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ON SUPER (a, d)-EDGE ANTIMAGIC TOTAL LABELING OF CERTAIN FAMILIES OF GRAPHS

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Abstract

A (p, q)-graph G is (a, d)-edge antimagic total if there exists a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}$ such that the edge weights $\Lambda(uv) = f(u) + f(uv) + f(v)$, $uv \in E(G)$ form an arithmetic progression with first term a and common difference d. It is said to be a super (a, d)-edge antimagic total if the vertex labels are $\{1, 2, \ldots, p\}$ and the edge labels are $\{p + 1, p + 2, \ldots, p + q\}$. In this paper, we study the super (a, d)-edge antimagic total labeling of special classes of graphs derived from copies of generalized ladder, fan, generalized prism and web graph.

Keywords: edge weight, magic labeling, antimagic labeling, ladder, fan graph, prism and web graph.

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1. Introduction

By a *graph G* we mean a finite, undirected, connected graph without any loops or multiple edges. Let $V(G)$ and $E(G)$ be the set of vertices and edges of a graph G, respectively. The *order* and *size* of a graph G is denoted as $p = |V(G)|$ and $q = |E(G)|$ respectively. For general graph theoretic notions we refer Harrary [6].

By a *labeling* we mean a one-to-one mapping that carries the set of graph elements onto a set of numbers (usually positive or non-negative integers), called *labels*. There are several types of labelings and a detailed survey of many of them can be found in the dynamic survey of graph labeling by Gallian [5].

Kotzig and Rosa [9] introduced the concept of *magic labeling*. They define an *edge magic total labeling* of a (p, q)-graph G as a bijection f from $V(G) \cup E(G)$ to the set $\{1, 2, \ldots, p + q\}$ such that for each edge $uv \in E(G)$, the edge weight $f(u) + f(uv) + f(v)$ is a constant.

Enomoto *et al.* [3] defined a *super edge magic labeling* as an edge magic total labeling such that the vertex labels are $\{1, 2, \ldots, p\}$ and edge labels are $\{p + 1, p + 2, \ldots, p + q\}$. They have proved that if a graph with p vertices and q edges is super edge magic then, $q \leq 2p - 3$. They also conjectured that every tree is super edge magic.

As a natural extension of the notion of edge magic total labeling, Hartsfield and Ringel [7] introduced the concept of an *antimagic labeling* and they defined an *antimagic labeling* of a (p, q)-graph G as a bijection f from $E(G)$ to the set $\{1, 2, \ldots, q\}$ such that the sums of label of the edges incident with each vertex $v \in V(G)$ are distinct.

Simanjuntak *et al.* [10] defined an (a, d)-*edge antimagic total labeling* as a one to one mapping f from $V(G) \cup E(G)$ to $\{1, 2, \ldots, p + q\}$ such that the set of edge weight $\{f(u) + f(uv) + f(v) : uv \in E(G)\}$ is equal to $\{a, a + d, a + 2d, \ldots, a + (q - 1)d\}$ for any two integers $a > 0$ and $d \geq 0$.

An (a, d)-edge antimagic total labeling of a (p, q)-graph G is said to be *super* (a, d)-*edge antimagic total* if the vertex labeles are $\{1, 2, \ldots, p\}$ and the edge labeles are $\{p + 1, p + 2, \ldots, p + q\}$. The super $(a, 0)$-edge antimagic total labeling is usually called as super edge magic in the literature (see [3, 4]).

An (a, d)-*edge antimagic vertex labeling* of a (p, q)-graph G is defined as a one to one mapping f from $V(G)$ to the set $\{1, 2, \ldots, p\}$ such that the set of edge weight $\{f(u) + f(v) : uv \in E(G)\}$ is equal to $\{a, a + d, a + 2d, \ldots, a + (q - 1)d\}$ for any two integers $a > 0$ and $d \geq 0$.

In [2] Bača *et al.* proved that if a (p, q)-graph G has an (a, d)-edge antimagic vertex labeling then $d(q - 1) \leq 2p - 1 - a \leq 2p - 4$.

