COLLOQUIUM MATHEMATICUM

XVIII DÉDIÉ À M. TADEUSZ WAŻEWSKI 1967

ON THE ABSOLUTE NÖRLUND SUMMABILITY OF ORTHOGONAL SERIES FOR A SYSTEM OF H-TYPE

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Let

(A) $$a_0+a_1+\ldots+a_n+\ldots$$

be a given series with partial sums s_n, and let us form the Nörlund transform

$$t_n = \frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}s_k \qquad (P_n \neq 0),$$

where $\{p_n\}$ is a sequence of real numbers and $P_n = p_0+p_1+\ldots+p_n$.

The series (A) will be said to be *absolutely summable* (N, p_n) or shortly $|N, p_n|$-*summable* provided that the series $\sum_{n=1}^{\infty}|t_n-t_{n-1}|$ is convergent.

The $|N, p_n|$-summability implies the (N, p_n)-summability, but not conversely (see [1], p. 169).

The conditions for regularity of the method (N, p_n) are

$$\text{(a)}\quad \frac{p_n}{P_n}\to 0 \qquad\text{and}\qquad \text{(b)}\quad \sum_{v=0}^{n}|p_v| = O(P_n).$$

In particular, if $\{p_n\}$ is non-negative, then

$$\lim_{n\to\infty}\frac{p_n}{P_n} = 0$$

is the necessary and sufficient condition for the regularity of the method (N, p_n) of summation.

In the present note we shall deal with the $|N, p_n|$-summability of orthogonal series for a system of H-type, which has been defined recently by L. Leindler (see [3], p. 244). The below obtained results establish conditions under which Leindler's theorem (see [3], Satz V, p. 261-265) can be carried over to the case of some classes of Nörlund's means. The idea and the proof of the theorem presented here are similar to the Leindler's result.

In the sequel, we shall limit ourselves mainly to the special classes $\overline{M}^{\alpha}(\alpha > -1)$ of Nörlund's means:

A sequence $\{p_n\}$ will be said to *belong to the class* $\overline{M}^{\alpha}$ $(-1 < \alpha < 0)$, if

(i) $$p_0 > 0 \text{ and } p_n < 0 \quad \text{for} \quad n = 1, 2, \ldots,$$

(ii) $$p_1 < p_2 < \ldots < p_n < p_{n+1} < \ldots,$$

(iii) $$\lim_{n\to\infty} \frac{n(p_n - p_{n-1})}{p_n} = \alpha - 1.$$

A sequence $\{p_n\}$ will be said to *belong to the class* $\overline{M}^{\alpha}$ with $\alpha \geqslant 0$, if

(j) $$0 < p_{n+1} < p_n \quad \text{or} \quad 0 < p_n < p_{n+1} \quad (n = 0, 1, 2, \ldots),$$

(jj) $$p_0 + p_1 + \ldots + p_n = P_n \nearrow +\infty,$$

(jjj) $$\lim_{n\to\infty} \frac{n(p_n - p_{n-1})}{p_n} = \alpha - 1.$$

An orthonormal system $\{\chi_n(x)\}$, defined in the interval $(0, 1)$, is said to be a *system of H-type*, if for every $x \in (0, 1)$ the relation

$$\chi_n(x)\,\chi_m(x) = 0 \qquad (2^k < n, m \leqslant 2^{k+1}, n \neq m;\ k = 0, 1, 2, \ldots)$$

holds true.

In the sequel, we shall use the following lemmas:

LEMMA 1. *If* $\{p_n\} \in \overline{M}^{\alpha}$ *with* $\alpha > -1$ *and* $\alpha \neq 0$, *then*

$$C_1(\alpha) k p_n p_{n-k} < |p_{n-k} P_n - p_n P_{n-k}| < C_2(\alpha) k p_n p_{n-k}$$
$$(n = N, N+1, \ldots;\ k = 1, 2, \ldots, n),$$

where $C_1(\alpha)$ *and* $C_2(\alpha)$ *are positive constants dependent only on* α, *and* N *denotes a sufficiently large natural number.*

Remark. The expression under the sign of absolute value is always positive if $\alpha > 0$, and always negative if $-1 < \alpha < 0$.

