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AN EXISTENCE THEOREM FOR AN HYPERBOLIC DIFFERENTIAL INCLUSION IN BANACH SPACES

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Abstract

In this paper, we investigate the existence of solutions on unbounded domain to a hyperbolic differential inclusion in Banach spaces. We shall rely on a fixed point theorem due to Ma which is an extension to multivalued between locally convex topological spaces of Schaefer's theorem.

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1. Introduction

This note deals with the existence of solutions defined on unbounded domain to the following hyperbolic differential inclusion (Darboux problem):

$$(1) \qquad \frac{\partial^2 u(x,y)}{\partial x \partial y} \in F(x,y,u(x,y)), \quad (x,y) \in J \times J = [0,\infty) \times [0,\infty)$$

(2) $$u(x,0) = f(x), \;\; u(0,y) = g(y),$$

where $F : J \times J \times E \longrightarrow 2^E$ is a multivalued map with nonempty compact and convex values, $f, \; g : J \to E$ and $(E, |\cdot|)$ a Banach space.

The single and multivalued finite dimensional versions of the problem (1) – (2) on compact domains were considered by DeBlasi and Myjak [9], [10] who established the topological regularity of the solutions set. Kubiaczyk [16] considered on a compact domain the single-valued infinite dimensional version of the problem, where a Kneser-type theorem was proved for the solutions set. Using a compactness type condition, involving the measure of noncompactness, Papageorgiou gives in [20] existence results on compact domains for the problem (1) – (2). Recent results on compact domains for hyperbolic differential equations and inclusions can be found in the papers of Dawidowski and Kubiaczyk [6], [7], [8] and Kubiaczyk and Mostafa [17].

In this note, we shall give an existence result on unbounded domain for the problem (1) – (2). The method we are going to use is to reduce the existence of solutions to problem (1) – (2) to the search for fixed points of a suitable multivalued map on the Fréchet space $C(J \times J, E)$. In order to prove the existence of fixed points, we shall rely on a fixed point theorem of Ma [19], which is an extension of Schaefer's theorem [21] to multivalued maps between locally convex topological spaces.

2. Preliminaries

In this section, we introduce notations, definitions, and preliminary facts from multivalued analysis which are used throughout the paper. In the sequel, we will note $\mathcal{J} = J \times J$, $\mathcal{J}_m = J_m \times J_m$ where J_m is the compact real interval $[0, m]$ $(m \in \mathbb{N})$.

$C(\mathcal{J}, E)$ is the linear metric Fréchet space of continuous functions from $\mathcal{J}$ into E with the metric (see Dugundji and Granas [12], Corduneanu [5])

$$d(u, \overline{u}) = \sum_{m=0}^{\infty} \frac{2^{-m}\|u - \overline{u}\|_m}{1 + \|u - \overline{u}\|_m} \quad \text{for each} \qquad u, \overline{u} \in C(\mathcal{J}, E),$$

where

$$\|u\|_m := \sup\{|u(t,s)| : (t,s) \in \mathcal{J}_m\}.$$

A measurable function $u : \mathcal{J} \longrightarrow E$ is Bochner integrable if and only if $|u|$ is Lebesgue integrable. (For properties of the Bochner integral see Yosida [22]).

$L^1(\mathcal{J}, E)$ denotes the Banach space of functions $u : \mathcal{J} \longrightarrow E$ which are Bochner integrable.

U_p denotes the neighbourhood of 0 in $C(\mathcal{J}, E)$ defined by

$$U_p := \{u \in C(\mathcal{J}, E) : \|u\|_m \le p \text{ for each } m \in \mathbb{N}\}.$$

The convergence in $C(\mathcal{J}, E)$ is the uniform convergence on compacts, i.e. $u_j \longrightarrow u$ in $C(\mathcal{J}, E)$ if and only if for each $m \in \mathbb{N}$, $\|u_j - u\|_m \longrightarrow 0$ in $C(\mathcal{J}_m, E)$ as $j \longrightarrow \infty$.