Also in [1] Bača and Barrientos proved the following: if a graph with q edges and $q + 1$ vertices has an α-labeling, then it has an $(a, 1)$-edge antimagic vertex labeling. A tree has $(3, 2)$-edge antimagic vertex labeling if and only if it has an α-labeling and the number of vertices in its two partite set differ by at most 1. If a tree with at least two vertices has a super (a, d)-edge antimagic total labeling, then d is at most 3. If a graph has an $(a, 1)$-edge antimagic vertex labeling, then it also has a super $(a_1, 0)$-edge antimagic total labeling and a super $(a_2, 2)$-edge antimagic total labeling.

In [12] Sugeng *et al.* studied the super (a, d)-edge antimagic total properties of ladders, generalized prisms and antiprisms.

We make use of the following lemmas for our further discussion.

Lemma 1. *If a (p, q)-graph G is super (a, d)-edge antimagic total, then* $d \leq \frac{2p+q-5}{q-1}$.

Lemma 2. *If a (p, q)-graph G has an $(a, 1)$-edge antimagic vertex labeling and odd number of edges, then it has a super $(a', 1)$-edge antimagic total labeling, where* $a' = a + p + \frac{q+1}{2}$.

Lemma 3. *If a (p, q)-graph G has an (a, d)-edge antimagic vertex labeling, then G has a super (a', d')-edge antimagic total labeling, where $a' = a + p + 1$ and $d' = d + 1$ or $a' = a + p + q$ and $d' = d - 1$.*

Lemma 2 appeared in [11] and Lemma 3 appeared in [2].

In this paper, we study the super (a, d)-edge antimagic total labeling of special classes of graphs derived from copies of generalized ladder, fan, generalized prism and web graph.

2. A Graph Derived from Copies of Generalized Ladder

Let $(u_{i,1}, u_{i,2}, \ldots, u_{i,n}, v_{i,1}, v_{i,2}, \ldots, v_{i,n})$, $1 \leq i \leq t$, be a collection of t disjoint copies of the *generalized ladder* $£_n$, $n \geq 2$, such that $u_{i,j}$ is adjacent to $u_{i,j+1}$, $v_{i,j+1}$ and $v_{i,j}$ is adjacent to $v_{i,j+1}$ for $1 \leq j \leq n-1$ and $u_{i,j}$ is adjacent to $v_{i,j}$ for $1 \leq j \leq n$. We denote the graph obtained by joining $u_{i,n}$ to $u_{i+1,1}, u_{i+1,2}, v_{i+1,1}$, $1 \leq i \leq t-1$, as $£_n^{(t)}$. Clearly, the vertex set V and the edge set E of the graph $£_n^{(t)}$ are given by

$V(£_n^{(t)}) = \{u_{i,j}, v_{i,j} : 1 \leq i \leq t, 1 \leq j \leq n\}$ and $E(£_n^{(t)}) = E_1 \cup E_2 \cup E_3$ where

$E_1 = \{u_{i,j}u_{i,j+1}, v_{i,j}v_{i,j+1}, u_{i,j}v_{i,j+1} : 1 \leq i \leq t, 1 \leq j \leq n-1\}$,

$E_2 = \{u_{i,j}v_{i,j} : 1 \leq i \leq t, 1 \leq j \leq n\}$,

$E_3 = \{u_{i,n}u_{i+1,1}, u_{i,n}u_{i+1,2}, u_{i,n}v_{i+1,1} : 1 \leq i \leq t-1\}$.

It is easy to see that for $£_n^{(t)}$, we have $p = 2nt$ and $q = 4nt - 3$.

Lemma 4. *The graph $£_n^{(t)}$, $n, t \geq 2$ has an $(a, 1)$-edge antimagic vertex labeling.*

Proof. Let us define a bijection $f_1 : V(£_n^{(t)}) \to \{1, 2, \ldots, 2nt\}$ as follows:

$f_1(u_{i,j}) = 2(i-1)n + 2j - 1$ if $1 \leq i \leq t$ and $1 \leq j \leq n$,

$f_1(v_{i,j}) = 2(i-1)n + 2j$ if $1 \leq i \leq t$ and $1 \leq j \leq n$.