Lemma 1 is known (see [8], Lemma 4).

LEMMA 2. *Let* $\{p_n\} \in \overline{M}^{\alpha}$, $\alpha \geqslant 0$, *and let* $\{q_n\}$ *be a convex or concave sequence such that* $\{q_n\} \in \overline{M}^{\beta}$ *with an arbitrary* $\beta > 0$. *If* $\sum u_n$ *is* $|N, p_n|$-*summable, then it is* $|N, r_n|$-*summable, where*

$$r_n = \sum_{k=0}^{n} p_k q_{n-k} \quad \text{and} \quad \{r_n\} \in \overline{M}^{\alpha+\beta}.$$

This lemma is also known (see [9], Theorem 1).

LEMMA 3. *Let $\{R_n(x)\}$ denote a system of step-functions defined in the interval $(0,1)$. Moreover, let $J_s(n)$ $(s=1,2,\ldots,s_n)$ denote the intervals in which $R_n(x)$ is a constant function, $n=1,2,\ldots$*

If for any $m>n$ the condition

$$\int_{J_s(n)} \operatorname{sign} R_m(x)\,dx = 0 \qquad (s=1,2,\ldots,s_n),$$

is satisfied, then for every real numerical sequence $d_1, d_2, \ldots, d_N$ there exists a simple set E_k (1) such that for $x\in E_k$ the following inequalities hold:

$$\Big|\sum_{l=1}^{N} d_l R_l(x)\Big| \geqslant |d_{N-k}R_{N-k}(x)| \qquad (k=0,1,\ldots,N-1)$$

and

$$|E_k \cap J_s(N-k-1)| = \frac{|J_s(N-k-1)|}{2^{k+1}} \ (2)$$

$$(k=0,1,2,\ldots,N-1;\ s=1,2,\ldots,s_{N-k-1};\ J_1(0)=(0,1)).$$

This lemma is known (see [3], Hilfssatz I, p. 246-249). In the proof of this lemma L. Leindler has given a construction of a system $\{\chi_n(x)\}$ of H-type. When taking $\chi_n(x)=r_n(x)$ $(n=0,1,2)$, where $\{r_n(x)\}$ is the Rademacher system, it will be assumed that the step-functions $\chi_n(x)$ have been already defined for $n=0,1,\ldots,2^s$; $s\geqslant 1$, so that they form a system of H-type. Next, it will be proved that they form a system of H-type for $n=0,1,\ldots,2^{s+1}$. Hence it follows by complete induction that $\{\chi_n(x)\}$ is an infinite system of H-type.

Let $\{a_v\}$, $v=0,1,\ldots$, be a sequence of real numbers and let $\{\varphi_v(x)\}$, $v=0,1,\ldots$, be an orthonormal system defined in the interval $(0,1)$. Let us denote by $t_n(x)$ the n-th (N,p_n)-mean, with $\{p_n\}\in\overline{M}^a$, $a>-1$, of the series

(1) $$\sum_{v=0}^{\infty} a_v\varphi_v(x).$$

Further let

$$A_m = \Big\{\sum_{n=2^m+1}^{2^{m+1}} a_n^2\Big\}^{1/2} \qquad (m=0,1,\ldots).$$

Now, we can prove the

(1) i.e. E_k is a sum of a finite number of intervals.

(2) $|E|$ denotes the Lebesgue's measure of the set E.

