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BOOLEAN FILTERS IN PSEUDO-COMPLEMENTED ALMOST DISTRIBUTIVE LATTICES

N. Rafi[1]

Department of Mathematics, Bapatla Engineering College
Bapatla, Andhra Pradesh, India – 522101

e-mail: rafimaths@gmail.com

Ravi Kumar Bandaru

Department of Engineering Mathematics, GITAM University
Hyderabad Campus, India – 502329

e-mail: ravimaths83@gmail.com

and

G.C. Rao

Department of Mathematics, Andhra University
Visakhapatnam, Andhra Pradesh, India – 530003

e-mail: gcraomaths@yahoo.co.in

Abstract

In this paper we have introduced the concept of Boolean filters in a pseudo-complemented Almost Distributive Lattice (pseudo-complemented ADL) and studied their properties. Finally, a Boolean filter is characterized in terms of filter congruences.

Keywords: Almost Distributive Lattice (ADL), pseudo-complemented ADL, Boolean filter, maximal filter, congruence, Boolean algebra.

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[1]Corresponding author.

1. Introduction

After Booles axiomatization of two valued propositional calculus as a Boolean algebra, a number of generalizations both, ring theoretically and lattice theoretically have come into being. The concept of an Almost Distributive Lattice (ADL) was introduced by Swamy and Rao [5] as a common abstraction of many existing ring theoretic generalizations of a Boolean algebra on one hand and the class of distributive lattices on the other. In that paper, the concept of an ideal in an ADL was introduced analogous to that in a distributive lattice and it was observed that the set $PI(L)$ of all principal ideals of L forms a distributive lattice. This enables us to extend many existing concepts from the class of distributive lattices to the class of ADLs. Swamy, G.C. Rao and G.N. Rao introduced the concept of Stone ADL and characterized it in terms of its ideals. U.M. Swamy, G.C. Rao and G. Nanaji Rao introduced the concept of a pseudocomplementation in an ADL [6] and they observed that an ADL L can have more than one pseudo-complementation. In fact, they proved that there is a one-to-one correspondence between the set of all maximal elements of an ADL L and the set of all pseudo-complementations on L. Also, they proved that if $*$ is a pseudo-complementation on an ADL A, then the set $A^* = \{a^* \mid a \in L\}$ is a Boolean algebra under suitable operations and that the pseudo-complementation $*$ on A is equationally definable. In [4], Sambasiva Rao and Shum introduced Boolean filters in Pseudo-complemented distributive lattices and proved their properties. In this paper we extend the concept of Boolean filters to a Pseudo-complemented ADL. Some important results are established. We proved that the dense set is the smallest Boolean filter of ADL. It is also observed that every prime filter of a relatively complemented ADL is a Boolean filter. Finally, we characterized the Boolean filters in terms of filter congruences. Throughout this paper ADL L stands for ADL with zero.

2. Preliminaries

Definition 2.1 [5]**.** An Almost Distributive Lattice with zero or simply ADL is an algebra $(L, \vee, \wedge, 0)$ of type $(2, 2, 0)$ satisfying:

1. $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$
2. $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
3. $(x \vee y) \wedge y = y$
4. $(x \vee y) \wedge x = x$
5. $x \vee (x \wedge y) = x$

6. $0 \wedge x = 0$
7. $x \vee 0 = x$, for all $x, y, z \in L$.

Every non-empty set X can be regarded as an ADL as follows. Let $x_0 \in X$. Define the binary operations $\vee, \wedge$ on X by

$$x \vee y = \begin{cases} x & \text{if } x \neq x_0 \\ y & \text{if } x = x_0 \end{cases} \qquad x \wedge y = \begin{cases} y & \text{if } x \neq x_0 \\ x_0 & \text{if } x = x_0. \end{cases}$$

Then $(X, \vee, \wedge, x_0)$ is an ADL (where x_0 is the zero) and is called a discrete ADL. If $(L, \vee, \wedge, 0)$ is an ADL, for any $x, y \in L$, define $x \leq y$ if and only if $x = x \wedge y$ (or equivalently, $x \vee y = y$), then $\leq$ is a partial ordering on L.

