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SOLUTIONS OF THE HAMMERSTEIN EQUATIONS IN $BV_\varphi(I_A^B, \mathbb{R})$

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Abstract

In this paper we study existence and uniqueness of solutions for the Hammerstein equation

$$u(x) = v(x) + \lambda \int_{I_a^b} K(x,y) f(y, u(y)) dy$$

in the space of function of bounded total φ-variation in the sense of Hardy-Vitali-Tonelli, where $\lambda \in \mathbb{R}$, $K : I_a^b \times I_a^b \longrightarrow \mathbb{R}$ and $f : I_a^b \times \mathbb{R} \longrightarrow \mathbb{R}$ are suitable functions. The existence and uniqueness of solutions are proved by means of the Leray-Schauder nonlinear alternative and the Banach contraction mapping principle.

Keywords: Hammerstein integral equation, Banach spaces, bounded φ-variation in the sense of Hardy-Vitali-Tonelli, Banach's contraction principle, Leray-Schauder nonlinear alternative principle.

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