$M \subseteq C(\mathcal{J}, E)$ is a bounded set if and only if there exists a positive function $\varphi \in C(\mathcal{J}, \mathbb{R})$ such that

$$|u(x, y)| \le \varphi(x, y) \text{ for all } (x, y) \in \mathcal{J} \text{ and all } u \in M.$$

Let $(X, |\cdot|)$ be a Banach space. A multivalued map $G : X \longrightarrow 2^X$ is convex (closed) valued if $G(x)$ is convex (closed) for all $x \in X$. G is bounded on bounded sets if $G(B) = \cup_{x\in B} G(x)$ is bounded in X for any bounded set B of X (i.e. $\sup_{x\in B}\{\sup\{|y| : y \in G(x)\}\} < \infty$).

G is called upper semicontinuous (u.s.c.) on X if, for each $x_* \in X$, the set $G(x_*)$ is a nonempty, closed subset of X, and if, for each open set B of X containing $G(x_*)$, there exists an open neighbourhood V of x_* such that $G(V) \subseteq B$. G is said to be completely continuous if $G(B)$ is relatively compact for every bounded subset $B \subseteq X$. If the multivalued map G is completely continuous with nonempty compact values, then G is u.s.c. if and only if G has a closed graph (i.e. $x_n \longrightarrow x_*$, $y_n \longrightarrow y_*$, $y_n \in G(x_n)$ imply $y_* \in G(x_*)$). G has a fixed point if there is $x \in X$ such that $x \in G(x)$.

In the following, $CC(X)$ denotes the set of all nonempty compact and convex subsets of X. A multivalued map $G : \mathcal{J} \longrightarrow CC(E)$ is said to be measurable if, for each $w \in E$ the function $Y : \mathcal{J} \longrightarrow \mathbb{R}$, defined by

$$Y(x, y) = d(w, G(x, y)) = \inf\{|w - v| : v \in G(x, y)\},$$

is measurable.

Definition 2.1. A multivalued map $F : \mathcal{J} \times E \longrightarrow 2^E$ is said to be an L^1-Carathéodory if

(i) $(x, y) \longmapsto F(x, y, u)$ is measurable for each $u \in E$;

(ii) $u \longmapsto F(x, y, u)$ is upper semicontinuous for almost all $(x, y) \in \mathcal{J}$;

(iii) For each $k > 0$, there exists $h_k \in L^1(\mathcal{J}, \mathbb{R}_+)$ such that

$$\|F(x,y,u)\| = \sup\{|v| : v \in F(x,y,u)\} \le h_k(t) \quad \text{for all} \quad |u| \le k$$
$$\text{and for almost all} \quad (x,y) \in \mathcal{J}.$$

For more details on multivalued maps see Deimling [11], Górniewicz [13] and Hu and Papageorgiou [15].

We will need the following hypotheses:

(H1) $F : \mathcal{J} \times E \longrightarrow CC(E)$ is an L^1-Carathéodory multivalued map and for each fixed $u \in C(\mathcal{J}, E)$ the set

$$S_{F,u} := \Big\{ v \in L^1(\mathcal{J}, E) : \ v(x,y) \in F(x,y,u(x,y)) \text{ for a.e. } (x,y) \in \mathcal{J} \Big\}$$

is nonempty;

(H2) There exist $H \in L^1(\mathcal{J}, \mathbb{R}^+)$ and $\psi : [0,\infty) \to (0,\infty)$ continuous and nondecreasing with
$$\int_0^\infty \frac{d\tau}{\psi(\tau)} = \infty$$
such that
$$\|F(x,y,u)\| := \sup\{|v| : v \in F(x,y,u)\} \le H(x,y)\psi(|u|)$$
for almost all $(x,y) \in \mathcal{J}$ and all $u \in C(\mathcal{J}, E)$;

(H3) The functions $f, \ g : J \to E$ are continuous with $f(0) = g(0)$;

(H4) For each bounded set $B \subseteq C(\mathcal{J}, E)$ and for each $(x,y) \in \mathcal{J}$ the set

$$\left\{ f(x) + g(y) - f(0) + \int_0^x \int_0^y v(t,s)dtds : v \in S_{F,B} \right\}$$

is relatively compact in E, where $S_{F,B} = \cup\{S_{F,u} : u \in B\}$.

