal Franklin basis (f_i), $i = 0, 1, \ldots, n$(6), and show that for linear transformations $T\colon X_n \to X_n$ for which $Tf_i = \pm f_i$ there exist constants $A(p, q)$ and $B(p, q)$ such that for $x \in X_n$

$$A(p, q)\int_0^1 |Tx(t)|^p\,dt \leqslant \int_0^1 |x(t)|^p\,dt \leqslant B(p, q)\int_0^1 |Tx(t)|^p\,dt.$$

New Scottish Book, Probl. 819, 9. V. 1968.

(6) Z. Ciesielski, *Properties of the orthonormal Franklin system*, Studia Mathematica 23 (1963), p. 141-157; especially p. 143.

J. SICIAK (KRAKÓW)

P 675. Let E be a compact subset of the complex plane with the positive logarithmic capacity. Does there exist a compact subset F of E which is not thin (in the sense of the theory of potential) at any point of F?

New Scottish Book, Probl. 820, 1. VI. 1968.

W. MLAK (KRAKÓW)

P 676. Let $T_1, \ldots, T_n$ be contractions (i.e. $\|T_i\| \leqslant 1$ for $i = 1, 2, \ldots, n$) in a complex Hilbert space and let T be a given polynom $w(z_1, \ldots, z_n)$ of variables $z_1, \ldots, z_n$. Is it true that if $T_iT_k = T_kT_i$ for $i, k = 1, 2, \ldots, n$, then

$$\|w(T_1, \ldots, T_n)\| \leqslant \sup_{\substack{|z_i| \leqslant 1 \\ i=1,\ldots,n}} |w(z_1, \ldots, z_n)|\,?$$

For $n = 1, 2$ the answer is positive (7).

New Scottish Book, Probl. 817, 8. V. 1968.

(7) B. Sz.-Nagy et C. Foiaş, *Analyse harmonique des opérateurs de l'espace de Hilbert*, Budapest 1967; see chapters I and III.