

COMPLICATED BE-ALGEBRAS AND CHARACTERIZATIONS OF IDEALS

YILMAZ ÇEVEN

Department of Mathematics, Faculty of Arts and Sciences
Süleyman Demirel University, 32260 Isparta, Turkey

e-mail: yilmazceven@sdu.edu.tr

AND

ZEKIYE ÇİLOĞLU

Department of Mathematics, Faculty of Arts and Sciences
Süleyman Demirel University, 32260 Isparta, Turkey

e-mail: zekiyeciloglu@sdu.edu.tr

Abstract

In this paper, using the notion of upper sets, we introduced the notions of complicated BE-Algebras and gave some related properties on complicated, self-distributive and commutative BE-algebras. In a self-distributive and complicated BE-algebra, characterizations of ideals are obtained.

Keywords: BE-algebras, complicated BE-algebras, ideals in BE-algebras.

2010 Mathematics Subject Classification: 03G25, 06F35.

1. INTRODUCTION

Y. Imai and K. Iséki introduced two classes of abstract algebras called BCK-algebras and BCI-algebras [8, 10]. It is known that the class of BCK-algebras is a proper subclass of BCI-algebras. In [5, 6], Q.P. Hu and X. Li introduced a wide class of abstract algebras called BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of BCH-algebras. J. Neggers and H.S. Kim ([16]) introduced the notion of a d-algebra which is a generalization of BCK-algebras, and also they introduced the notion of B-algebras ([17, 18]). Y.B. Jun, E.H. Roh and H.S. Kim ([11]) introduced a new notion called BH-algebra which

is another generalization of BCH/BCI/BCK-algebras. A. Walendziak obtained another equivalent axioms for B-algebras ([20]). C.B. Kim and H.S. Kim ([13]) introduced the notion of BM-algebra which is a specialization of B-algebras. They proved that the class of BM-algebras is a proper subclass of B-algebras and also showed that a BM-algebra is equivalent to a 0-commutative B-algebra. In [14], H.S. Kim and Y.H. Kim introduced the notion of BE-algebra as a generalization of a BCK-algebra. Using the notion of upper sets they gave an equivalent condition of the filter in BE-algebras. In [2] and [3], S.S. Ahn and K.S. So introduced the notion of ideals in BE-algebras, and proved several characterizations of such ideals. Also they generalized the notion of upper sets in BE-algebras and discussed some properties of the characterizations of generalized upper sets related to the structure of ideals in transitive and self distributive BE-algebras. In [4], S.S. Ahn, Y.H. Kim and J.M. Ko are introduced the notion of terminal section of BE-algebras and provided the characterization of a commutative BE-algebras.

B.M. Schein [19] considered systems of the form $(\phi; \circ, \setminus)$, where ϕ is a set of functions closed under the composition " \circ " of functions (and hence $(\phi; \circ)$ is a function semigroup) and the set theoretic subtraction " \setminus " (and hence $(\phi; \setminus)$ is a subtraction algebra in the sense of [1]). B. Zelinka [22] discussed a problem proposed by B.M. Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. Y.B. Jun *et al.* [12] introduced the complicated subtraction algebras and investigated several properties on it.

In this paper, using the notion of upper sets, we introduced the notions of complicated BE-Algebras and gave some related properties on complicated, self-distributive and commutative BE-algebras. In a self-distributive and complicated BE-algebra, characterizations of ideals are obtained.

2. PRELIMINARIES

Definition 2.1 [14]. An algebra $(X; *, 1)$ of type $(2, 0)$ is called a BE-algebra if, for all a, b, c in X , the following identities hold:

$$(BE1) \quad a * a = 1,$$

$$(BE2) \quad a * 1 = 1,$$

$$(BE3) \quad 1 * a = a,$$

$$(BE4) \quad a * (b * c) = b * (a * c).$$

In a BE-algebra X , the relation " \leq " is defined by $a \leq b$ if and only if $a * b = 1$.

Proposition 2.2 [14]. *If $(X; *, 1)$ is a BE-algebra, then*

$$(i) \quad a * (b * a) = 1,$$

(ii) $a * ((a * b) * b) = 1$
for any $a, b \in X$.