Remark 2.1. (i) If $\dim E < \infty$ and $\mathcal{J}$ is compact, then for each $u \in C(\mathcal{J}, E)$ the set $S_{F,u}$ is nonempty (see Lasota and Opial [18]).

(ii) If $\dim E = \infty$ then $S_{F,u}$ is nonempty if and only if the function $Y : \mathcal{J} \longrightarrow \mathbb{R}^+$ defined by

$$Y(x,y) := \inf\{|v| : v(x,y) \in F(x,y,u(x,y))\}$$

is measurable (see Hu and Papageorgiou [15]).

Definition 2.2. By a solution to (1) – (2) we mean a function $u(\cdot,\cdot) \in C(\mathcal{J}, E)$ such that there exists $v \in L^1(\mathcal{J}, E)$ for which we have

$$u(x,y) = f(x) + g(y) - f(0) + \int_0^x \int_0^y v(t,s)dtds \ \text{ for each } (x,y) \in \mathcal{J}$$

and $v(t,s) \in F(t,s,u(t,s))$ a.e. on $\mathcal{J}$.

Our considerations are based on the following lemmas.

Lemma 2.1 [18]**.** *Let F be a multivalued map, satisfying* (H1) *and let Γ be a linear continuous mapping from $L^1(\mathcal{J}, E)$ to $C(\mathcal{J}, E)$. Then the operator*

$$\Gamma \circ S_F : C(\mathcal{J}, E) \longrightarrow CC(C(\mathcal{J}, E)), \ u \longmapsto (\Gamma \circ S_F)(u) := \Gamma(S_{F,u}),$$

is a closed graph operator in $C(\mathcal{J}, E) \times C(\mathcal{J}, E)$.

Lemma 2.2 (Lemma 1.5.3 [14])**.** *Let I be a compact real interval. If $p \in L^1(I, \mathbb{R})$ and $\psi : \mathbb{R}_+ \to (0, +\infty)$ is increasing with*

$$\int_0^\infty \frac{du}{\psi(u)} = \infty,$$

then the integral equation

$$z(t) = z_0 + \int_0^t p(s)\psi(z(s))ds, \quad t \in I,$$

has for each $z_0 \in \mathbb{R}$ a unique solution z. If $u \in C(I, E)$ satisfies the integral inequality

$$|u(t)| \le z_0 + \int_0^t p(s)\psi(|u(s)|)ds, \quad t \in I,$$

then $|u| \le z$.

Lemma 2.3 [19]**.** *Let X be a locally convex space and let $N : X \longrightarrow 2^X$ be a compact convex valued, u.s.c. multivalued map such that there exists a closed neighbourhood U_p of 0 for which $N(U_p)$ is a relatively compact set for each $p, m \in \mathbb{N}$. If the set*

$$\Omega := \{y \in X : \lambda y \in N(y) \ \ \textit{for some} \ \ \lambda > 1\}$$

is bounded, then N has a fixed point.

3. Main result

Now, we are able to state and prove our main theorem.

Theorem 3.1. *Assume that hypotheses* (H1) – (H4) *hold. Then the problem* (1) – (2) *has at least one solution on* $\mathcal{J}$.

Proof. Let $C(\mathcal{J}, E)$ be the Fréchet space endowed with the seminorms

$$\|u\|_m := \sup\{|u(x,y)| : (x,y) \in \mathcal{J}_m\}, \text{ for } u \in C(\mathcal{J}, E).$$

Transform the problem into a fixed point problem. Consider the multivalued map, $N : C(\mathcal{J}, E) \longrightarrow 2^{C(\mathcal{J},E)}$, defined by:

$$N(u) := \left\{ h \in C(\mathcal{J}, E) : h(x,y) = f(x) + g(y) - f(0) + \int_0^x \int_0^y v(t,s)dtds \right\},$$

where

$$v \in S_{F,u} = \left\{ v \in L^1(\mathcal{J}, E) : v(t,s) \in F(t,s,u(t,s)) \text{ for a.e. } (t,s) \in \mathcal{J} \right\}.$$

Remark 3.1. It is clear that the fixed points of N are solutions to (1) – (2).