Theorem 2.2 [5]**.** *If $(L, \vee, \wedge, 0)$ is an ADL, for any $x, y, z \in L$, we have the following:*

(1) $x \vee y = x \Leftrightarrow x \wedge y = y$

(2) $x \vee y = y \Leftrightarrow x \wedge y = x$

(3) $\wedge$ *is associative in* L

(4) $x \wedge y \wedge z = y \wedge x \wedge z$

(5) $(x \vee y) \wedge z = (y \vee x) \wedge z$

(6) $x \wedge y = 0 \Leftrightarrow y \wedge x = 0$

(7) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

(8) $x \wedge (x \vee y) = x$, $(x \wedge y) \vee y = y$ *and* $x \vee (y \wedge x) = x$

(9) $x \leq x \vee y$ *and* $x \wedge y \leq y$

(10) $x \wedge x = x$ *and* $x \vee x = x$

(11) $0 \vee x = x$ *and* $x \wedge 0 = 0$

(12) *If* $x \leq z$, $y \leq z$ *then* $x \wedge y = y \wedge x$ *and* $x \vee y = y \vee x$

(13) $x \vee y = (x \vee y) \vee x$.

It can be observed that an ADL L satisfies almost all the properties of a distributive lattice except the right distributivity of $\vee$ over $\wedge$, commutativity of $\vee$, commutativity of $\wedge$. Any one of these properties make an ADL L a distributive lattice. That is

Theorem 2.3 [5]**.** *Let $(L, \vee, \wedge, 0)$ be an ADL with 0. Then the following are equivalent:*

1) $(L, \vee, \wedge, 0)$ *is a distributive lattice*

2) *$a \vee b = b \vee a$, for all $a, b \in L$*

3) *$a \wedge b = b \wedge a$, for all $a, b \in L$*

4) *$(a \wedge b) \vee c = (a \vee c) \wedge (b \vee c)$, for all $a, b, c \in L$.*

As usual, an element $m \in L$ is called maximal if it is a maximal element in the partially ordered set $(L, \leq)$. That is, for any $a \in L$, $m \leq a \Rightarrow m = a$.

Theorem 2.4 [5]**.** *Let L be an ADL and $m \in L$. Then the following are equivalent:*

1) *m is maximal with respect to $\leq$*

2) *$m \vee a = m$, for all $a \in L$*

3) *$m \wedge a = a$, for all $a \in L$*

4) *$a \vee m$ is maximal, for all $a \in L$.*

Definition 2.5. a non-empty subset I of an ADL L is called an ideal of L if $a \vee b \in I$ and $a \wedge x \in I$ for any $a, b \in I$ and $x \in L$. Also, a non-empty subset F of L is said to be a filter of L if $a \wedge b \in F$ and $x \vee a \in F$ for $a, b \in F$ and $x \in L$.

The set $I(L)$ of all ideals of L is a bounded distributive lattice with least element $\{0\}$ and greatest element L under set inclusion in which, for any $I, J \in I(L)$, $I \cap J$ is the infimum of I and J while the supremum is given by $I \vee J := \{a \vee b \mid a \in I, b \in J\}$. A proper ideal P of L is called a prime ideal if, for any $x, y \in L$, $x \wedge y \in P \Rightarrow x \in P$ or $y \in P$. A proper ideal M of L is said to be maximal if it is not properly contained in any proper ideal of L. It can be observed that every maximal ideal of L is a prime ideal. Every proper ideal of L is contained in a maximal ideal. For any subset S of L the smallest ideal containing S is given by $(S] := \{(\bigvee_{i=1}^{n} s_i) \wedge x \mid s_i \in S, x \in L$ and $n \in N\}$. If $S = \{s\}$, we write $(s]$ instead of $(S]$. We call $(s]$, the principal ideal of L generated by 's'. Similarly, for any $S \subseteq L$, $[S) := \{x \vee (\bigwedge_{i=1}^{n} s_i) \mid s_i \in S, x \in L$ and $n \in N\}$. If $S = \{s\}$, we write $[s)$ instead of $[S)$. We call $[s)$, the principal filter of L generated by 's'.

Theorem 2.6 [5]**.** *For any x, y in an ADL L the following are equivalent:*

1) *$(x] \subseteq (y]$*

2) *$y \wedge x = x$*

3) *$y \vee x = y$*

4) *$[y) \subseteq [x)$.*

For any $x, y \in L$, it can be verified that $(x] \vee (y] = (x \vee y]$ and $(x] \wedge (y] = (x \wedge y]$. Hence the set $PI(L)$ of all principal ideals of L is a sublattice of the distributive lattice $I(L)$ of ideals of L.