Example 2.1. [14] Let $X = \{1, a, b, c, d, 0\}$ be a set with the following table:

$*$	1	a	b	c	d	0
1	1	a	b	c	d	0
a	1	1	a	c	c	d
b	1	1	1	c	c	c
c	1	a	b	1	a	b
d	1	1	a	1	1	a
0	1	1	1	1	1	1

Then $(X; *, 1)$ is a BE-algebra.

Definition 2.3 [14]. A BE-algebra $(X; *, 1)$ is said to be self-distributive if $a * (b * c) = (a * b) * (a * c)$ for all $a, b, c \in X$.

Example 2.2 [14]. Let $X = \{1, a, b, c, d\}$ be a set with the following table:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	a	1	c	c
c	1	1	b	1	b
d	1	1	1	1	1

Then $(X; *, 1)$ is a self-distributive BE-algebra.

Proposition 2.4 ([2, 4]). Let $(X; *, 1)$ be a self-distributive BE-algebra. If $a \leq b$, then, for all a, b, c in X , the following hold:

- (i) $c * a \leq c * b$,
- (ii) $b * c \leq a * c$,
- (iii) $a * b \leq (b * c) * (a * c)$.

Definition 2.5 [21]. Let X be a BE-algebra. We say that X is commutative if (C) $(a * b) * b = (b * a) * a$ for all $a, b \in X$.

Proposition 2.6 [21]. If $(X; *, 1)$ is a commutative BE-algebra, then for all $a, b \in X$,

$$a * b = 1 \text{ and } b * a = 1 \text{ imply } a = b.$$

Definition 2.7 [2]. Let X be a BE-algebra. A nonempty subset I of X is called an ideal of X if

- (I1) $\forall x \in X$ and $\forall a \in I$ imply $x * a \in I$,
 (I2) $\forall x \in X$ and $\forall a, b \in I$ imply $(a * (b * x)) * x \in I$.

Corollary 2.8 [2]. Let I be an ideal of X . If $a \in I$ and $a \leq x$, then $x \in I$.

Corollary 2.9 [2]. Let X be a self-distributive BE-algebra. A nonempty subset I of X is an ideal of X if and only if it satisfies the following conditions

- (I3) $1 \in I$,
 (I4) $x * (y * z) \in I$ and $y \in I$ imply $x * z \in I$ for all $x, y, z \in X$.

3. COMPLICATED BE-ALGEBRAS

Definition 3.1. Let $(X; *, 1)$ be a BE-algebra and $a, b \in X$. The set

$$A(a, b) = \{x \in X : a * (b * x) = 1\}$$

is called an upper set of a and b . It is easy to see that $1, a, b \in A(a, b)$.

Proposition 3.2. Let $(X; *, 1)$ be a BE-algebra. Then $A(a, b) = A(b, a)$ for all $a, b \in X$.

Proof. It is clear by (BE4). ■

Example 3.1. Let $X = \{1, a, b, c\}$ be a set with the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	1	1	c
c	1	1	1	1

It is clear that X is a BE-algebra and $A(1, 1) = \{1\}$, $A(1, a) = A(a, a) = \{1, a\}$, $A(1, b) = A(a, b) = A(b, b) = \{1, a, b\}$ and $A(1, c) = A(a, c) = A(b, c) = A(c, c) = X$.

Example 3.2. Let $X = \{1, a, b, c\}$ be a set with the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	a	1	c
c	1	1	1	1

It is clear that X is a BE-algebra and $A(1, 1) = \{1\}$, $A(1, a) = A(a, a) = \{1, a\}$, $A(1, b) = A(b, b) = \{1, b\}$, $A(a, b) = \{1, a, b\}$ and $A(1, c) = A(a, c) = A(b, c) = A(c, c) = X$.

Definition 3.3. A BE-algebra $(X; *, 1)$ is called a complicated BE-algebra (c-BE-algebra, shortly) if for all $a, b \in X$, the set $A(a, b)$ has the smallest element. The smallest element of $A(a, b)$ is denoted by $a \circledast b$.