We shall show that N satisfies the assumptions of Lemma 2.3. The proof will be given in several steps.

Step 1. $N(u)$ is convex for each $u \in C(\mathcal{J}, E)$.

Indeed, if h_1, h_2 belong to $N(u)$, then there exist $v_1, v_2 \in S_{F,u}$ such that for each $(x,y) \in \mathcal{J}$ we have

$$h_i(x,y) = f(x) + g(y) - f(0) + \int_0^x \int_0^y v_i(t,s)dtds, \quad i = 1, 2.$$

Let $0 \leq \alpha \leq 1$. Then, for each $(x,y) \in \mathcal{J}$, we have

$$\begin{aligned}&(\alpha h_1 + (1-\alpha)h_2)(x,y)\\ &= f(x) + g(y) - f(0) + \int_0^x \int_0^y [\alpha v_1(t,s) + (1-\alpha)v_2(t,s)]dtds.\end{aligned}$$

Since $S_{F,u}$ is convex (because F has convex values) then

$$\alpha h_1 + (1-\alpha)h_2 \in N(u).$$

Step 2. N is bounded on bounded sets of $C(\mathcal{J}, E)$.

Indeed, it is enough to show that for each $m \in \mathbb{N}$ there exists a positive constant c_m such that for each $h \in N(u), u \in U_q$ one has $\|h\|_m \le c_m$.

If $h \in N(u)$, then there exists $v \in S_{F,u}$ such that for each $(x, y) \in \mathcal{J}$ we have

$$h(x, y) = f(x) + g(y) - f(0) + \int_0^x \int_0^y v(t, s)dtds.$$

By (H1) we have, for each $(x, y) \in \mathcal{J}_m$, that

$$|h(x, y)| \le |f(x)| + |g(y)| + |f(0)| + \int_0^x \int_0^y h_r(t, s)dtds.$$

Then

$$\|h\|_m \le \|f\|_m + \|g\|_m + |f(0)| + \int_0^m \int_0^m h_r(t, s)dtds = c_m.$$

Step 3. For each $q \in \mathbb{N}$, $N(U_q)$ is equicontinuous for $U_q \in C(\mathcal{J}, E)$.

Let $(x_1, y_1),\ (x_2, y_2) \in \mathcal{J}_m,\ x_1 < x_2,\ y_1 < y_2$ and U_q be a neighbourhood of 0 in $C(\mathcal{J}, E)$ for $q \in \mathbb{N}$. For each $u \in U_q$ and $h \in N(u)$, there exists $v \in S_{F,u}$ such that

$$h(x, y) = f(x) + g(y) - f(0) + \int_0^x \int_0^y v(t, s)dtds.$$

Thus we obtain

$$\begin{aligned}
&\|h(x_2, y_2) - h(x_1, y_1)\|_m \\
&\le |f(x_2) - f(x_1)| + |g(y_2) - g(y_1)| + \int_{x_1}^{x_2} \int_{y_1}^{y_2} |v(t, s)|dtds \\
&\le |f(x_2) - f(x_1)| + |g(y_2) - g(y_1)| + \int_{x_1}^{x_2} \int_{y_1}^{y_2} h_q(t, s)dtds.
\end{aligned}$$

As $(x_2, y_2) \longrightarrow (x_1, y_1)$ the right-hand side of the above inequality tends to zero.

As a consequence of Step 2, Step 3 and (H4) together with the metric of the Fréchet space we can conclude that $N(U_q)$ is relatively compact in $C(\mathcal{J}, E)$.

Step 4. N has a closed graph.