Definition 2.7 [3]**.** An equivalence relation θ on an ADL L is called a congruence relation on L if $(a \wedge c, b \wedge d), (a \vee c, b \vee d) \in \theta$, for all $(a, b), (c, d) \in \theta$.

Theorem 2.8 [3]**.** *An equivalence relation θ on an ADL L is a congruence relation if and only if for any $(a, b) \in \theta,\ x \in L, (a \vee x, b \vee x), (x \vee a, x \vee b), (a \wedge x, b \wedge x), (x \wedge a, x \wedge b)$ are all in θ*

Definition 2.9 [5]**.** An ADL L with 0 is called relatively complemented if each interval $[a, b],\ a \leq b$, in L is a complemented lattice.

Theorem 2.10 [5]**.** *Let L be an ADL with 0. Then L is relatively complemented if and only if every prime filter of L is maximal.*

The following definition was introduced by U.M. Swamy, G.C. Rao and G.N. Rao.

Definition 2.11 [6]**.** Let $(L, \vee, \wedge, 0)$ be an ADL. Then a unary operation $a \longrightarrow a^*$ on L is called a pseudo-complementation on L if, for any $a, b \in L$, it satisfies the following conditions:

(1) $a \wedge b = 0 \Rightarrow a^* \wedge b = b$

(2) $a \wedge a^* = 0$

(3) $(a \vee b)^* = a^* \wedge b^*$.

Then $(L, \vee, \wedge, ^*, 0)$ is called a pseudo-complemented ADL.

Theorem 2.12 [6]**.** *Let L be an ADL and $*$ a pseudo-complementation on L. Then, for any $a, b \in L$, we have the following:*

(1) $0^{**} = 0$

(2) $0^* \wedge a = a$

(3) $a^{**} \wedge a = a$

(4) $a^{***} = a^*$

(5) $a \leq b \Rightarrow b^* \leq a^*$

(6) $a^* \wedge b^* = b^* \wedge a^*$

(7) $(a \wedge b)^{**} = a^{**} \wedge b^{**}$

(8) $a^* \wedge b = (a \wedge b)^* \wedge b^*$.

3. Boolean filters in Pseudo-complemented ADLs

In this section, Boolean filters in Pseudo-complemented ADL is introduced with help of the concept of ADL. Some important properties are studied thoroughly. Finally, a Boolean filter is characterized in terms of filter congruences.

Now, we begin with the following definition and necessary results.

Definition 3.1. Let L be a pseudo-complemented ADL. A filter F of L is called a Boolean filter if $x \vee x^* \in F$ for each $x \in L$.

We derive the following example of Boolean filter.

Example 3.2. Let $L = \{0, a, b, c\}$. Define two binary operations $\vee$ and $\wedge$ on L as follows

$\vee$	0	a	b	c
0	0	a	b	c
a	a	a	a	a
b	b	b	b	b
c	c	a	b	c

$\wedge$	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	a	b	c
c	0	c	c	c

Now define $x^* = 0$, for all $x \neq 0$ and $0^* = a$. Then $(L, \vee, \wedge, 0)$ is an ADL and $*$ is a pseudo-complementation on L. But which is not a lattice. Take a filter $F = \{a, b, c\}$, clearly which is a Boolean filter of L. A filter $F_1 = \{a, b\}$, which is not a Boolean filter of L because $c \vee c^* = c \notin F_1$.

For any pseudo-complemented ADL L, let us denote the set of all elements of the form $x^* = 0$ by D.

Now we have the following lemma.