THEOREM. *Let $\{p_n\}\in\overline{M}^{\alpha}$, where $\alpha > -1$. The series* (1) *is $|N, p_n|$-summable almost everywhere for every system $\{\varphi_n(x)\}$ of H-type if and only if*

$$\sum_{m=0}^{\infty}\frac{A_m}{P_{2^m}} < \infty \quad \textit{if} \quad -1<\alpha<0, \tag{2}$$

and

$$\sum_{m=0}^{\infty} A_m < \infty \quad \textit{if} \quad \alpha \geqslant 0. \tag{3}$$

Proof. Sufficiency. Let $-1<\alpha<0$. Writing

$$W_{n,v} = p_{n-v}P_n - p_nP_{n-v} \qquad (v = 0, 1, \ldots, n;\ n = 1, 2, \ldots),$$

we get

$$\sum_{n=2}^{\infty}\int_0^1 |t_n(x)-t_{n-1}(x)|\,dx$$

$$\leqslant \sum_{m=0}^{\infty}\sum_{n=2^m+1}^{2^{m+1}}\sum_{l=0}^{m}\int_0^1\Big|\sum_{v=2^l+1}^{\min(2^{l+1},n)}\frac{W_{n,v}}{P_nP_{n-1}}a_v\varphi_v(x)\Big|\,dx$$

$$\leqslant O(1)\sum_{m=0}^{\infty}\sum_{n=2^m+1}^{2^{m+1}}\sum_{l=0}^{m}\sum_{v=2^l+1}^{\min(2^{l+1},n)}\frac{|W_{n,v}|}{P_nP_{n-1}}|a_v|\int_0^1|\varphi_v(x)|\,dx$$

$$= O(1)\sum_{m=0}^{\infty}\sum_{l=0}^{m}\sum_{v=2^l+1}^{2^{l+1}}|a_v|\int_0^1|\varphi_v(x)|\,dx\sum_{n=\max(2^m+1,v)}^{2^{m+1}}\frac{|W_{n,v}|}{P_nP_{n-1}}$$

$$\leqslant O(1)\sum_{m=0}^{\infty}\sum_{l=0}^{m}A_l\Big\{\sum_{v=2^l+1}^{2^{l+1}}\Big(\int_0^1|\varphi_v(x)|\,dx\Big)^2\Big\}^{1/2}\sum_{n=\max(2^m+1,v)}^{2^{m+1}}\frac{|W_{n,v}|}{P_nP_{n-1}}.$$

Now, we examine two cases: 1) $1<v\leqslant 2^{m-1}$ and 2) $2^{m-1}<v\leqslant n$. In the first case we have $2^l<v\leqslant 2^{l+1}$ $(l = 0, 1, \ldots, m-2)$ so that $2v\leqslant 2^m<n$, whence $n-v>v$. It is easy to verify that

a) $nP_n \nearrow$ for $n>N$ $(\alpha>-1)$ [3],

b) $P_n/n \searrow$ for each natural n $(-1<\alpha<1)$.

[3] The letter N denotes here a sufficiently large natural number.

In view of these properties of the sequence $\{P_n\}$ and by Lemma 1, it follows that

$$\sum_{n=\max(2^m+1,v)}^{2^{m+1}} \frac{|W_{n,v}|}{P_nP_{n-1}} = O(1) \sum_{n=2^m+1}^{2^{m+1}} \frac{vp_np_{n-v}}{P_nP_{n-1}} = O(1) \sum_{n=2^m+1}^{2^{m+1}} \frac{vP_{n-v}}{nP_n(n-v+1)}$$

$$< O(1)2^l \sum_{n=2^m+1}^{2^{m+1}} \frac{P_{2^{m-1}}}{2^{m-1}} \cdot \frac{1}{nP_n} < O(1)2^{l-m}$$

because $n-v > 2^{m-1}$.

In the second case we have $2^{m-1} < v \leqslant n$ so that $0 \leqslant n-v < 3 \cdot 2^{m-1}$, and, by the Raabe criterion, it follows that

$$\sum_{n=\max(2^m+1,v)}^{2^{m+1}} \frac{|W_{n,v}|}{P_nP_{n-1}} < O(1) \frac{1}{P_{2^{m-1}}} \sum_{n=\max(2^m+1,v)}^{2^{m+1}} \frac{P_{n-v}}{n-v+1}$$

$$\leqslant O(1) \frac{1}{P_{2^{m-1}}} \sum_{k=0}^{\infty} \frac{P_k}{k+1} = O(1) \frac{1}{P_{2^{m-1}}}.$$