Let $u_n \longrightarrow u_*$, $h_n \in N(u_n)$, and $h_n \longrightarrow h_*$. We shall prove that $h_* \in N(u_*)$. $h_n \in N(u_n)$ means that there exists $v_n \in S_{F,u_n}$ such that

$$h_n(x,y) = f(x) + g(y) - f(0) + \int_0^x \int_0^y v_n(t,s)dtds,\ (x,y) \in \mathcal{J}.$$

We have to prove that there exists $v_* \in S_{F,u_*}$ such that

$$h_*(x,y) = f(x) + g(y) - f(0) + \int_0^x \int_0^y v_*(t,s)dtds,\ (x,y) \in \mathcal{J}. \tag{3}$$

The idea is then to use the facts that

(i) $h_n \longrightarrow h_*$;

(ii) $h_n(x,y) - f(x) - g(y) + f(0) \in \Gamma(S_{F,u_n})$, where

$$\Gamma : L^1(\mathcal{J}, E) \longrightarrow C(\mathcal{J}, E) \quad \text{is defined by}$$

$$v \longmapsto \Gamma(v)(x,y) = \int_0^x \int_0^y v(t,s)dtds,\ (x,y) \in \mathcal{J}.$$

If $\Gamma \circ S_F$ is a closed graph operator, we would be done. But we do not know whether $\Gamma \circ S_F$ is a closed graph operator. So, we cut the functions y_n, h_n, v_n and we consider them defined on the compact $[k, k+1] \times [k, k+1]$ for any $k \in \mathbb{N} \cup \{0\}$. Then, using Lemma 2.1, in this case we are able to affirm that (3) is true on the compact $[k, k+1] \times [k, k+1]$, i.e.

$$(h_*(x,y) - f(x) - g(y) + f(0))\Big|_{[k,k+1]\times[k,k+1]} = \int_0^x \int_0^y v_*^k(t,s)dtds$$

for a suitable L^1-selection v_*^k of $F(x, y, u_*(x,y))$ on the compact $[k, k+1] \times [k, k+1]$.

At this point we can paste the functions v_*^k obtaining the selection v_* defined by

$$v_*(t,s) = v_*^k(t,s) \quad \text{for} \quad (t,s) \in [k, k+1) \times [k, k+1).$$

We obtain then that v_* is an L^1-selection and (3) will be satisfied.

We give now the details.

By hypothesis we have that

$$\|(h_n(x,y) - f(x) - g(y) + f(0)) - (h_* - f(x) - g(y) + f(0))\|_m \longrightarrow 0, \text{ as } n \longrightarrow \infty.$$

Now, we consider for all $k \in \mathbb{N} \cup \{0\}$, the mapping

$$S_F^k : C([k,k+1] \times [k,k+1], E) \longrightarrow L^1([k,k+1] \times [k,k+1], E)$$

$$u \longmapsto S_{F,u}^k := \{v \in L^1([k,k+1] \times [k,k+1], E) : v(x,y) \in F(x,y,u(x,y)) \text{ for a.e. } (x,y) \in [k,k+1] \times [k,k+1]\}.$$

Also, we consider the linear continuous operators

$$\Gamma_k : L^1([k,k+1] \times [k,k+1], E) \longrightarrow C([k,k+1] \times [k,k+1], E)$$

$$v \longmapsto \Gamma_k(v)(x,y) = \int_0^x \int_0^y v(t,s)dtds.$$

From Lemma 2.1, it follows that $\Gamma_k \circ S_F^k$ is a closed graph operator for all $k \in \mathbb{N} \cup \{0\}$.

Moreover, we have that

$$(h_n(x,y) - f(x) - g(y) + f(0))\Big|_{[k,k+1]\times[k,k+1]} \in \Gamma_k(S_{F,u_n^k}).$$

Since $u_n \longrightarrow u_*$, it follows from Lemma 2.1 that

$$(h_*(x,y) - f(x) - g(y) + f(0))\Big|_{[k,k+1]\times[k,k+1]} = \int_0^x \int_0^y v_*^k(t,s)dtds$$

for some $v_*^k \in S_{F,u^*}^k$. So the function v_* defined on J by

$$v_*(x,y) = v_*^k(x,y) \quad \text{for} \quad (x,y) \in [k,k+1) \times [k,k+1)$$

is in S_{F,u^*} since $v_*(x,y) \in F(x,y,u_*(x,y))$ for a.e. $(x,y) \in \mathcal{J}_m$.