Lemma 3.3. *Let L be an pseudo-complemented ADL. Then D is the smallest Boolean filter of L.*

Proof. Clearly, D is a Boolean filter of L. Suppose A is any Boolean filter of L. We prove that $D \subseteq A$. Let $x \in D$. Then $x^* = 0$. Since A is a Boolean filter of L, we have $x \vee x^* \in A$ and hence $x \in A$. Therefore D is the smallest Boolean filter of L. ■

Lemma 3.4. *Every maximal filter of a pseudo-complemented ADL L is a Boolean filter.*

Proof. Let M be a maximal filter of L. We prove that $x \vee x^* \in M$, for all $x \in L$. Suppose $x \vee x^* \notin M$, for some $x \in L$. Then $M \vee [x \vee x^*) = L$. That implies $0 = a \wedge b$, for some $a \in M$ and $b \in [x \vee x^*)$. Now, $(a \wedge x) \vee (a \wedge x^*) = a \wedge (x \vee x^*) = a \wedge b \wedge (x \vee x^*) = 0$ and hence $a \wedge x = 0$ and $a \wedge x^* = 0$. This implies that $a = x^* \wedge a = 0$. Therefore $0 \in M$, which is a contradiction to proper filter M. Hence $x \vee x^* \in M$ for all $x \in L$. Therefore, M is a Boolean filter of L. ■

The following result can be verified easily.

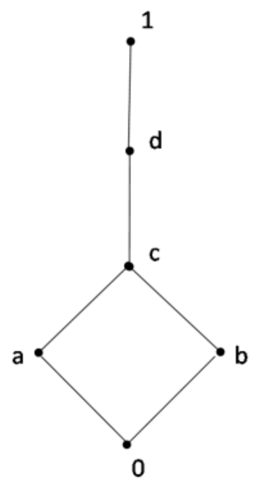
Lemma 3.5. *In a relatively complemented ADL, every prime filter is a Boolean filter.*

Theorem 3.6. *A proper filter of a pseudo-complemented ADL L which contains either x or x^* for all $x \in L$ is a Boolean filter.*

Proof. Let F be a proper filter of L such that either $x \in F$ or $x^* \in F$. We show that F is maximal. Suppose G is a proper filter of L such that $F \subsetneqq G$. Choose $a \in G \setminus F$. Since $a \notin F$, by the condition, we get $a^* \in F \subsetneqq G$. Since $a \in G$ and $a^* \in G$, we get $0 = a \wedge a^* \in G$, which is a contradiction. Therefore, F is a maximal filter. Thus by lemma 3.4, F is a Boolean filter. ∎

The converse of lemma 3.4 is not true in general. For,

Example 3.7. Let $L = \{0, a, b, c, d, 1\}$ be a distributive lattice whose Hasse diagram is given in the following figure. Consider the filter $F_1 = \{a, c, d, 1\}; F_2 = \{b, c, d, 1\}; F_3 = \{c, d, 1\}$ Then clearly F_1, F_2 and F_3 are Boolean filters, but F_3 is not a maximal filter of L.



A set of equivalent conditions are derived for a Boolean filter to become a maximal filter.

Theorem 3.8. *Let F be a proper filter of a pseudo-complemented ADL L. Then the following conditions are equivalent:*

(1) *F is maximal.*

(2) *$x \notin F$ implies $x^* \in F$ for all $x \in L$.*

(3) *F is prime Boolean.*

Proof. (1)$\Rightarrow$(2): Assume that F is a maximal filter of L. Let $x \notin F$. Then $F \vee [x) = L$. That implies $a \wedge x = 0$, for some $a \in F$. Hence $x^* \wedge a = a$, which implies that $x^* \in F$, since $x^* \vee a \in F$.

(2)$\Rightarrow$(3): Assume that condition (2). Let $x \in L$. Suppose $x \vee x^* \notin F$. Then it implies that $x \notin F$ and $x^* \notin F$, which is a contradiction to our assumption. Therefore $x \vee x^* \in F$ and hence F is a Boolean filter of L. Let $x, y \in L$ with $x \vee y \in F$. We prove that either $x \in F$ or $y \in F$. Suppose $x \notin F$. Then by our assumption, we get that $x^* \in F$. Hence $x^* \wedge y = 0 \vee (x^* \wedge y) = (x^* \wedge x) \vee (x^* \wedge y) = x^* \wedge (x \vee y) \in F$. Since $x^* \wedge y \leq y$, we get that $y \in F$. Therefore, F is a prime Boolean filter of L.

(3)$\Rightarrow$(1): Assume that F is a prime Boolean filter of L. Suppose F is not maximal. There exists a proper filter F' of L such that $F \subsetneqq F'$. Choose $x \in F' \backslash F$. Since F is Boolean, we get $x \vee x^* \in F$. Since F is prime and $x \notin F$, we get $x^* \in F \subsetneqq F'$. Since $x, x^* \in F$ we get that $0 = x \wedge x^* \in F'$, which is a contradiction. Therefore, F is a maximal filter. ∎

We have the following result.