Example 3.3. The BE-algebra X in Example 3.1 is a c-BE-algebra since $1 \circledast 1 = 1$, $1 \circledast a = a$, $a \circledast a = a$, $1 \circledast b = a \circledast b = b \circledast b = b$ and $1 \circledast c = a \circledast c = b \circledast c = c \circledast c = c$. But the BE-algebra in Example 3.2 is not a c-BE-algebra since $A(a, b) = \{1, a, b\}$ has no the smallest element.

Proposition 3.4. Let $(X; *, 1)$ be a c-BE-algebra. Then, for all $a, b \in X$,

- (i) $a \circledast b \leq a$ and $a \circledast b \leq b$,
- (ii) $a \circledast 1 = a$,
- (iii) $a \circledast b = b \circledast a$,
- (iv) $a \circledast (a * b) \leq b$.

Proof. (i) and (ii) are easily seen by the definition of the c-BE algebra.

(iii) is clear since $A(a, b) = A(b, a)$.

(iv) From Proposition 2.1 (i), since $a * ((a * b) * b) = 1$, we have $b \in A(a, a * b)$ and hence $a \circledast (a * b) \leq b$. ■

Proposition 3.5. Let $(X; *, 1)$ be a self-distributive BE-algebra. If, for all $a, b, c \in X$, $a \leq b$ and $b \leq c$ then $a \leq c$.

Proof. Since $a * c = 1 * (a * c) = (a * b) * (a * c) = a * (b * c) = a * 1 = 1$, we have $a \leq c$. ■

Proposition 3.6. Let $(X; *, 1)$ be a self-distributive c-BE-algebra. Then, for all $a, b, c \in X$,

- (i) $a \leq b$ implies $a \circledast c \leq b \circledast c$,
- (ii) $(a * b) \circledast (b * c) \leq a * c$.

Proof. (i) Let $a \leq b$. Since X is self-distributive, by Proposition 2.4 (ii), we have $b * (b \circledast c) \leq a * (b \circledast c)$. Also since $b \circledast c \in A(b, c)$, we have $c \leq b * (b \circledast c)$. Then by Proposition 3.5, we get $c \leq a * (b \circledast c)$. Hence we obtain $b \circledast c \in A(a, c)$ and $a \circledast c \leq b \circledast c$.

(ii) By Proposition 2.4. (iii), we have $a * b \leq (b * c) * (a * c)$. Hence we see that $a * c \in A(a * b, b * c)$ and $(a * b) \circledast (b * c) \leq a * c$. ■

Theorem 3.7. *Let $(X; *, 1)$ be a self-distributive and commutative c -BE-algebra. Then $(X; \mathbb{S})$ is a commutative monoid.*

Proof. By Proposition 3.4 (ii) and (iii), we need only to show that $(X; \mathbb{S})$ is associative. Say $(a \mathbb{S} b) \mathbb{S} c = u$. Then, since $u \in A(a \mathbb{S} b, c)$ and $A(a \mathbb{S} b, c) = A(c, a \mathbb{S} b)$, we know that

$$(3.1) \quad a \mathbb{S} b \leq c * u$$

and

$$(3.2) \quad c \leq (a \mathbb{S} b) * u.$$

Hence using the equation (3.1), we have, by Proposition 2.4 (i) and (BE4),

$$(3.3) \quad b * (a \mathbb{S} b) \leq b * (c * u) = c * (b * u).$$

Since $a \leq b * (a \mathbb{S} b)$, using the equation (3.3) and Proposition 3.5, we obtain

$$(3.4) \quad a \leq c * (b * u).$$

From the equation (3.4), we have $b * u \in A(a, c)$ and $a \mathbb{S} c \leq b * u$. So we see that $u \in A(a \mathbb{S} c, b)$, that is,

$$(3.5) \quad (a \mathbb{S} c) \mathbb{S} b \leq (a \mathbb{S} b) \mathbb{S} c = u.$$

Since the equation (3.5) is true for all $a, b, c \in X$, the following inequality is true:

$$(3.6) \quad (a \mathbb{S} b) \mathbb{S} c \leq (a \mathbb{S} c) \mathbb{S} b.$$