Step 5. The set

$$\Omega := \{u \in C(\mathcal{J}, E) : \lambda u \in N(u) \quad \text{for some} \quad \lambda > 1\}$$

is bounded.

Let $u \in \Omega$. Then $\lambda u \in N(u)$ for some $\lambda > 1$. Thus there exists $v \in S_{F,u}$ such that

$$u(x,y) = \lambda^{-1}f(x) + \lambda^{-1}g(y) - \lambda^{-1}f(0) + \lambda^{-1}\int_0^x \int_0^y v(t,s)dtds, \quad (x,y) \in \mathcal{J}.$$

This implies by (H2) that for each $(x,y)\in\mathcal{J}$ we have

$$|u(x,y)|\le\|f(x)\|_m+\|g(y)\|_m+|f(0)|+\int_0^x\int_0^y H(t,s)\psi(|u(t,s)|)dtds.$$

As a consequence of Lemma 2.2 we obtain

$$\|u\|_m\le\|z\|_m,$$

where z is the unique solution on $\mathcal{J}_m$ to the integral equation

$$z(x,y)-\|f(x)\|_m-\|g(y)\|_m-|f(0)|=\int_0^x\int_0^y H(t,s)\psi(z(t,s))dtds. \tag{4}$$

This shows that Ω is bounded. Set $X:=C(\mathcal{J},E)$. As a consequence of Lemma 2.3 we deduce that N has a fixed point which is a solution to (1) – (2) on $\mathcal{J}$.

Remark 3.2. Hypothesis (H2) and Lemma 2.2 imply the existence and the uniqueness of the solution to the integral equation (4).

4. Nonlocal hyperbolic problem

In this section, we indicate some generalizations of the problem (1) – (2). By using the same method as in Theorem 3.1 (with obvious modifications), we can prove existence results for the following nonlocal hyperbolic problem

$$\frac{\partial^2 u(x,y)}{\partial x\partial y}\in F(x,y,u(x,y)),\quad (x,y)\in J\times J=[0,\infty)\times[0,\infty) \tag{5}$$

$$u(x,0)+Q(u)=f(x),\quad x\in J \tag{6}$$

$$u(0,y)+K(u)=g(y),\quad y\in J \tag{7}$$

where F,f,g are as in the problem (1) – (2) and $Q,\ K: C(\mathcal{J},E)\to E$, are continuous functions under the following additional assumptions:

(H5) There exist constants $\overline{k}>0$ and $\overline{q}>0$ such that

$$|Q(u)\le\overline{q},\quad |K(u)|\le\overline{k}\quad\text{for each}\ \ u\in C(\mathcal{J},E);$$

(H4)' For each bounded set $B \subseteq C(\mathcal{J}, E)$ and for each $(x, y) \in \mathcal{J}$ the set

$$\left\{ f(x) + g(y) - Q(u) - K(u) - f(0) + \int_0^x \int_0^y v(t,s)dtds : v \in S_{F,B} \right\}$$

is relatively compact in E, where $S_{F,B} = \cup\{S_{F,u} : u \in B\}$.

By a solution to the nonlocal problem (5) – (7) we mean a function $u(\cdot,\cdot) \in C(\mathcal{J}, E)$ such that there exists $v \in L^1(\mathcal{J}, E)$ for which we have

$$u(x,y) = f(x) + g(y) - Q(u) - K(u) - f(0) + \int_0^x \int_0^y v(t,s)dtds$$

for each $(x, y) \in \mathcal{J}$ and $v(t, s) \in F(t, s, u(t, s))$ a.e. on $\mathcal{J}$.

For results on nonlocal problems the interested reader is referred to [1], [2], [3], [4] and the references cited therein.

Theorem 4.1. *Assume that hypotheses* (H1) – (H3), (H5) – (H4)' *hold. Then the nonlocal problem* (5) – (7) *has at least one solution on* $\mathcal{J}$.