Theorem 3.9. *Let F, G be two filters of a pseudo-complemented ADL such that $F \subseteq G$. If F is a Boolean filter then so is G.*

Proof. Let F be a Boolean filter of L. Suppose G is any filter of L with $F \subseteq G$. We prove that G is a Boolean filter of L. Clearly, we have $x \vee x^* \in F$, for all $x \in L$. Since $F \subseteq G$, we get that $x \vee x^* \in G$, for all $x \in L$. Hence G is a Boolean filter of L. ∎

We now characterize the Boolean filters in the following:

Theorem 3.10. *Let F be a proper filter of a pseudo-complemented ADL L. Then the following conditions are equivalent:*

(1) *F is a Boolean filter.*

(2) *$x^{**} \in F$ implies $x \in F$.*

(3) *For $x, y \in L$, $x^* = y^*$ and $x \in F$ imply $y \in F$.*

Proof. (1)$\Rightarrow$(2): Assume that F is a Boolean filter of L. Suppose $x^{**} \in F$. Since F is a Boolean filter, we get $x \vee x^* \in F$. Now, $x = (x^{**} \wedge x) \vee 0 = (x^{**} \wedge x) \vee (x^{**} \wedge x^*) = x^{**} \wedge (x \vee x^*) \in F$. Therefore $x \in F$.

(2)$\Rightarrow$(3): Let $x, y \in L$ and $x^* = y^*$. Suppose $x \in F$. Then $x^{**} \in F$. That implies $y^{**} \in F$.

(3)$\Rightarrow$(1): Assume that condition (3). We prove that F is a Boolean filter of L. For that it is enough to prove that $D \subseteq F$. Let $x \in D$. Then $x^* = 0 \leq a^*$ for any $a \in F$. Hence $a^{**} \leq x^{**}$ and $a^{**} \in F$. Hence $x^{**} \in F$. Since $x^* = x^{***}$ and

$x^{**} \in F$, by the condition (3), we get $x \in F$. Hence $D \subseteq F$. Since D is a Boolean filter, by theorem3.9, we get that F is a Boolean filter of L. ∎

Now, we discuss about the homomorphic images of Boolean filters of pseudo-complemented ADLs. By a homomorphism f on a pseudo-complemented ADL, we mean a bounded homomorphism which also preserves the pseudo-complementation, that is, $f(x^*) = f(x)^*$ for all $x \in L$.

Theorem 3.11. *Let $(L, \vee, \wedge, ^*, 0)$ and $(L', \vee, \wedge, ^*, 0')$ be two pseudo-complemented ADLs and ψ, a homomorphism from L onto L'. Then we have the following conditions:*

(1) *$\psi(F)$ is a Boolean filter of L' whenever F is a Boolean filter of L.*

(2) *$\psi^{-1}(G)$ is a Boolean filter of L whenever G is a Boolean filter of L'.*

Proof. (1) Suppose F is a Boolean filter of L. It is known that $\psi(F)$ is a filter of L'. Let $y \in L'$. Since is onto, there exists $x \in L$ such that $\psi(x) = y$. Since F is a Boolean filter of L, we get $x \vee x^* \in F$. Now, $y \vee y^* = \psi(x) \vee \psi(x)^* = \psi(x) \vee \psi(x^*) = \psi(x \vee x^*) \in \psi(F)$. Therefore, $\psi(F)$ is a Boolean filter of L'.

(2) Let G be a Boolean filter of L'. Clearly $\psi^{-1}(G)$ is a filter of L. Let $x \in L$. Then $\psi(x \vee x^*) = \psi(x) \vee \psi(x^*) = \psi(x) \vee \psi(x)^* \in G$, since $\psi(x) \in L'$. Hence we get $x \vee x^* \in \psi^{-1}(G)$. Therefore, $\psi^{-1}(G)$ is a Boolean filter of L. ∎

Let L be an ADL and F a filter in L. Then the relation $\psi_F = \{(x,y) \in L \times L \mid x \wedge t = y \wedge t$, for some $t \in F\}$ is a congruence relation on L. Then the set $L/\psi_F = \{x/\psi_F \mid x \in L\}$ is an ADL. Let $\prod$ be the natural homomorphism from L onto L/ψ_F defined by $\prod(x) = x/\psi_F$, for all $x \in L$. Note that for any filter F of L, L/ψ_F need not be a lattice. For, consider the following example.