Basing on these estimations, we write (omitting the argument x)

$$\sum_{n=2}^{\infty} \int_0^1 |t_n - t_{n-1}|\, dx \leqslant O(1) \Bigg[\sum_{m=0}^{\infty} \sum_{l=0}^{m-2} 2^{l-m} A_l \Big\{ \sum_{v=2^m+1}^{2^{m+1}} I_v \Big\}^{1/2} +$$

$$+ \sum_{m=0}^{\infty} \left(\frac{A_{m-1}}{P_{2^{m-1}}} + \frac{A_m}{P_{2^m}} \right) \Big\{ \sum_{v=2^m+1}^{2^{m+1}} I_v \Big\}^{1/2} \Bigg]$$

$$= O(1) \left[\sum_{l=0}^{\infty} 2^l A_l \sum_{m=l}^{\infty} 2^{-m} + \sum_{m=0}^{\infty} \frac{A_m}{P_{2^m}} \right] = O(1) \sum_{m=0}^{\infty} \frac{A_m}{P_{2^m}} < \infty,$$

where I_n $(2^m < n \leqslant 2^{m+1})$ denote the subsets of $(0, 1)$ on which $\varphi_n(x) \neq 0$. Thus we have proved the sufficiency of (2) when $-1 < \alpha < 0$.

Passing to the case of $\alpha = 0$, we notice that $0 < W_{n,v}/P_nP_{n-1} < p_{n-v}/P_{n-1}$. Hence we get in the first case that

$$\sum_{n=\max(2^m+1,v)}^{2^{m+1}} \frac{|W_{n,v}|}{P_nP_{n-1}} < \sum_{n=2^m+1}^{2^{m+1}} \frac{p_{n-v}}{P_{n-1}} < O(1) \frac{1}{P_{2^m}} \sum_{n=2^m+1}^{2^{m+1}} \frac{P_v}{v}$$

$$< O(1) \frac{P_{2^l}}{2^l} \cdot \frac{1}{P_{2^m}} \cdot 2^m < O(1) 2^{m-l}$$

$$(2^l < v \leqslant 2^{l+1},\ l = 0, 1. ,\ldots, m-2).$$

In the other case, we have

$$\sum_{n=\max(2^m+1,v)}^{2^{m+1}} \frac{|W_{n,v}|}{P_nP_{n-1}} < O(1)\frac{1}{P_{2^{m-1}}}\sum_{v=0}^{3\cdot 2^{m-1}} p_v = O(1).$$

In view of the last two estimations, we find that

$$\sum_{n=2}^{\infty}\int_0^1 |t_n - t_{n-1}|\,dx$$

$$< O(1)\Big[\sum_{m=0}^{\infty}\Big\{\sum_{v=2^m+1}^{2^{m+1}} I_v\Big\}^{1/2}\sum_{l=0}^{m-2} 2^{l-m}A_l + \sum_{m=0}^{\infty}(A_{m-1}+A_m)\Big\{\sum_{v=2^m+1}^{2^{m+1}} I_v\Big\}^{1/2}\Big]$$

$$= O(1)\sum_{m=0}^{\infty} A_m < +\infty.$$

Thus we have proved the sufficiency of condition (3) for $a = 0$. Hence, applying Lemma 2, condition (3) follows for $a \geqslant 0$; it suffices to choose a convex or concave sequence $\{q_n\}$ such that $\{q_n\}\in\overline{M}^{\beta}$ with an arbitrary $\beta > 0$, and to apply Lemma 2.

Necessity. Let $\{\chi_n(x)\}$ be a system of H-type, and let $\bar{t}_n(x)$ denote the n-th (N, p_n)-mean of the series $\sum_{n=0}^{\infty} a_n\chi_n(x)$, with $\{p_n\}\in\overline{M}^{a}$, $-1 < a < 0$. Moreover, let $\sum_{n=2}^{\infty}|\bar{t}_n(x) - \bar{t}_{n-1}(x)| < \infty$ almost everywhere in the interval $(0, 1)$.