REFERENCES

[1] L. Byszewski, *Existence and uniqueness of solutions of nonlocal problems for hyperbolic equation* $u''_{xt} = F(x, t, u, u_x)$, J. Appl. Math. Stoch. Anal. **3** (3) (1990), 163–168.

[2] L. Byszewski, *Theorem about existence and uniqueness of continuous solution of nonlocal problem for nonlinear hyperbolic equation*, Appl. Anal. **40** (1991), 173–180.

[3] L. Byszewski and V. Lakshmikantham, *Monotone iterative technique for nonlocal hyperbolic differential problem*, J. Math. Phys. Sci. **26** (4) (1992), 345–359.

[4] L. Byszewski and N.S. Papageorgiou, *An application of a noncompactness technique to an investigation of the existence of solutions to nonlocal multivalued Darboux problem*, J. Appl. Math. Stoch. Anal. **12** (2) (1999), 179–190.

[5] C. Corduneanu, Integral Equations and Applications, Cambridge Univ. Press, New York 1990.

[6] M. Dawidowski and I. Kubiaczyk, *An existence theorem for the generalized hyperbolic equation* $z''_{xy} \in F(x, y, z)$ *in Banach space*, Ann. Soc. Math. Pol. Ser. I, Comment. Math. **30** (1) (1990), 41–49.

[7] M. Dawidowski and I. Kubiaczyk, *Existence theorem for hyperbolic differential inclusion with Carathéodory right hand side*, Discuss. Math. Differ. Incl. **10** (1990), 69–75.

[8] M. Dawidowski and I. Kubiaczyk, *On bounded solutions of hyperbolic differential inclusion in Banach spaces*, Demonstr. Math. **25** (1–2) (1992), 153–159.

[9] F. De Blasi and J. Myjak, *On the structure of the set of solutions of the Darboux problem for hyperbolic equations*, Proc. Edinburgh Math. Soc. **29** (1986), 7–14.

[10] F. De Blasi and J. Myjak, *On the set of solutions of a differential inclusion*, Bull. Inst. Math., Acad. Sin. **14** (1986), 271–275.

[11] K. Deimling, Multivalued Differential Equations, Walter de Gruyter, Berlin-New York 1992.

[12] J. Dugundji and A. Granas, Fixed Point Theory, Monografie Mat. PWN, Warsaw 1982.

[13] L. Górniewicz, *Topological Fixed Point Theory of Multivalued Mappings*, Mathematics and its Applications, 495, Kluwer Academic Publishers, Dordrecht 1999.

[14] S. Heikkila and V. Lakshmikantham, Monotone Iterative Techniques for Discontinuous Nonlinear Differential Equations, Marcel Dekker, New York 1994.

[15] Sh. Hu and N. Papageorgiou, Handbook of Multivalued Analysis, Volume I: Theory, Kluwer, Dordrecht, Boston, London, 1997, Volume II: Applications, Kluwer, Dordrecht, Boston, London 2000.

[16] I. Kubiaczyk, *Kneser's theorem for hyperbolic equations*, Funct. Approx. Comment. Math. **14** (1984), 183–196.

[17] I. Kubiaczyk and A.N. Mostafa, *On the existence of weak solutions of the Darboux problem for the hyperbolic partial differential equations in Banach spaces*, Fasc. Math. **28** (1998), 93–99.

[18] A. Lasota and Z. Opial, *An application of the Kakutani-Ky Fan theorem in the theory of ordinary differential equations*, Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys. **13** (1965), 781–786.

[19] T.W. Ma, *Topological degrees for set-valued compact vector fields in locally convex spaces*, Dissertationess Math. **92** (1972), 1–43.

[20] N.S. Papageorgiou, *Existence of solutions for hyperbolic differential inclusions in Banach spaces*, Arch. Math. (Brno) **28** (1992), 205–213.

[21] H. Schaefer, *Über die methode der a priori schranken*, Math. Ann. **129** (1955), 415–416.

[22] K. Yosida, Functional Analysis, 6^{th} edn. Springer-Verlag, Berlin 1980.

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