Example 3.12. Let D be a discrete ADL. Let $F = D \setminus \{0\}$. Then $\psi_F = \{(a,b) \in D \times D \mid x \wedge a = x \wedge b$ for some $x \neq 0\} = \Delta$.

Therefore $D/\psi_F (\cong D)$ which is not a lattice, unless $|D| \leq 2$.

Theorem 3.13. *Let L be an ADL. For any filter F of L and $x \in L$, we have the following:*

(1) *x/ψ_F is the largest element in L/ψ_F if and only if $x \in F$.*

(2) *$x/\psi_F = 0/\psi_F$ if and only if $(x)^* \cap F \neq \emptyset$.*

Proof. (1) Let $x \in L$ such that x/ψ_F is the largest element of L/ψ_F. Since $F \neq \emptyset$, we can choose $y \in F$. Then $x/\psi_F \wedge y/\psi_F = y/\psi_F$. Therefore $(x \wedge y,\ y) \in \psi_F$. Hence, there exists $a \in F$ such that $x \wedge y \wedge a = y \wedge a \in F$ and hence $x \in F$. On the other hand, let $x \in F$ and $y \in L$. Then $(y,\ y \wedge x) \in \psi_F$. Therefore $y/\psi_F = y/\psi_F \wedge x/\psi_F$. Thus x/ψ_F is the largest element of L/ψ_F.

(2) Let $x \in L$. Suppose $x/\psi_F = 0/\psi_F$. Then $(x, 0) \in \psi_F$. Therefore $x \wedge t = 0 \wedge t = 0$, for some $t \in F$. Hence $t \in (x)^* \cap F$ and hence $(x)^* \cap F \neq \emptyset$. Conversely suppose $(x)^* \cap F \neq \emptyset$. Choose $y \in (x)^* \cap F$. Then $x \wedge y = 0 = 0 \wedge y$, $y \in F$. Therefore $(x, 0) \in \psi_F$. Hence $x/\psi_F = 0/\psi_F$. ■

Now, Boolean filters are characterized in terms of congruence ψ_F.

Theorem 3.14. *Let F be a filter of a pseudo-complemented ADL L. Then the following conditions are equivalent:*

(1) *F is a Boolean filter.*

(2) *L/ψ_F is a Boolean algebra.*

Proof. (1)⇒(2): Assume that F is a Boolean filter of L. for any $x \in L$, we have always $x \wedge x^* = 0$ and hence $x/\psi_F \wedge x^*/\psi_F = (x \wedge x^*)/\psi_F = 0/\psi_F$. Since F is a Boolean filter, we get that $x \vee x^* \in F$. Hence we have $x/\psi_F \vee x^*/\psi_F = (x \vee x^*)/\psi_F$ is the largest element of L/ψ_F. Therefore, L/ψ_F is a Boolean algebra.

(2)⇒(1): Assume that L/ψ_F is a Boolean algebra. Let $x \in L$. Then $x/\psi_F \in L/\psi_F$. Since L/ψ_F is a Boolean algebra, there exists $y \in L$ such that $(x \wedge y)/\psi_F = x/\psi_F \wedge y/\psi_F = 0/\psi_F$ and $(x \vee y)/\psi_F = x/\psi_F \vee y/\psi_F$ is the largest element of L/ψ_F. Hence it follows that $(x \wedge y, 0) \in \psi_F$ and $x \vee y \in F$. Since $(x \wedge y, 0) \in \psi_F$, there exists $f \in F$ such that $x \wedge y \wedge f = 0$ and thus we get $y \wedge f = x^* \wedge y \wedge f$. Therefore, we get the following consequence. $x \vee y \in F$ and $f \in F \Rightarrow (x \vee y) \wedge f \in F \Rightarrow (x \wedge f) \vee (y \wedge f) \in F \Rightarrow (x \wedge f) \vee x^* \in F$ since $y \wedge f = x^* \wedge y \wedge f \Rightarrow (x \vee x^*) \wedge (f \vee x^*) \in F \Rightarrow x \vee x^* \in F$. Therefore, F is a Boolean filter of L. ■

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