By direct computation, we observe that the edge weights of all the edges of $£_n^{(t)}$, constitute an arithmetic sequence $\{3, 4, \ldots, 4nt-1\}$. Thus f_1 is an $(3, 1)$-edge antimagic vertex labeling of $£_n^{(t)}$. ■

Theorem 5. *The graph $\pounds_n^{(t)}$, $n, t \geq 2$, has a super (a, d)-edge antimagic total labeling if and only if $d \in \{0, 1, 2\}$.*

Proof. If the graph $\pounds_n^{(t)}$, $n, t \geq 2$, is super (a, d)-edge antimagic total, then by Lemma 1, we get $d \leq 2$.

Conversely, by Lemma 4 and Lemma 3, we see that the graph $\pounds_n^{(t)}$, $n, t \geq 2$ has a super $(6nt, 0)$-edge antimagic total labeling and a super $(2nt + 4, 2)$-edge antimagic total labeling.

Also by Lemma 2, we conclude that the graph $\pounds_n^{(t)}$, $n, t \geq 2$, has a super $(4nt + 2, 1)$-edge antimagic total labeling, since $q = 4nt - 3$, which is odd for all n and t. ■

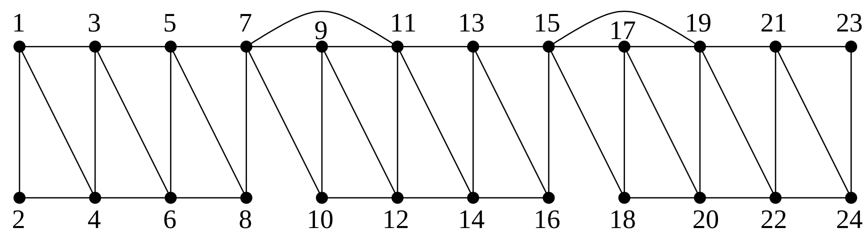

Figure 1. $(a, 1)$-edge antimagic vertex labeling of $\pounds_4^{(3)}$.

3. A Graph Derived from Copies of Fan Graph

Let $(u_i, w_i, v_{i,1}, v_{i,2}, \ldots, v_{i,m})$, $1 \leq i \leq t$, be a collection of t disjoint copies of the *fan graph* $\mathcal{F}_{m,2}$, $m \geq 2$, such that u_i is adjacent to w_i and $v_{i,j}$ is adjacent to both u_i and w_i for $1 \leq j \leq m$. We denote the graph [8] obtained by joining $v_{i,m}$ to $u_{i+1}, v_{i+1,1}, v_{i+1,2}$, $1 \leq i \leq t-1$, as $\mathcal{F}_{m,2}^{(t)}$. Clearly, the vertex set V and the edge set E of the graph $\mathcal{F}_{m,2}^{(t)}$ are given by

$$V(\mathcal{F}_{m,2}^{(t)}) = \{u_i, w_i, v_{i,j} : 1 \leq i \leq t, 1 \leq j \leq m\} \text{ and}$$
$$E(\mathcal{F}_{m,2}^{(t)}) = \{u_i w_i, u_i v_{i,j}, w_i v_{i,j} : 1 \leq i \leq t, 1 \leq j \leq m\} \cup \{v_{i,m} u_{i+1}, v_{i,m} v_{i+1,1}, v_{i,m} v_{i+1,2} : 1 \leq i \leq t-1\}.$$

It is easy to see that for $\mathcal{F}_{m,2}^{(t)}$, we have $p = (m+2)t$ and $q = (m+2)2t - 3$.