Let us take $\varepsilon = 3^{-2}2^{-8}C_2^{-2}(a)C_1^2(a)$, where $C_1(a)$ and $C_2(a)$ denote the constants mentioned in Lemma 1. In view of Egoroff's theorem there exist a measurable set $E\subset(0,1)$ of measure $|E|\geqslant 1-\varepsilon$ and a constant M such that

$$\sum_{n=2}^{\infty}|\bar{t}_n(x) - \bar{t}_{n-1}(x)| < M \qquad (x\in E).$$

Hence it follows that

$$\sum_{n=2}^{\infty}\int_E |\bar{t}_n(x) - \bar{t}_{n-1}(x)|\,dx \leqslant M|E|. \tag{4}$$

Let m be an arbitrary natural number and let $2^m < n \leqslant 2^{m+1}$. We take

$$R_l(x) = \sum_{v=2^l+1}^{2^{l+1}} \frac{W_{n,v}}{P_nP_{n-1}} a_v\chi_v(x) \qquad (l = 0, 1, \ldots, m-1),$$

$$R_m(x) = \sum_{v=2^m+1}^{n} \frac{W_{n,v}}{P_nP_{n-1}} a_v\chi_v(x).$$

Applying Lemma 3 to the functions $R_l(x)$ $(l = 0, 1, \ldots, m)$ with $N = m+1$ and $k = 1$, and taking into account the fact that $W_{n,v}$ $(v = 0, 1, \ldots, n;\ n = 1, 2, \ldots)$ is of constant sign, we find that

$$\left|\sum_{v=0}^{n}\frac{W_{n,v}}{P_nP_{n-1}}a_v\chi_v(x)\right| = \left|\sum_{l=1}^{m}R_l(x)\right| \geqslant |R_m(x)| = \left|\sum_{v=2^m+1}^{n}\frac{|W_{n,v}|}{P_nP_{n-1}}a_v\chi_v(x)\right|.$$

Denoting by E_1 the corresponding set defined in Lemma 3, we can write

$$\sum_{n=2}^{\infty}\int_{E}|\bar{t}_n(x)-\bar{t}_{n-1}(x)|\,dx$$

$$\geqslant \sum_{m=0}^{\infty}\sum_{n=2^m+1}^{2^{m+1}}\int_{E_1\cap E}\left|\sum_{v=2^m+1}^{n}\frac{W_{n,v}}{P_nP_{n-1}}a_v\chi_v(x)\,dx\right|$$

$$\geqslant \sum_{m=0}^{\infty}\sum_{n=2^m+1}^{2^{m+1}}\Big(\int_{E_1}-\int_{E_1-E_1\cap E}\Big)\left|\sum_{v=2^m+1}^{n}\frac{W_{n,v}}{P_nP_{n-1}}a_v\chi_v(x)\right|dx$$

$$\geqslant \sum_{m=0}^{\infty}\sum_{n=2^m+1}^{2^{m+1}}\Bigg[\int_{E_1}\left|\sum_{v=2^m+1}^{n}\frac{W_{n,v}}{P_nP_{n-1}}a_v\chi_v(x)\right|dx-$$

$$-\sqrt{\varepsilon}\left\{\int_0^1\Big(\sum_{v=2^m+1}^{n}\frac{W_{n,v}}{P_nP_{n-1}}a_v\chi_v(x)\Big)^2dx\right\}^{1/2}\Bigg]$$

$$\geqslant \sum_{m=0}^{\infty}\sum_{n=2^m+1}^{2^{m+1}}\Big(\sum_{v=2^m+1}^{n}\frac{|W_{n,v}|}{P_nP_{n-1}}a_v\int_{E_1}\chi_v(x)\,dx-\sqrt{\varepsilon}\,C_2(a)\,p_0\,|a_n|\frac{A_m}{P_{2^m}}\Big)$$

$$\geqslant \sum_{m=0}^{\infty}\Big(2^{-2}C_1(a)\sum_{n=2^m+1}^{2^{m+1}}\sum_{v=2^m+1}^{n}\frac{vp_np_{n-v}\cdot a_v^2}{P_nP_{n-1}\cdot A_m}-2^{-4}3^{-1}C_1(a)\,|a_n|\,p_0\frac{A_m}{P_{2^m}}\Big),$$