Hence by Proposition 2.6, using the equation (3.5) and (3.6), we get

$$(3.7) \quad (a \mathbb{S} b) \mathbb{S} c = (a \mathbb{S} c) \mathbb{S} b.$$

Then we obtain $(a \mathbb{S} b) \mathbb{S} c = (b \mathbb{S} a) \mathbb{S} c = (b \mathbb{S} c) \mathbb{S} a = a \mathbb{S} (b \mathbb{S} c)$. ■

Proposition 3.8. *If $(X; *, 1)$ is a self-distributive and commutative c -BE-algebra and $X \neq \{1\}$, then $(X; \mathbb{S})$ has no group structure.*

Proof. Let $1 \neq a \in X$. Hence we have $a \leq 1$. If there exists an element $b \in X$ such that $a \mathbb{S} b = b \mathbb{S} a = 1$, then since $1 = a \mathbb{S} b \leq a \leq 1$, we have $a = 1$. This is a contradiction. ■

Proposition 3.9. *Let $(X; *, 1)$ be a self-distributive and commutative c-BE-algebra. Then $a \leq b$ implies $a \circledast b = a$.*

Proof. (i) Let $a \leq b$. Hence we have $a * b = 1$. Then we get

$$\begin{aligned} a * (a \circledast b) &= 1 * (a * (a \circledast b)) \\ &= (a * b) * (a * (a \circledast b)) \\ &= a * (b * (a \circledast b)), && \text{by self-distributivity property} \\ &= 1 \end{aligned}$$

since $a \circledast b \in A(a, b)$. Hence $a \leq b * (a \circledast b)$. Then we have $a \leq a \circledast b$. Also we know that $a \circledast b \leq a$. Hence we obtain $a \circledast b = a$ by Proposition 2.6. \blacksquare

Now, in a c-BE-algebra, define the set

$$(3.8) \quad B(a, b) = \{x \in X : x \circledast a \leq b\}$$

Theorem 3.10. *Let $(X; *, 1)$ is a self-distributive c-BE-algebra. Then the set $B(a, b)$ in equation (3.8) has the greatest element and it is $a * b$.*

Proof. Since $a * b \leq a * b$, we have $b \in A(a * b, a)$. Hence we get $(a * b) \circledast a \leq b$. So, it is seen that $a * b \in B(a, b)$. If $c \in B(a, b)$, we write $c \circledast a \leq b$. By Proposition 2.4 (i), we have $a * (c \circledast a) \leq a * b$. Since $c \circledast a \in A(c, a)$, we have $c \leq a * (c \circledast a)$. Then we obtain $c \leq a * b$, by Proposition 3.5. Hence $a * b$ is the greatest element of $B(a, b)$. \blacksquare

Proposition 3.11. *Let $(X; *, 1)$ be a self-distributive and commutative c-BE-algebra. Then*

- (i) $a \circledast b \leq a * b \leq (a \circledast c) * (c \circledast b)$,
- (ii) $(a * b) \circledast a = a \circledast b$,
- (iii) $(a \circledast b) * c = a * (b * c)$,
- (iv) $a * (b \circledast c) = (a * b) \circledast (a * c)$,
- (v) $a \circledast b$ is the greatest lower bound of the set $\{a, b\}$.

Proof. (i) Using Proposition 3.4 (iv) and Proposition 3.6 (i), we have $c \circledast (a \circledast (a * b)) \leq c \circledast b$. We get $(c \circledast a) \circledast (a * b) \leq c \circledast b$ or by Proposition 3.4 (iii), $(a * b) \circledast (c \circledast a) \leq c \circledast b$. Hence since $a * b \in B(c \circledast a, c \circledast b)$, we obtain $a * b \leq (a \circledast c) * (c \circledast b)$. Also it is known that $a \circledast b \leq b \leq a * b$. By Proposition 3.5, we get $a \circledast b \leq a * b \leq (a \circledast c) * (c \circledast b)$.

(ii) Since $a * b \in B(a, b)$, we have $(a * b) \circledast a \leq b$. Using Proposition 3.4 (i), commutativity and associativity of the operation \circledast , we get $(a * b) \circledast (a \circledast a) \leq a \circledast b$. By Proposition 3.9, we see that $a \circledast a = a$ since $a \leq a$. Hence $(a * b) \circledast a \leq a \circledast b$.