Lemma 6. *The graph $\mathcal{F}_{m,2}^{(t)}$, $m, t \geq 2$, has an $(a, 1)$-edge antimagic vertex labeling.*

Proof. Let us define a bijection $f_2 : V(\mathcal{F}_{m,2}^{(t)}) \to \{1, 2, \ldots, (m+2)t\}$ as follows:

$$f_2(u_i) = (i-1)(m+2) + 1 \quad \text{if } 1 \leq i \leq t,$$
$$f_2(w_i) = (m+2)i \quad \text{if } 1 \leq i \leq t,$$
$$f_2(v_{i,j}) = f_2(u_i) + j \quad \text{if } 1 \leq i \leq t \text{ and } 1 \leq j \leq m.$$

By direct computation, we observe that the edge weights of all the edges of $\mathcal{F}_{m,2}^{(t)}$ constitute an arithmetic sequence $\{3,4,\ldots,2t(m+2)-1\}$. Thus f_2 is an $(3,1)$-edge antimagic vertex labeling of $\mathcal{F}_{m,2}^{(t)}$. ■

Theorem 7. *The graph $\mathcal{F}_{m,2}^{(t)}$, $m,t \geq 2$, has a super (a,d)-edge antimagic total labeling if and only if $d \in \{0,1,2\}$.*

Proof. If the graph $\mathcal{F}_{m,2}^{(t)}$, $m,t \geq 2$, is super (a,d)-edge antimagic total, then by Lemma 1, we get $d \leq 2$.

Conversely, by Lemmas 3 and 6, we see that the graph $\mathcal{F}_{m,2}^{(t)}$, $m,t \geq 2$, has a super $((m+2)\,3t, 0)$-edge antimagic total labeling and a super $((m+2)\,t+4, 2)$-edge antimagic total labeling.

Also by Lemma 2, we conclude that the graph $\mathcal{F}_{m,2}^{(t)}$, $m,t \geq 2$, has a super $((m+2)2t+2, 1)$-edge antimagic total labeling, since $q = (m+2)2t-3$, which is odd for all m and t. ■

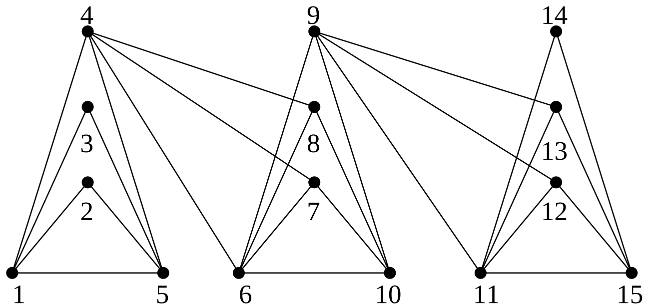

Figure 2. $(a,1)$-edge antimagic vertex labeling of $\mathcal{F}_{3,2}^{(3)}$.

4. A Graph Derived from Copies of Generalized Prism

Let $(v_{i,j}^{(k)}, 1 \leq i \leq m, 1 \leq j \leq n)$, $1 \leq k \leq t$, be a collection of t disjoint copies of the *generalized prism* $C_m \times P_n$, $m \geq 3$, $n \geq 2$, such that $v_{i,j}^{(k)}$ is adjacent to $v_{i+1,j}^{(k)}$ for $1 \leq i \leq m-1$, $1 \leq j \leq n$, $v_{m,j}^{(k)}$ is adjacent to $v_{1,j}^{(k)}$ for $1 \leq j \leq n$ and $v_{i,j}^{(k)}$ is adjacent to $v_{i,j+1}^{(k)}$ for $1 \leq i \leq m$, $1 \leq j \leq n-1$. We denote the graph obtained by joining $v_{m,n}^{(k)}$ to $v_{i,1}^{(k+1)}$ if n is odd or $v_{1,n}^{(k)}$ to $v_{i,1}^{(k+1)}$ if n is even for $1 \leq i \leq m$, $1 \leq k \leq t-1$ as $(C_m \times P_n)^{(t)}$. Clearly, the vertex set V and the edge set E of the graph $(C_m \times P_n)^{(t)}$ are given by $V((C_m \times P_n)^{(t)}) = \{v_{i,j}^{(k)} : 1 \leq k \leq t, 1 \leq i \leq$