where $a_n = (n+1)p_n/P_n$. Since

$$\sum_{n=2^m+1}^{2^{m+1}}\sum_{v=2^m+1}^{n}va_v^2\frac{p_np_{n-v}}{P_nP_{n-1}} = \sum_{n=2^m+1}^{2^{m+1}}va_v^2\sum_{n=v}^{2^{m+1}}\frac{|a_n|\,|p_{n-v}|}{nP_n}$$

$$> \frac{2^m|a|}{2\cdot 2^{m+1}P_{2^{m+1}}}\sum_{n=2^m+1}^{2^{m+1}}a_v^2\sum_{n=v}^{2^{m+1}}|p_{n-v}| > \frac{2^{-2}|a|\,A_m^2}{P_{2^m}}\,p_0$$

for $m > m_0$, where m_0 denotes a sufficiently large natural number, we can finally write

$$\sum_{n=2}^{\infty} \int_E |\bar{t}_n(x) - \bar{t}_{n-1}(x)|\, dx > \sum_{m=m_0}^{\infty} (2^{-4} - 2^{-5})\, |a|\, p_0 C_1(a) \frac{A_m}{P_{2^m}}$$

$$= \frac{p_0 |a| C_1(a)}{2^5} \sum_{m=m_0}^{\infty} \frac{A_m}{P_{2^m}}.$$

Thus by (4)

$$\sum_{m=0}^{\infty} \frac{A_m}{P_{2^m}} < \infty,$$

which proves the necessity of condition (2) for $-1 < a < 0$. Now, we pass to the case of $a > \frac{1}{2}$. Here the situation is similar to that in the last case. Suppose that the functions $R_l(x)$ $(l = 0, 1, \ldots, m)$ have the same meaning as in the former estimations. Applying Lemma 3, with $N = m$ and $k = 2$, and denoting by E_2 the corresponding set, we find after many abbreviations that

$$M|E| \sum_{n=2}^{\infty} \int_E |\bar{t}_n(x) - \bar{t}_{n-1}(x)|\, dx$$

$$> \sum_{m=0}^{\infty} \sum_{n=2^m+1}^{2^{m+1}} \int_E \left| \sum_{v=0}^{n} \frac{W_{n,v}}{P_n P_{n-1}} a_v \chi_v(x) \right| dx$$

$$> \sum_{m=0}^{\infty} \sum_{n=2^m+1}^{2^{m+1}} \int_{E_2 \cap E} \left| \sum_{n=2^{m-2}+1}^{2^{m-1}} \frac{W_{n,v}}{P_n P_{n-1}} a_v \chi_v(x) \right| dx$$

$$> \frac{p_0 |a| C_1(a)}{2^5} \sum_{m=m_0}^{\infty} A_{m-2} = O(1) \sum_{m=0}^{\infty} A_m.$$

Hence it follows that $\sum_{m=0}^{\infty} A_m < +\infty$.

This proves the necessity of condition (3) in the case of $a > \frac{1}{2}$. Similarly, the necessity of condition (3) follows also for $0 \leqslant a \leqslant \frac{1}{2}$. Indeed, let us suppose that the series (1) is $|N, p_n|$-summable $(0 \leqslant a \leqslant \frac{1}{2})$ almost everywhere for all systems of H-type. By Lemma 2, it is also $|N, r_n|$-summable with $\{r_n\} \in \overline{M}^{a+\beta}$ and $a + \beta > \frac{1}{2}$, where $r_n = \sum_{k=0}^{n} p_k q_{n-k}$, and

$\{q_n\}$ satisfies the assumptions of Lemma 2. Thus $\sum_{m=0}^{\infty} A_m < \infty$, and we conclude that (3) is the necessary condition for $0 \leqslant a \leqslant \frac{1}{2}$. This fact completes the proof of the Theorem.

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Reçu par la Rédaction le 10. 2. 1966