Secondly, since $b \leq a * b$, by commutativity of the operation $\textcircled{\text{S}}$ and Proposition 3.6 (i), we have $a \textcircled{\text{S}} b \leq (a * b) \textcircled{\text{S}} a$. So we obtain $(a * b) \textcircled{\text{S}} a = a \textcircled{\text{S}} b$ by Proposition 2.6.

(iii) $a \textcircled{\text{S}} b \in A(a, b)$ implies $a \leq b * (a \textcircled{\text{S}} b)$. Also from Proposition 2.4 (iii), we have $b * (a \textcircled{\text{S}} b) \leq ((a \textcircled{\text{S}} b) * c) * (b * c)$. So we get $a \leq ((a \textcircled{\text{S}} b) * c) * (b * c)$ by Proposition 3.5. Then we have $(a \textcircled{\text{S}} b) * c \leq a * (b * c)$. Secondly, using Proposition 2.2 (ii), Proposition 2.4 (iii) and (BE4), since

$$\begin{aligned} b &\leq (b * c) * c \\ &\leq (a * (b * c)) * (a * c) \\ &= a * ((a * (b * c)) * c), \end{aligned}$$

we have $b \leq a * ((a * (b * c)) * c)$ or $a \leq b * ((a * (b * c)) * c)$. Then we obtain $a \textcircled{\text{S}} b \leq (a * (b * c)) * c$ or $a * (b * c) \leq (a \textcircled{\text{S}} b) * c$. Consequently we see that $a * (b * c) = (a \textcircled{\text{S}} b) * c$.

(iv) By (i), we have $a * c \leq (a \textcircled{\text{S}} b) * (b \textcircled{\text{S}} c)$ or $a \textcircled{\text{S}} b \leq (a * c) * (b \textcircled{\text{S}} c)$. Then we get $a * (a \textcircled{\text{S}} b) \leq a * ((a * c) * (b \textcircled{\text{S}} c))$ by Proposition 2.4 (i). We can write $a * b \leq (a \textcircled{\text{S}} a) * (a \textcircled{\text{S}} b) \leq (a * c) * (a * (b \textcircled{\text{S}} c))$ by (i). Hence since $a * (b \textcircled{\text{S}} c) \in A(a * b, a * c)$, we have

$$(3.9) \quad (a * b) \textcircled{\text{S}} (a * c) \leq a * (b \textcircled{\text{S}} c).$$

Secondly, since $b \textcircled{\text{S}} c \leq b$, we have $a * (b \textcircled{\text{S}} c) \leq a * b$ by Proposition 2.4 (i). Hence we get

$$(3.10) \quad (a * (b \textcircled{\text{S}} c)) \textcircled{\text{S}} (a * c) \leq (a * b) \textcircled{\text{S}} (a * c).$$

Also since $b \textcircled{\text{S}} c \leq c$, we have $a * (b \textcircled{\text{S}} c) \leq a * c$ and so we get $(a * (b \textcircled{\text{S}} c)) \textcircled{\text{S}} (a * (b \textcircled{\text{S}} c)) \leq (a * (b \textcircled{\text{S}} c)) \textcircled{\text{S}} (a * c)$, that is

$$(3.11) \quad a * (b \textcircled{\text{S}} c) \leq (a * (b \textcircled{\text{S}} c)) \textcircled{\text{S}} (a * c).$$

Hence from the equation (3.10) and (3.11) and by Proposition 3.5, we obtain

$$(3.12) \quad a * (b \textcircled{\text{S}} c) \leq (a * (b \textcircled{\text{S}} c)) \textcircled{\text{S}} (a * c).$$

The equations (3.9) and (3.12) show that $a * (b \textcircled{\text{S}} c) = (a * (b \textcircled{\text{S}} c)) \textcircled{\text{S}} (a * c)$ by Proposition 2.6.