$m, 1 \leq j \leq n\}$ and $E((C_m \times P_n)^{(t)}) = E_1 \cup E_2 \cup E_3$ where

$$\begin{aligned} E_1 &= \{v_{i,j}^{(k)} v_{i+1,j}^{(k)} : 1 \leq k \leq t, 1 \leq i \leq m-1, 1 \leq j \leq n\} \\ &\cup \{v_{m,j}^{(k)} v_{1,j}^{(k)} : 1 \leq k \leq t, 1 \leq j \leq n\}, \\ E_2 &= \{v_{i,j}^{(k)} v_{i,j+1}^{(k)} : 1 \leq k \leq t, 1 \leq i \leq m, 1 \leq j \leq n-1\}, \\ E_3 &= \{v_{m,n}^{(k)} v_{i,1}^{(k+1)} : \text{if } n \text{ is odd and } 1 \leq k \leq t-1, 1 \leq i \leq m\} \\ &\cup \{v_{1,n}^{(k)} v_{i,1}^{(k+1)} : \text{if } n \text{ is even and } 1 \leq k \leq t-1, 1 \leq i \leq m\}. \end{aligned}$$

It is easy to see that for $(C_m \times P_n)^{(t)}$, we have $p = mnt$ and $q = m(2nt-1)$.

Lemma 8. *For odd m, $m \geq 3$ and $n, t \geq 2$, the graph $(C_m \times P_n)^{(t)}$ has an $(a, 1)$-edge antimagic vertex labeling.*

Proof. Let us define a bijection $f_3 : V((C_m \times P_n)^{(t)}) \to \{1, 2, \ldots, mnt\}$ as follows.

If j is odd and $1 \leq i \leq m$, $1 \leq j \leq n$, $1 \leq k \leq t$, then

$$f_3(v_{i,j}^{(k)}) = \begin{cases} (k-1)mn + (j-1)m + \frac{i+1}{2} & \text{if } i \text{ is odd,} \\ (k-1)mn + (j-1)m + \frac{m+i+1}{2} & \text{if } i \text{ is even.} \end{cases}$$

If j is even and $1 \leq i \leq m$, $2 \leq j \leq n$, $1 \leq k \leq t$, then

$$f_3(v_{i,j}^{(k)}) = \begin{cases} (k-1)mn + (j-1)m + \frac{m+i}{2} & \text{if } i \text{ is odd,} \\ (k-1)mn + (j-1)m + \frac{i}{2} & \text{if } i \text{ is even.} \end{cases}$$

By direct computation, we observe that the edge weights of all the edges of $(C_m \times P_n)^{(t)}$ constitute an arithmetic sequence $\left\{\frac{m+3}{2}, \frac{m+5}{2}, \ldots, \frac{m+4mnt-3}{2}\right\}$. Clearly $\frac{m+3}{2}$ is an integer only when m is odd. Thus f_3 is an $\left(\frac{m+3}{2}, 1\right)$-edge antimagic vertex labeling of $(C_m \times P_n)^{(t)}$, for odd m. ∎

Theorem 9. *For odd m, $m \geq 3$ and $n, t \geq 2$, the graph $(C_m \times P_n)^{(t)}$ has a super (a, d)-edge antimagic total labeling if and only if $d \in \{0, 1, 2\}$.*

Proof. If the graph $(C_m \times P_n)^{(t)}$, $m \geq 3$ and $n, t \geq 2$, is super (a, d)-edge antimagic total, then by Lemma 1 we get

$$d \leq \frac{2p+q-5}{q-1} = \frac{2mnt+m(2nt-1)-5}{m(2nt-1)-1} = 2 + \frac{m-3}{2mnt-m-1}.$$

Since $2mnt - m - 1 > 0$, for $m \geq 3$, $n, t \geq 2$, it follows that $\frac{m-3}{2mnt-m-1} < 1$ and hence $d < 3$.

Conversely, by Lemma 8 and Lemma 3, we obtain that for odd m, the graph $(C_m \times P_n)^{(t)}$, $m \geq 3$, $n, t \geq 2$, is both super $\left(\frac{m+3}{2} + p + q, 0\right)$-edge antimagic total and super $\left(\frac{m+3}{2} + p + 1, 2\right)$-edge antimagic total.

Also by Lemma 2, we conclude that the graph $(C_m \times P_n)^{(t)}$, $m \geq 3$, $n, t \geq 2$, has a super $\left(\frac{m+3}{2} + p + \frac{q+1}{2}, 1\right)$-edge antimagic total labeling, since $q = m(2nt - 1)$, which is odd for odd m. ∎

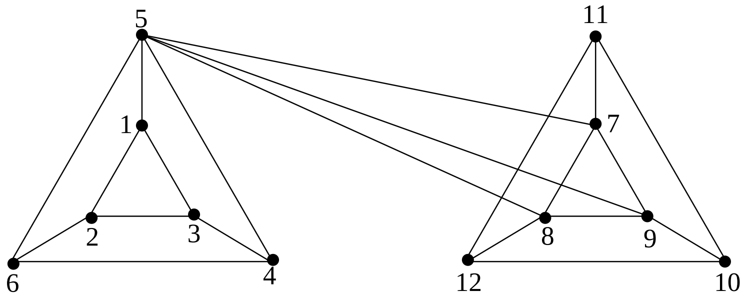

Figure 3. $(a,1)$-edge antimagic vertex labeling of $(C_3 \times P_2)^{(2)}$.

5. A Graph Derived from Copies of Generalized Web Graph

Let $(v_{i,j}^{(k)}, 1 \le i \le m, 1 \le j \le n+1)$, $1 \le k \le t$, be a collection of t disjoint copies of the *generalized web graph* $W(m,n)$, $m \ge 3$, $n \ge 2$, such that $v_{i,j}^{(k)}$ is adjacent to $v_{i+1,j}^{(k)}$ for $1 \le i \le m-1$, $1 \le j \le n$, $v_{m,j}^{(k)}$ is adjacent to $v_{1,j}^{(k)}$ for $1 \le j \le n$ and $v_{i,j}^{(k)}$ is adjacent to $v_{i,j+1}^{(k)}$ for $1 \le i \le m$, $1 \le j \le n$. We denote the graph obtained by joining $v_{1,n}^{(k)}$ to $v_{i,1}^{(k+1)}$ and $v_{i,2}^{(k+1)}$ for $1 \le i \le m$, $1 \le k \le t-1$ as $(W(m,n))^{(t)}$. Clearly, the vertex set V and the edge set E of the graph $(W(m,n))^{(t)}$ are given by $V((W(m,n))^{(t)}) = \{v_{i,j}^{(k)} : 1 \le k \le t, 1 \le i \le m, 1 \le j \le n+1\}$ and $E((W(m,n))^{(t)}) = E_1 \cup E_2 \cup E_3$ where

$$\begin{aligned} E_1 &= \{v_{i,j}^{(k)} v_{i+1,j}^{(k)} : 1 \le k \le t, 1 \le i \le m-1, 1 \le j \le n\} \\ &\quad \cup \{v_{m,j}^{(k)} v_{1,j}^{(k)} : 1 \le k \le t, 1 \le j \le n\}, \\ E_2 &= \{v_{i,j}^{(k)} v_{i,j+1}^{(k)} : 1 \le k \le t, 1 \le i \le m, 1 \le j \le n\}, \\ E_3 &= \{v_{1,n}^{(k)} v_{i,1}^{(k+1)}, v_{1,n}^{(k)} v_{i,2}^{(k+1)} : 1 \le k \le t-1, 1 \le i \le m\}. \end{aligned}$$

It is easy to see that for $(W(m,n))^{(t)}$, we have $p = mt(n+1)$ and $q = 2m(nt+t-1)$.

Lemma 10. *For odd m, $m \ge 3$, $n,t \ge 2$, the graph $(W(m,n))^{(t)}$ has an $(a,1)$-edge antimagic vertex labeling.*

Proof. Let us define a bijection $f_4 : V(W(m,n))^{(t)}) \to \{1,2,\dots,mt(n+1)\}$ as follows:

Case (i): n is even.