(v) Since $a \textcircled{\text{S}} b \leq a$ and $a \textcircled{\text{S}} b \leq b$, $a \textcircled{\text{S}} b$ is the lower bound of the set $\{a, b\}$. Let c be another lower bound of the set $\{a, b\}$. Then we know that $c * a = 1$ and $c * b = 1$. So since $c * (a \textcircled{\text{S}} b) = (c * a) \textcircled{\text{S}} (c * b) = 1 \textcircled{\text{S}} 1 = 1$, we have $c \leq a \textcircled{\text{S}} b$. ■

Remark 3.1. Let $(X; *, 1)$ be a BE-algebra. In [21], the binary operation "+" on X was defined as the following: for any $a, b \in X$,

$$a + b = (a * b) * b.$$

Also the author proved that if $(X; *, 1)$ is a commutative BE-algebra, then $(X; +)$ is a semilattice. By Proposition 3.11 (v), we proved that a self-distributive and commutative c-BE-algebra X is a semilattice under the operation " \textcircled{S} ". In a self-distributive and commutative c-BE-algebra, since $a \leq a + b$ by Proposition 2.2 (ii) and using Proposition 3.9, we see that $a \textcircled{S}(a + b) = a$. Also, since $a \leq b$ implies $a + b = b$ and since $a \textcircled{S}b \leq a$, we have $(a \textcircled{S}b) + a = a$. Therefore any self-distributive and commutative c-BE-algebra is a lattice with respect to operations " \textcircled{S} " and "+".

Now we provide characterizations of ideals in a self-distributive c-BE-algebra.

Corollary 3.12 [2]. *Let $(X; *, 1)$ be a self-distributive BE-algebra. A nonempty subset I of X is an ideal of X if and only if $A(u, v) \subseteq I$ for all $u, v \in I$.*

Theorem 3.13. *Let $(X; *, 1)$ be a self-distributive c-BE-algebra. A nonempty subset I of X is an ideal of X if and only if it satisfies the following conditions:*

- (i) $\forall a \in I, \forall x \in X, a \leq x \implies x \in I$,
- (ii) $\forall a, b \in I, \exists c \in I, c \leq a$ and $c \leq b$.

Proof. Let I be an ideal of X . (i) follows from the Corollary 2.8. Let $a, b \in I$. From Corollary 3.12, we have $A(a, b) \subseteq I$. Then we get $a \textcircled{S}b \in I$. If we take $a \textcircled{S}b = c$, then we have $c \leq a$ and $c \leq b$ by Proposition 3.4 (i) which proves (ii). Conversely, let I be a non-empty subset of X satisfying (i) and (ii). Since for $a \in I, a \leq 1$ by (BE2), we have $1 \in I$ by (i). For any $a, b, c \in X$, let $b \in I$ and $a * (b * c) \in I$. By (ii), there exists $d \in I$ such that $d \leq b$ and $d \leq a * (b * c)$. Then using (BE3), (BE4), and self-distributivity, we have

$$1 = d * (a * (b * c)) = d * (b * (a * c)) = (d * b) * (d * (a * c)) = d * (a * c).$$

Hence, we get $d \leq a * c$. By (i), it is obtained $a * c \in I$. So I is an ideal of X by Corollary 2.9. ■

Theorem 3.14. *Let $(X; *, 1)$ be a self-distributive c-BE-algebra. A non-empty subset I of X is an ideal of X if and only if it satisfies the following conditions:*

- (i) $\forall a \in I, \forall x \in X, a \leq x \implies x \in I$,
- (ii) $\forall a, b \in I, a \textcircled{S}b \in I$.

Proof. The necessity is given in the proof of Theorem 3.13. Conversely, since for $a \in I, a \leq 1$ by (BE2), we have $1 \in I$ by (i). Let I be a non-empty subset of X satisfying (i) and (ii). We know that $x * y \in B(x, y)$ in a self-distributive c -BE-algebra. So $(x * y) \odot x \leq y$ and hence

$$(3.13) \quad x \odot (x * y) \leq y.$$

Now let $y \in I$ and $x * (y * z) \in I$. By (ii) and (BE4), we get $y \odot (x * (y * z)) = y \odot (y * (x * z)) \in I$. From the equation 3.13, it is clear that $y \odot (y * (x * z)) \leq x * z$. Hence it is obtained $x * z \in I$ by (i). Consequently, I is an ideal of X by Corollary 2.9. ■

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Received 11 November 2014

Revised 10 January 2015