If j is odd and $1 \le i \le m$, $1 \le j \le n+1$, $1 \le k \le t$, then

$$f_4(v_{i,j}^{(k)}) = \begin{cases} (k-1)(mn+m) + (j-1)m + \frac{i+1}{2} & \text{if } i \text{ is odd,} \\ (k-1)(mn+m) + (j-1)m + \frac{m+i+1}{2} & \text{if } i \text{ is even.} \end{cases}$$

If j is even and $1 \le i \le m$, $2 \le j \le n$, $1 \le k \le t$, then

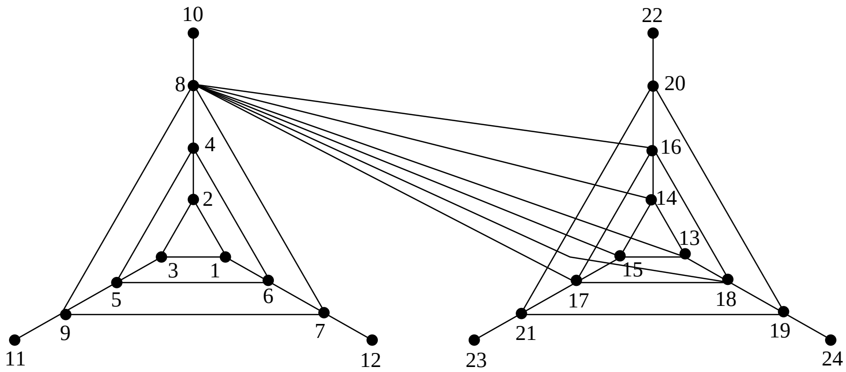

Figure 4. $(a,1)$-edge antimagic vertex labeling of $(W(3,3))^{(2)}$.

$$f_4(v_{i,j}^{(k)}) = \begin{cases} (k-1)(mn+m) + (j-1)m + \frac{m+i}{2} & \text{if } i \text{ is odd ,} \\ (k-1)(mn+m) + (j-1)m + \frac{i}{2} & \text{if } i \text{ is even.} \end{cases}$$

Case (ii): n is odd.

If j is odd and $1 \le i \le m$, $1 \le j \le n$, $1 \le k \le t$, then

$$f_4(v_{i,j}^{(k)}) = \begin{cases} (k-1)(mn+m) + (j-1)m + \frac{m+i}{2} & \text{if } i \text{ is odd,} \\ (k-1)(mn+m) + (j-1)m + \frac{i}{2} & \text{if } i \text{ is even.} \end{cases}$$

If j is even and $1 \le i \le m$, $2 \le j \le n+1$, $1 \le k \le t$, then

$$f_4(v_{i,j}^{(k)}) = \begin{cases} (k-1)(mn+m) + (j-1)m + \frac{i+1}{2} & \text{if } i \text{ is odd,} \\ (k-1)(mn+m) + (j-1)m + \frac{m+i+1}{2} & \text{if } i \text{ is even.} \end{cases}$$

In both the cases, we observe that under the bijection f_4, the edge weights of all the edges of $(W(m,n))^{(t)}$ constitute an arithmetic sequence $\{\frac{m+3}{2}, \frac{m+5}{2}, \ldots, \frac{m+4mnt+4m(t-1)+1}{2}\}$. Clearly $\frac{m+3}{2}$ is an integer only when m is odd. Hence the vertex labeling f_4 is an $\left(\frac{m+3}{2}, 1\right)$-edge antimagic vertex labeling of $(W(m,n))^{(t)}$, for odd m. ∎

Theorem 11. *For odd m, $m \ge 3$, $n, t \ge 2$ and $d \in \{0, 2\}$, the graph $(W(m,n))^{(t)}$, has a super (a,d)-edge antimagic total labeling.*

Proof. By Lemmas 3 and 10, we see that for odd m, the graph $(W(m,n))^{(t)}$, $m \ge 3$, $n, t \ge 2$ has a super $\left(\frac{m+3}{2} + p + q, 0\right)$-edge antimagic total labeling and a super $\left(\frac{m+3}{2} + p + 1, 2\right)$-edge antimagic total labeling. ∎

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